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Heterogeneity and Labor Demand in an Equilibrium Search Model

Norface conference
06./07.12.2012, Nürnberg

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Motivation

Aim of the paper:
Development of a *theoretical* framework to discuss employment and wage effects of minimum wages in *frictional* labor markets with heterogenous labour.

→ „Two sides of the same coin"-interpretation (Krugman, 1994; Blank, 1997); BUT: frictions neglected.

Motivation: Two Strands of literature...

- Unemployment...
  - → Structural unemployment: In neo-classical labor demand models because wages are too high
  - → Frictional unemployment: In equilibrium search theory because of imperfect information
- Wage dispersion...
  - → Between skill groups: explained by supply and demand factors in neo-classical models
  - → Within skill groups: explained by search frictions in search equilibrium models
- Effects of minimum wages on wages and employment...
  - → Neo-classical models: Compress wages (spike) and increase unemployment
  - → Equilibirum search models: Compress wages (?), redistribute rents and do not affect unemployment,

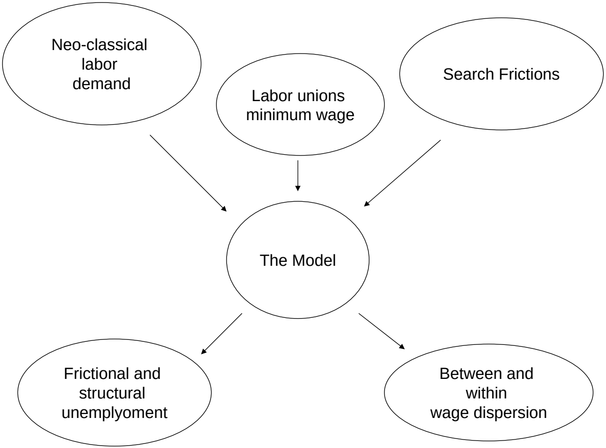
Aim of the Paper: Bring together both strands of the literature.

Literature

Introducing heterogenous labour and production functions in frictional labour market models - literature

- Postel-Vinay/Robin (2002), IER/Ecta: Value of unemployment proportional to own productivity, firms always pay reservation wages (perfect information). (similar Bagger/Fontaine/Postel-Vinay/Robin, 2011)
- Burdett/Carrillo-Tudela/Coles, 2011, IER; Carillo-Tudela, 2012, accumulation of human capital in an eq. search model à la Postel-Vinay/Robin (2002)
- Ridder/Van den Berg (1997), non-linear production (one type of labour); proove masspoint can exist (upper bound).
- Holzner/Launov (2010), EER : Cobb-Douglas production and supermodularity. Supermodularity implies that a firm occupies exactly the same position in each skill group wage distribution.
- Teulings/Gautier/van Vuuren (2006): Sorting literature. (similar: Menzio/Telyukova/Visschers, 2012)

Theory: Combining Equilibrium Search and Production Functions for heterogenous labour



The model – individuals income

- Structure of the model,
- Drawing from the wage offer distribution means drawing from the joint distribution (w_1, w_2). Individuals base their decision on the marginals, only.
- Employment dynamics from individual optimization under stationarity (no mass point at w_1),

$$(\lambda_1 U_1 + \lambda_{1,L} L_1(w_1))(1 - H_1(w_1)) = \delta_1(N_1 - U_1 - L_1(w_1))$$

- "FOC"(under differentiability)

$$\frac{l_1'(w_1)}{l_1(w_1)} = \frac{2\lambda_{1,L} h_1(w_1)}{(\lambda_{1,L}(1 - H_1(w_1)) + \delta_1)} \qquad (1)$$

The model – firms profits

- Firms profit at expected employment:

$$\Pi(w_1, w_2) = y(l_1(w_1), l_2(w_2)) - w_1 l_1(w_1) - w_2 l_2(w_2)$$

- FOC:

$$\frac{l_1'(w_1)}{l_1(w_1)} = \frac{1}{\left(\frac{\partial y()}{\partial l_1} - w_1\right)} = \frac{1}{y'(l_1) - w_1} \tag{2}$$

- Result: Firms cover the same position in both wage distributions (under supermodularity, see Holzner/Launov, 2010)

Solution strategy

- Equating eq. (2) and (1), imposing Cobb-Douglas production and using the same position result yields

$$h_1(w_1) = \frac{-H_1(w_1) + \frac{\lambda_{1,L}+\delta_1}{\lambda_{1,L}}}{2\left(\alpha_1 A \left(\frac{(N_1-U_1)\delta_1\left(\lambda_{1,L}+\delta_1\right)}{\left(\lambda_{1,L}(1-H_1(w_1))+\delta_1\right)^2}\right)^{\alpha_1-1}\left(\frac{(N_2-U_2)\delta_2\left(\lambda_{2,L}+\delta_2\right)}{\left(\lambda_{2,L}(1-H_1(w_1))+\delta_2\right)^2}\right)^{\alpha_2} - w_1\right)} \quad (3)$$

- Solution $h_1^*(w_1)$ that solves eq. (3)?
- No solution strategy available that solves this (non-autonomous, non-linear ordinary) differential equation analytically, in general!

