Workers and Firms Sorting into Temporary Jobs*

Fabio Berton
University of Eastern Piedmont and LABORatorio R. Revelli

Pietro Garibaldi
University of Turin and Collegio Carlo Alberto

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Abstract

The liberalization of fixed term contracts in Europe has led to a two tier regime, with a growing share of jobs covered by temporary contracts. The paper proposes a matching model with direct search in which temporary and permanent jobs coexist in a long run equilibrium. When temporary contracts are allowed, firms are willing to open permanent jobs in as much as their job filling rate is faster than that of temporary jobs. From the labour demand standpoint, a simple *trade-off* emerges between an ex-ante job filling rate and ex-post flexible dismissal rate. The model thus features a natural sorting of firms and workers into permanent and temporary jobs. It is also consistent with the observation that workers hired on a permanent contract receive more training. Empirically, we test with Italian longitudinal data whether non employment spells that lead to a temporary job are shorter on average. We find that, other things being equal, the transition intensity of exit towards temporary jobs is higher than to permanent jobs. The other empirical implications, and notably the effects of training, are coherent with the existing literature.

- Key Words: Matching Models, Temporary Jobs

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1 Introduction

The liberalization of temporary contracts, or fixed term contracts as are often defined in the policy debate, is the main labour market reform in continental Europe. The liberalization applies only to new hires, so that only new jobs and new vacancies can *potentially* be advertised and filled with temporary contracts. Existing jobs, covered by open ended contracts, are not directly affected by the reform. As a result, a two tier regime has emerged in many continental European markets, with a growing share of temporary contract, which reached 13.6% in 2005 [European Commission 2005]. As the stock of open ended jobs dies out by natural turnover, many observers and policy analysts wonder whether the share of temporary contracts will eventually absorb the entire labor market. This paper shows that the latter implication is not likely, and that permanent and temporary workers are likely to coexist in the long run, even with homogeneous labor from the labor demand standpoint.

In the existing literature, the long run implication of a labor market with both temporary and permanent contracts are not fully understood. In a pure labor demand setting with risk neutral-homogeneous workers and without market frictions, temporary jobs should indeed take over the entire labour market. Boeri and Garibaldi [2007] study theoretically and empirically the transition from a rigid system with only permanent contracts to a dual system with temporary and permanent contracts. In the aftermath of the liberalization, no vacancies covered by permanent contracts are posted, and the stock of temporary contracts absorb the entire workforce. Similar implications are held by various papers [Cahuc and Postel Vinay 2002; Blanchard and Landier 2002][1] and ad hoc assumptions ensure that temporary and permanent contract coexist in equilibrium.

This paper studies firms and workers' sorting into permanent and temporary contracts in an imperfect labor market. Specifically, it studies vacancy posting in permanent and temporary jobs in a world with matching frictions and direct search. From the labor demand standpoint, a filled job with a temporary and flexible contract is more profitable to a firm, since it allows the firm to easily adjust labour in the face of adverse productivity shocks. Free entry in each submarket implies that in equilibrium jobs advertised with permanent contracts display a larger job filling rate. From a labour demand standpoint, a simple *trade-off* emerges between an ex-ante slower job filling rate and ex-post more flexible dismissal rate. In other words, firms that post jobs with temporary contracts face longer job filling rate. This mechanism is akin to wage posting and to the competitive search equilibrium initially proposed by Moen [1997].

From the labour supply stand point, a similar mechanism emerges. For a given wage within the bargaining set, in the spirit of Hall [2005], risk neutral workers with heterogeneous and unobservable reservation utility, prefer to search in the permanent submarket. Yet, in as much as job search in the submarket for temporary

[1] In Cahuc and Postel Vinay [2002] temporary and permanent contracts coexist in light of a random and exogenous state permission to fill jobs with temporary contracts. In Blanchard and Landier [2001] all jobs start with a temporary contract, and only a fraction is endognouesly converted into a permanent job. Garibaldi and Violante [2005] have similar implications

workers leads to larger job filling rate, a simple *labor supply trade-off* emerges between an ex-ante lower job finding rate and an ex-post larger retention rate[2]. As a result the model features a natural sorting of firms and workers into permanent and temporary jobs.