Solving non-linear, non-autonomous differential equations

- Analytical Methods: Special cases for which an analytical solution can be found
- Use numerical methods in cases where no analytical solution can be found

Analytical solution for special case

Assume: $\alpha_1 + \alpha_2 = 1$ (CRS), $\lambda_{1,L} = \lambda_{2,L} = \lambda$ and $\delta_1 = \delta_2 = \delta$. ($\frac{\delta_1}{\lambda_{1,L}} = \frac{\delta_2}{\lambda_{2,L}}$ is sufficient for the results)

- The problem can be written in the following way:

$$h_1(w_1) = \frac{-H_1(w_1) + \frac{\lambda_L + \delta}{\lambda_L}}{2\left(\alpha_1 A\left(\frac{\delta(\lambda_L+\delta)}{(\lambda_L(1-H_1(w_1))+\delta)^2}\right)^{\alpha_1 - 1}\left(\frac{\delta(\lambda_L+\delta)}{(\lambda_L(1-H_1(w_1))+\delta)^2}\right)^{1-\alpha_1}(N_1-U_1)^{\alpha_1-1}(N_2-U_2))^{\alpha_2} - w_1\right)}$$

- Solve by separation of variables and obtain:

$$H_1^*(w_1) = \frac{\delta + \lambda_L}{\lambda_L}\left(1 - \sqrt[2]{\frac{(\alpha_1 A(N_1-U_1)^{\alpha_1-1}(N_2-U_2)^{1-\alpha_1} - w_1)}{(\alpha_1 A(N_1-U_1)^{\alpha_1-1}(N_2-U_2)^{1-\alpha_1} - w_1^R)}}\right) \quad (4)$$

- Moments, reservation wage and upper bound of the wage distribution can be calculated.
- Marginal productivities are identical over the support of the distributions.

Simulation for more general results

- Use German labor market data for „realistic value" of search frictions (Calibration) and start from model that is analytically solvable,
- Use Mathematicas Routine NDSolve to obtain a function $H_1^*(w_1)$ which solves the system.
- Results: Analytical case, param. deviations, decreasing returns,
- Summary of results:
  - Always: Continuous part for wage density.
  - Often: Obtain mass points
    - Low Frictions: Obtain mass points for small parameter deviations.
    - High frictions: Persistence of solution, no mass points.
  - Can obtain partly decreasing functions of $h_1^*(w_1)$.

Summary: Analytical solutions and simulation

Properties of the solution

- (A part of) the wage distribution is below marginal productivity.
- Wage dispersion between and within groups of identical individuals.
- Wage offer and wage densities are (mostly) increasing.
- Introducing a minimum wage for one of the skill groups...
  - ...does neither affect employment of this skill group nor of the other skill group if its below marginal productivity.
  - ... that is above marginal productivity does not give clear results. Still optimal to employ some members of this skill group...

$\rightarrow$ Model extension necessary

Model extension: Introducing contact costs

Endogenize λ_1 by introducing contact cost c (Mortensen, 2003)

- Idea: Firms contact individuals at cost c, a priori no distinction possible btw. employees and unemployed: $\lambda_{i,L} = \lambda_i$. Here: $\lambda_1 = \lambda_{1,L}$ determined by aggregate search effort of firms (λ_2 exogenous).
- Acceptance probability γ upon contact, given by:

$$\gamma(w_1) = \frac{U_1 + (N_1 - U_1)(G_1(w_1))}{N_1}$$

- Profit function with contact costs:

$$\Pi(w_1, w_2) = y(l_1(w_1), l_2(w_2)) - w_1 l_1(w_1) - w_2 l_2(w_2) - \frac{(\lambda_1(1 - H_1(w_1)) + \delta_1) l_1(w_1)}{\gamma(w_1)} c_1 \tag{5}$$

- Rewrite contact cost:

$$\frac{(\lambda_{1,L}(1 - H_1(w_1)) + \delta_1) l_1(w_1)}{\gamma(w_1)} c_1 = \lambda_1 N_1 c_1$$

which is independent of the wage. (FOC unchanged)

Determining λ_1

- Equilibrium requires: Profit per (additional) worker contact equal cost
- Profit per (additional) worker contact (skill group 1):

$$P(w_1) = \gamma(w_1)\frac{1}{r+\delta_1+\lambda_1(1-H_1(w_1))}\left(\frac{\partial y}{\partial l_1}-w_1\right)=c_1$$

which holds at $w_1 = z_1$ and thus:

$$G(z_1)=\frac{\delta_1}{\delta_1+\lambda_1}\frac{1}{r+\delta_1+\lambda_1}\left(\alpha A\left(\frac{(N_1-U_1)\delta_1}{\lambda_1+\delta_1}\right)^{\alpha-1}\left(\frac{(N_2-U_2)\delta_2}{\lambda_2+\delta_2}\right)^{1-\alpha}-z_1\right)-c_1=0 \qquad (6)$$

This uniquely defines λ_1!