The simple theory has several implications. First, the coexistence of temporary and permanent contract implies that in equilibrium temporary jobs lead to faster job finding rate for workers. This is true even when workers can graduate to a permanent position via a temporary job. Second, the steady state of the model displays both temporary and permanent jobs, with an equilibrium share of temporary jobs that crucially depends on the average duration of temporary contracts and the structure of productivity shocks. Third, the liberalization of temporary contracts does not crowd out permanent contracts, and the labour market moves smoothly toward a long run dual system. Fourth, when firms have the option to undertake costly training in the aftermath of adverse productivity shocks, the theory clearly implies that workers covered with permanent contracts are more likely to be trained.

While the existing empirical literature on temporary/permanent jobs is large, the basic implicit mechanism proposed by the model has not been directly tested. Empirically, we use Italian longitudinal data to test whether non employment spells that lead to a temporary job are shorter on average. We run a competing risks model on a sample of prime age male workers attached to the labour market in 1998 and find that, other things being equal, the transition intensity of exit towards temporary jobs is higher than to permanent. We also review the rest of the empirical implications, and notably the effects of training, and we find it fully coherent with the existing literature

The paper proceeds as follows. Section 2 highlights the structure of the model and the basic equations. Section 3 defines and solves the equilibrium. Section 4 studies analytically the transition toward a dual regime and presents a simple set of simulations. Section 5 introduces the option to train workers in the aftermath of adverse shocks. Section 6 studies the model with on the job search. Section 7 presents the empirical analysis. Section 8 discusses other implications of our theory vis-à-vis the existing empirical evidence. Section 9 concludes.

2 The matching framework

The labour market consists of a mass one of risk neutral workers. Workers are fully attached to the labor market and if they are out of work, they actively search for a job. Employed workers are subject to natural turnover and separate from their existing job with a Poisson process with arrival rate equal to s.

Workers differ in their idiosyncratic income from non employed. The outside flow utility is indicated with z, and we assume that z is time invariant and not observable to the firms. z is drawn from a continuous

[2] A similar implication, at least from the labor supply standpoint, emerges in the quantitative general equilirbium model proposed by Alonso-Borrego et al. [2005]. The free entry condition in both markets, a key feature of the mechanism of this paper, is not modeled by Alonso-Borrego et al.

cumulative distribution $F(z)$ with upper support z^u. Since z is not observable, workers are identical vis-à-vis the firms.

Firms produce with a constant returns to scale technology with labor productivity equal to y_h. Each job has an instantaneous probability λ of experiencing a (permanent) adverse shock. Conditional on an adverse shock, the productivity falls to $y_l < y_h$. We further assume that the wage paid is strictly larger than z^u so that the labour market is viable for each worker.

Two types of contracts exist in the economy. Temporary contracts and permanent contracts. Temporary contracts can be broken by the firm at will. Firm initiated separation is not possible with permanent contracts[3]. Firms that hire workers on permanent contracts must rely on workers' natural turnover for downsizing. Firms create jobs by posting costly vacancies, and firms can freely decide to open either temporary or permanent jobs. Keeping open a vacancy, either temporary or permanent, involves a flow cost equal to c. For simplicity, we assume that the vacancy cost is identical for both contracts.

Temporary and permanent contracts are offered in different *submarkets*. In each submarket, the meeting of unemployed workers and vacant firms is described by a well defined matching function m with constant returns to scale. Submarkets are indexed by $i \in [p,t]$ where p stands for *permanent* and t for *temporary*. Unemployed workers can freely move across submarkets but can not search simultaneously across submarkets. In this respect, search is directed toward a specific submarket (this hypothesis will be relaxed in section 6). Unemployed workers searching for a permanent job enjoy a fixed exogenous benefit $b > 0$. b is not enjoyed when the worker searches in the temporary submarket. In real life labor markets, unemployed income often requires a specific on the job tenure, and our assumption is fully consistent with this fact.