Frictional and structural unemployment

Assume no mass point at z_1

- Negative effect of an increasing minimum wage on the job offer rate:

$$\frac{d\lambda_1}{dz_1} = -\frac{\frac{\partial G}{\partial z_1}}{\frac{\partial G}{\partial \lambda_1}} < 0 \tag{7}$$

- Increasing the minimum wage z_1 decreases λ_1 and thus employment.
- Minimum wages cause structural unemployment (not for skill group 2)

Within group and between group wage dispersion

Assume in addition: $\lambda_i = \lambda_{i,L} = \lambda$ and $\delta_i = \delta$

- Effect of z_1 on $E_{G_1}(w_1)$. Expected positive.

$$\frac{\partial}{\partial z_1} E_{G_1}(w_1) = \frac{\delta}{\delta + \lambda} \left(1 + \frac{(y_1' - z_1)}{\delta + \lambda} \frac{\partial \lambda}{\partial z_1} \right)$$

  ...depends on the parameters.
- Effect of z_1 on $VAR_{G_1}(w_1 - z_1)$. Expected negative! ...depends on the parameters.
- Effect of z_1 on moments of $G_2(w_2)$ expected

Conclusion

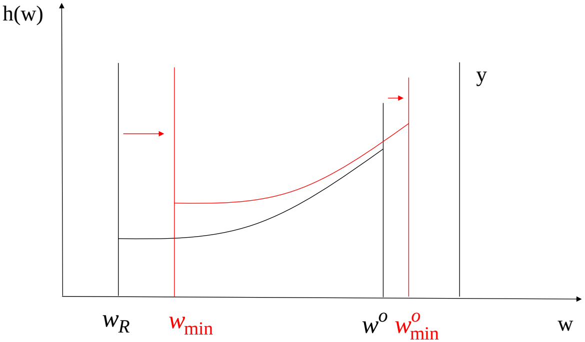
- Have constructed and solved an equilibrium search model with two skill groups, combined with a CD production function.
- Have introduced labor demand effects by endogenizing λ.
- Model involves frictional unemployment caused by search frictions and structural unemployment caused by a minimum wage.
- Model yields interesting results concerning the effect of minimum wages on expected wages for both skill groups and for changes in wage dispersion within and between groups.

Open questions

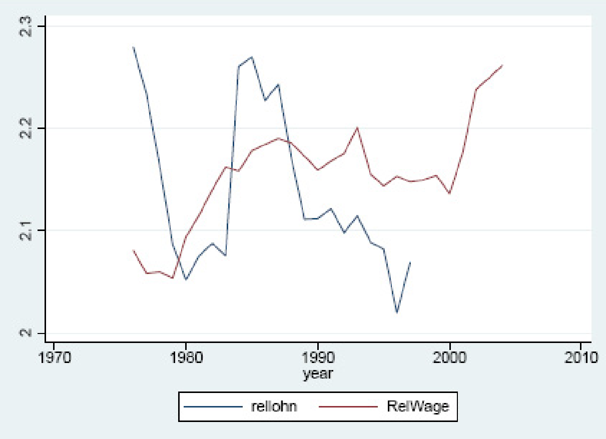
- Model without contact costs:
  - Is the equal position hypothesis as general as we pretend it to be?
  - Is the analytical model solution a too particular case? Can we generalize the results? To more general production functions?
  - Why is the model so sensitive wrt variations of the parameters? Are the results relevant?
- Model with contact costs:
  - Generalization: Are the derived results more than just an example?
  - Can the model be used to structurally estimate the effect of a (sectoral) minimum wage on labour market outcomes.
- Model might be useful for understanding effects of minimum wages on wages and employment! Also useful for understanding effects of SBTC?

Thank you for your attention!