There are matching frictions in each submarket. We let $m(u_i, v_i)$ be the flow of new matches, where u_i denotes the measure of unemployed workers in submarket i searching for the measure v_i of vacancies; following standard assumptions, we assume that m is concave and homogeneous of degree one in (u_i, v_i) with continuous derivatives. Now define $h_i = m(u_i, v_i)/u_i = m(1, \theta_i) = h(\theta_i)$ as the transition rate from unemployment to employment for an unemployed worker in submarket i and $q_i = m(u_i, v_i)/v_i = q(\theta_i)$ as the arrival rate of workers for a vacancy in submarket i. $\theta_i = v_i/u_i$ is the submarket specific labour market tightness. The matching function m satisfies the following conditions:

$$\begin{aligned}
\lim_{\theta_i \to 0} h(\theta_i) &= \lim_{\theta_i \to \infty} q(\theta_i) = 0 \qquad i = p, t \\
\lim_{\theta_i \to \infty} h(\theta_i) &= \lim_{\theta_i \to 0} q(\theta_i) = \infty \qquad i = p, t
\end{aligned}$$

Upon the meeting of an unemployed workers and a vacant firm, each match signs a long term contract that fix a wage for the entire employment relationship without ex-post renegotiation. In the spirit of Hall [2005], any wage within the parties bargaining set, at the time of job creation, can be supported as an equilibrium. To make the problem interesting, we restrict our attention to wages such that $y_h > w_p > y_l$

[3] The interpretation of dismissal at will in the case of temporary workers is twofold: either firms are allowed to fire whenever the shock occurs, or they're able to set contracts whose duration is exactly $1/(s+\lambda)$

and $y_h > w_t > y_l$. This will ensure that, conditional on the realization of the adverse shock λ, permanent contracts involve a loss to the firm. Further, we will focus on a constant wage across submarket, such that $w_p = w_t = w$.

The equilibrium of the model is characterized by free entry of firms in each submarket, and workers' sorting condition across submarkets.

2.1 Value Functions and Job Creation in the Permanent Market

Let $U_p(z)$ and $E_p(z)$ denote, respectively, the expected discounted income for an unemployed worker and for an employed one in the permanent market. The Bellman equations are:

$$\begin{aligned} rU_p(z) &= z + b + h(\theta_p)[E_p(z) - U_p(z)] && (1) \\ rE_p(z) &= w + s[U_p(z) - E_p(z)] && (2) \end{aligned}$$

where r is the pure discount rate, z is the workers' specific outside option and b is the unemployment benefit. Let J_p^h and J_p^l denote, respectively, the present discounted value of a permanent job when productivity is high (y_h) or low (y_l); their formal expression read

$$\begin{aligned} rJ_p^h &= y_h - w + \lambda[J_p^l - J_p^h] + s[V_p - J_p^h] \\ rJ_p^l &= y_l - w + s[V_p - J_p^l] \end{aligned}$$

When productivity is high, the firm enjoys an operational profit equal to $y_h - w$. The worker leaves at rate s and the firm gets the expected value of a vacancy formally indicated with V_p. Conditional on a productivity shock λ, the firm has no margin of adjustment and experiences a capital loss equal to the difference between the value of a permanent job in high state and a value in bad state $J_p^l - J_p^h$. In the low state, the firm runs an operational loss $y_l - w$ as long as the worker separates at rate s. The asset equation of a vacancy reads

$$rV_p = -c + q(\theta_p)[J_p^h - V_p]$$

Assuming free entry in the permanent market, $V_p = 0$, we have that

$$c = q(\theta_p)J_p^h \tag{3}$$

The previous condition is one of the key equations of the model. It shows that the flow cost of vacancy posting is equal to expected benefit, where the latter is described as the product of the job filling rate into permanent contract time the value of a filled job.

Finally note that the value of a filled job can be written as

$$\begin{aligned} J_p^h &= \frac{y_h - w}{r + s + \lambda} + \frac{\lambda(y_l - w)}{(r+s)(r+s+\lambda)} && (4) \\ J_p^l &= \frac{y_l - w}{r+s} < 0 \end{aligned}$$

The latter expression represents the cost associated to having a permanent contract in case of adverse shock.

2.2 Value Functions and Job Creation in the Temporary Market

Workers employed with a temporary contract are dismissed conditional on the arrival rate λ, so that the value of employment reads

$$rE_t(z) = w + (s+\lambda)[U_t(z) - E_t(z)] \tag{5}$$

The value of unemployment depends on the specific outside income and faces a transition probability $h(\theta_t)$

$$rU_t(z) = z + h(\theta_t)[E_t(z) - U_t(z)] \tag{6}$$

Firms in temporary market are free to dismiss workers conditional on the adverse productivity shock; the value of a filled temporary job and of a temporary vacancy read

$$\begin{aligned} rJ_t^h &= y_h - w + (s+\lambda)[V_t - J_t^h] \\ rV_t &= -c + q(\theta_t)[J_t^h - V_t] \end{aligned}$$

Assuming free entry also in the temporary market, $V_t = 0$, we have that

$$c = q(\theta_t)J_t^h \tag{7}$$

Similarly to the condition above, equation (7) says that the flow cost of vacancy in the temporary market is equal to expected benefit, where the latter is described as product of the job filling rate into temporary contract time the value of a filled job.

Before turning to the equilibrium definition, we derive the second key condition of our analysis. Using the free entry condition into the temporary, one can easily show that a filled temporary job has larger value than a permanent job

$$J_{t,h} = \frac{y_h - w}{r+s+\lambda} > J_{p,h}$$

We are now in a position to establish a key result of our model. The expected value of vacancy depends on the job filling rate and on the value of a filled job. A labour market with both temporary and labour market is such that

$$q(\theta_t)J_t^h = q(\theta_p)J_p^h$$

where we have just proved that $J_t^h > J_p^h$. This result tells that the coexistence of temporary and permanent contract implies that

$$q(\theta_t) < q(\theta_p)$$

Once the job is filled, the firms prefer a flexible contract. They are thus willing to offer both temporary and permanent contract if the job filling rate for permanent contracts is larger than the job filling rate for temporary contracts. Conversely, this result suggests that the job finding rate of a temporary contract is larger, so that

$$h(\theta_t) > h(\theta_p)$$

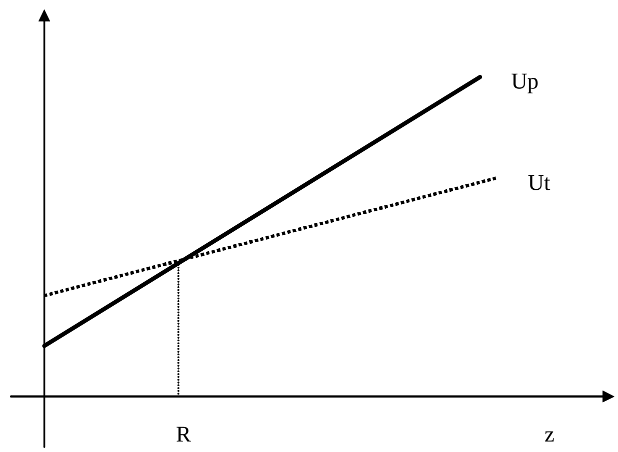

Figure 1: Workers' endogenous sorting

The previous result is very important for the results of the next section, where we discuss the workers' sorting condition between the two submarkets.

2.3 Workers' Sorting

Workers take as given the job finding rate[4] and optimally decide in which submarket to search for a job. Since workers can freely move across submarkets, the optimal allocation will be

$$U(z) = Max[U_p(z), U_t(z)]$$

where the expressions for $U_p(z)$ and $U_t(z)$ are obtained combinig (2) with (1) and (5) with (6)

$$rU_p(z) = \frac{(z+b)(r+s) + h(\theta_p)w}{r+s+h(\theta_p)} \tag{8}$$

$$rU_t(z) = \frac{z(r+s+\lambda) + h(\theta_t)w}{r+s+\lambda+h(\theta_t)} \tag{9}$$

The values of unemployment, for given job finding rates, are monotonically increasing in z. In what follows, we look for a reservation value of R such that the marginal worker (the one with idiosyncratic outside option $z = R$) is indifferent between searching for a temporary or a permanent job. If such R exist, workers endogenously sort between the two markets. Note that workers with low z place a larger willingness to work. Such workers are more willing to take up a job right away, even if such job has shorter duration. The formal value of R is

$$R = w - b\frac{(r+s)[r+s+\lambda+h(\theta_t)]}{(r+s)h(\theta_t) - (r+s+\lambda)h(\theta_p)}$$

[4] Once a functional form for the matching function is chosen, θ_i is completely determined by the behaviour of the firms.

Figure (1), plots the reservation value. As long as the existence condition holds[5], then $R < w$ and there exists a proportion of workers $1 - F(R)$ searching in the permanent market. It's easy to see that when $b = 0$ the reservation outside option is equal to the wage and all workers look for a temporary job.