Wage offer distribution and a binding minimum wage

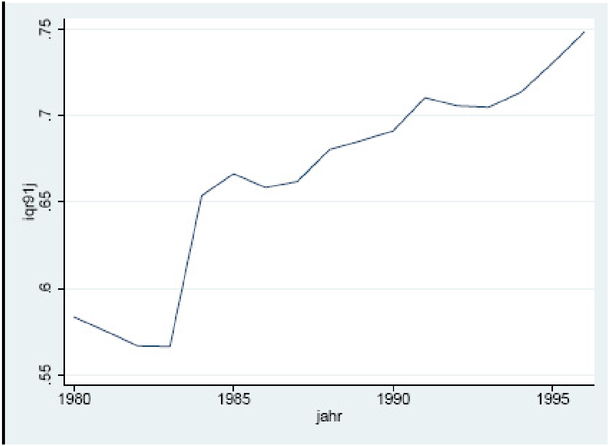


WI between high- and low-skilled, Germany, back, p.3

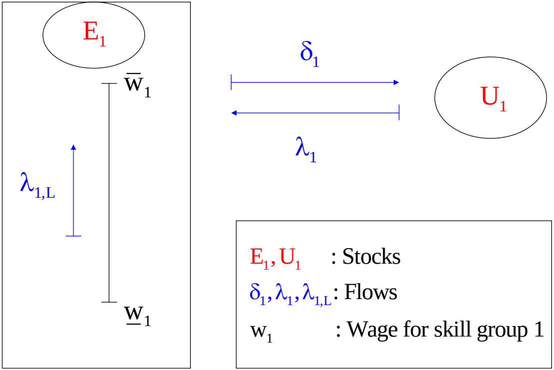


Wage inequality between high- and low-skilled, Germany, IABS97 and German statistical office

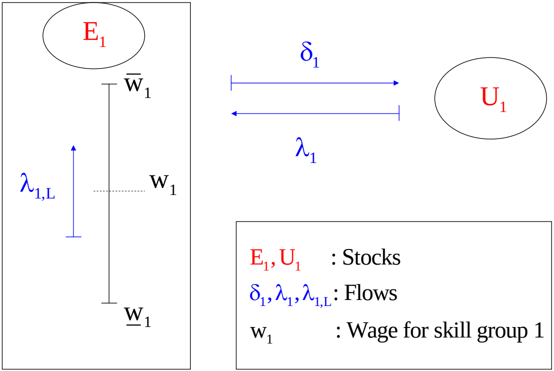
Residual WI, 90P-10P, men, Germany, back, p.3



Residual wage inequality, 90P-10P, men, Germany 1980-1996, IABS, conditional on education, age and sex


E_1
$\bar{w}_1$
$\lambda_{1,L}$
$\underline{w}_1$
δ_1
λ_1
U_1
E_1, U_1 : Stocks
$\delta_1, \lambda_1, \lambda_{1,L}$: Flows
w_1 : Wage for skill group 1

Flows, skill group 1, back, p.6



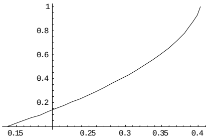
Simulation parameters, back, p.11

Table 2: Simulation parameters

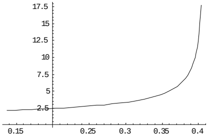
Parameter	Low frictions	High frictions
δ_1	0.004	0.008
λ_1	0.04	0.02
$\lambda_{1,L}$	0.03	0.015
N_1	1	3
α_1	0.6	0.3
δ_2	0.008	0.016
λ_2	0.12	0.08
$\lambda_{2,L}$	0.06	0.03
N_2	1	1
α_2	0.4	0.7
A	1	1
r	0.02	0.04
z_1	0.1	0.03
z_2	0.1	0.2

Analytical case:

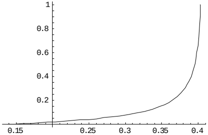
H1(w1)



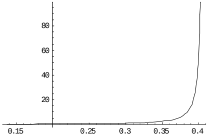
h1(w1)



G1(w1)



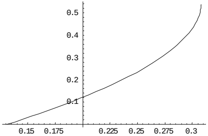
g1(w1)



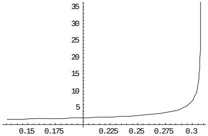
Simulation: Decreasing returns to scale, back, p.11

Decreasing returns to scale: Mass points, alpha1=0.2

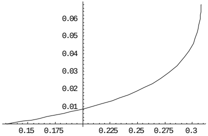
H1(w1)



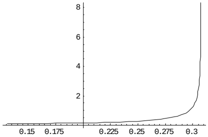
h1(w1)



G1(w1)

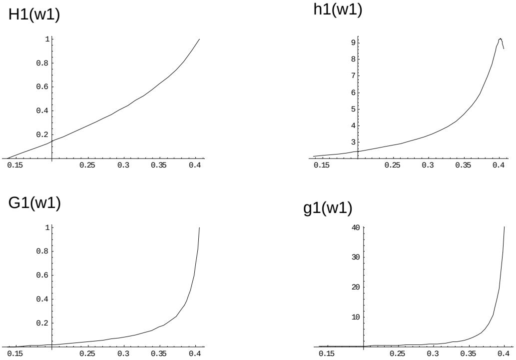


g1(w1)



Simulation: Decreasing wage offer density, back, p.11

Slight parameter deviations: Decreasing wage offer density



Thank you for your attention!

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