2.4 Labor Market Stock and Flows

Labour supply is the sum of unemployment and employment in each submarket

$$\begin{aligned} u_t + n_t &= F(R) \\ u_p + n_p &= 1 - F(R) \end{aligned}$$

The dynamic evolution of unemployment in the two submarket is given by difference between job creation and job destruction. This implies that

$$\begin{aligned} \dot{u}_p &= sn_p - h(\theta_p)u_p = s[1 - F(R) - u_p] - h(\theta_p)u_p \\ \dot{u}_t &= (s + \lambda)n_p - h(\theta_t)u_t = (s + \lambda)[F(R) - u_t] - h(\theta_t)u_t \end{aligned}$$

Unemployment in each submarket is constant when job creation is equal to job destruction; the steady state expressions for the stocks read

$$\begin{aligned} u_p &= \frac{s[1 - F(R)]}{s + h(\theta_p)} \\ n_p &= \frac{h(\theta_p)[1 - F(R)]}{s + h(\theta_p)} \\ u_t &= \frac{(s + \lambda)F(R)}{s + \lambda + h(\theta_t)} \\ n_t &= \frac{F(R)h(\theta_t)}{s + \lambda + h(\theta_t)} \end{aligned}$$

3 Equilibrium

The equilibrium is obtained by a triple $\{\theta_t, \theta_p, R\}$, and a distribution of employment across states that satisfy the set of value functions $\left\{J_i^h, J_p^l, V_i, E_i(z), U_i(z) \text{ with } i \in [p, t]\right\}$ and:

- Optimal vacancy posting in each submarket. The value of a vacancy is identical across submarkets and driven down to zero by free entry

  $$V_p = V_t = 0$$

  This in turn implies:

  – Job creation in the permanent market

  $$q(\theta_p)J_p^h = c \qquad \text{(JC, permanent)}$$

[5] See the appendix.

– Job creation in the temporary market

$$q(\theta_t)J_t^h = c \tag{JC, temporary}$$

which together say that in equilibrium the expected benefit of a permanent job must be equal to the expected benefit of a temporary job.

- Optimal workers' sorting. The marginal worker is indifferent between searching in the market for temporary or permanent jobs

$$U_p(R) = U_t(R) \tag{Sorting}$$

Once a functional form for $m(u_i, v_i)$ is chosen, θ_p and θ_t are determined through job creation conditions; the sorting equation yields R and the last equations in the previous section determine the stocks. The coexistence of the two submarkets depends on a simple condition, as we show next.

Proposition. *Temporary and Permanent submarkets coexist in equilibrium as long as the reservation utility* R exists**.** Further, if R exists, it is also lower than the wage**.**

Proof, see appendix.

3.1 Comparative Static

Qualitative aspects of the final equilibrium obviously depend on the values taken by the exogenous parameters. In this section we focus our attention upon the effects of changes from a relevant couple of them, namely the unemployment benefit b and the shock occurrence rate λ, on the unemployment rate, the labour market tightness, the reservation outside option and the value (for the firm) of a filled job.

- An increase in the wage w leads to a reduction in market tightness in both submarkets and an increase in total unemployment. The effect on the two market tightness follows directly from a simple differentiation of equations JC, permanent and JC, temporary, so that $\frac{\partial \theta_i}{\partial w} < 0$ while the effect on R is ambiguous. The latter follows from the fact that both transition rate fall, and it is not clear a priori which of the transition rate falls more

- From the point of view of the firms, the level of the unemployment benefit does not have any *direct* effect on the value of a filled job, and using the job creation conditions in the two submarkets, also on the labor market tightness. In symbols

$$q(\theta_i)J_i^h = c \Rightarrow \frac{\partial \theta_i}{\partial b} = \frac{-c[\partial J_i^h/\partial b]}{[J_i^h]^2[\partial q(\theta_i)/\partial \theta_i]} = 0 \text{ since } \partial J_i^h/\partial b = 0$$

  An increase in b makes the permanent submarket more attractive for the workers. Since market tightness does not change, permanent unemployment increases and temporary unemployment decreases. Formally, using the formal value of R it is immediate to see that, as long as $R < w$, $\partial R/\partial b < 0$. This result allows to evaluate the effect on the unemployment rates

$$\frac{\partial u_p}{\partial b} = -\frac{s}{s+h(\theta_p)} \frac{\partial F(z)}{\partial z|z=R} \frac{\partial R}{\partial b} > 0$$

and

$$\frac{\partial u_t}{\partial b} = \frac{s+\lambda}{s+\lambda+h(\theta_t)} \frac{\partial F(z)}{\partial z|z=R} \frac{\partial R}{\partial b} < 0$$

as expected. The effect on total unemployment is consequently ambiguous[6].

- An increase in the arrival rate λ has various effects, but the overall result is not as clear. If a shock to the productivity of a match becomes more likely, all firms enjoy the operational profit for a shorter period; the value of a filled job, either temporary or permanent, diminishes and firms are less prone to post new vacancies. In formal terms

$$\frac{\partial J_p^h}{\partial \lambda} = \frac{y_l - y_h}{(r+s+\lambda)^2} < 0 \text{ and } \frac{\partial J_t^h}{\partial \lambda} = -\frac{y_h - w}{(r+s+\lambda)^2} < 0$$

Using the result above with job creation conditions in both markets yields the negative reaction of the labor market tightness

$$\frac{\partial \theta_i}{\partial \lambda} = \frac{-c[\partial J_i^h/\partial \lambda]}{[J_i^h]^2[\partial q(\theta_i)/\partial \theta_i]} < 0$$

From the point of view of the workers a higher λ makes the duration of a temporary job shorter; a fraction of them would therefore move from the temporary to the permanent tier but, differently from the case of the unemployment benefit, the productivity shock negatively affects the tightness in both submarkets too. In other words a trade off emerges between a higher risk of being fired on the temporary market (which has a negative direct effect upon the reservation outside option) and a possibly too high unemployment duration on the permanent. The net effect of a change in the shock rate upon R is therefore a priori ambiguous and no prediction can be made upon the unemployment rates.

4 Liberalization of Temporary Contracts

While the steady state solution clearly implies a long run coexistence of the two type of contracts, the question linked to the liberalization of temporary contracts has not yet been discussed. In this section we consider the full transition from a *rigid regime*, a situation where only permanent contracts are allowed, to a *dual regime* where temporary and permanent contracts coexist in equilibrium.

The rigid regime is formally described as a labour market in which only the permanent submarket exists. We define the introduction of temporary jobs as a permanent unexpected shock to the steady state of the rigid market. The functioning of the liberalization is as follows. At time $\tau = 0$ when the shock occurs

[6] With some algebra it can be shown that an increase in the unemployment benefit increases total unemployment as long as $\lambda < [h(\theta_t) - h(\theta_p)]/h(\theta_p)$.

the stock of unemployed workers of the old regime is immediately split in two: workers with $z \leq R$ start searching in the temporary submarket, while workers with z above the reservation productivity R stay in the permanent one. Firms immediately post vacancies in order to fully absorb any rent. Thereafter, the stock of workers smoothly move toward a new steady state with two submarkets. Note that both the reservation utility R as well as market tightness in the two submarkets are time invariant, and the dynamics of the model can be described analytically.

To keep track of the dynamics of the model after the introduction of temporary contracts, we will consider separately the behavior of workers whose outside option is below or above the reservation threshold.

- $z \leq R$. At $\tau = 0$ all unemployed workers with an outside utility below the reservation utility start searching for a temporary job at the finding rate $h(\theta_t)$. On the demand side, firms post a number of temporary vacancies such that the tightness jumps to its equilibrium level and fill them at rate $q(\theta_t)$. In addition those workers employed with a permanent contract and idiosyncratic utility below R are gradually dismissed at rate s and become unemployed in the temporary submarket. The steady state is reached when all workers with outside utility below the reservation R move to the temporary submarket. Let's define with $n_p(z,\tau)$ and $n_t(z,\tau)$ the share of permanent and temporary contract with outside utility less or equal to z at transition time τ. $u_t(z,\tau)$ is similarly defined for the unemployment stock. This implies that at each point in time the distribution of workers with a low outside option reads

$$F(z) = n_p(z,\tau) + n_t(z,\tau) + u_t(z,\tau), \qquad z \leq R$$

  and the dynamics of the three functions is described by

$$\begin{aligned} \dot{n}_p(z,\tau) &= -s n_p(z,\tau), \qquad z \leq R \\ \dot{n}_t(z,\tau) &= h(\theta_t) u_t(z,\tau) - (s+\lambda) n_t(z,\tau), \qquad z \leq R \\ \dot{u}_t(z,\tau) &= s n_p(z,\tau) + (s+\lambda) n_t(z,\tau) - h(\theta_t) u_t(z,\tau), \qquad z \leq R \end{aligned}$$

  where it is clear that there is no inflow into $n_p(z,\tau)$ for $z \leq R$, but simply an outflow that dies out as all permanent jobs with outside utility below R are slowly destroyed at rate s. The flows of temporary contracts is governed by flows that are identical to those of the steady state. During the transition, the unemployment rate into the temporary submarket increases also because of the inflow of old permanent jobs.

- $z > R$. People with outside utility above the reservation R are either employed with a permanent contract or unemployed and searching for a permanent job. This is true both in the rigid and in the liberalized regime. Accordingly, the distribution of such workers reads

$$F(z) = u_p(z,\tau) + n_p(z,\tau), \qquad z > R$$

where $u_p(z,\tau)$ is the stock of unemployed at time τ and $n_p(z,\tau)$ is the stock of employed workers. The dynamics of these two components is given by

$$\begin{aligned}\dot{u}_p(z,\tau) &= sn_p(z,\tau) - h(\theta_p)u_p(z,\tau), \qquad z > R \\ \dot{n}_p(z,\tau) &= h(\theta_p)u_p(z,\tau) - sn_p(z,\tau), \qquad z > R\end{aligned}$$

The system of differential equations can be solved analytically. The details are in the appendix. The readers can find the final results below

$$\begin{aligned} n_t(z,\tau) = &\left\{ \frac{h(\theta_t)h(\theta_p)F(z)}{[s+h(\theta_p)][h(\theta_t)+\lambda]} - \frac{h(\theta_t)F(z)}{h(\theta_t)+s+\lambda} \right\} e^{-[h(\theta_t)+s+\lambda]\tau} + \\ &+ \frac{h(\theta_t)F(z)}{h(\theta_t)+s+\lambda} - \frac{h(\theta_t)h(\theta_p)F(z)}{[s+h(\theta_p)][h(\theta_t)+\lambda]} e^{-s\tau} \end{aligned}$$

$$\begin{aligned} u_t(z,\tau) = F(z) &\left\{ \frac{s}{s+h(\theta_p)} + \frac{\lambda h(\theta_p)}{[s+h(\theta_p)]\,[h(\theta_t)+\lambda]} - \frac{s+\lambda}{h(\theta_t)+s+\lambda} \right\} e^{-[h(\theta_t)+s+\lambda]\tau} + \\ &+ \frac{(s+\lambda)F(z)}{h(\theta_t)+s+\lambda} - \frac{\lambda h(\theta_p)F(z)}{[s+h(\theta_p)]\,[h(\theta_t)+\lambda]} e^{-s\tau} \end{aligned}$$

$$\begin{aligned} u_p(z,\tau) &= \frac{s[1-F(z)]}{s+h(\theta_p)} \\ n_p(z,\tau) &= \frac{h(\theta_p)F(z)}{s+h(\theta_p)} e^{-s\tau} + \frac{h(\theta_p)[1-F(z)]}{h(\theta_p)+s} \end{aligned}$$

Taking the limit as τ goes to infinity and using $z = R$, one gets easily the expressions for the two tiers steady state (see section 2.4).

4.1 Just a "honeymoon effect"?

Having derived the analytical solution to the transition, we now look into the effects of the liberalization of temporary contracts, with particular attention to the unemployment rate. Our solution distinguishes between a short run and a long run effect.

In the aftermath of the liberalization, immediately after the shock, the unemployment rate necessarily falls. The reasoning is as follows. At $\tau = 0$ the stock of unemployed workers is as large as in the rigid regime, but a fraction $F(R)$ of workers starts searching into the temporary submarket where the job finding rate $h(\theta_t)$ is larger. Indeed, market tightness and vacancy posting are a forward looking variable, and immediately jump to exhaust all the rents. While it is true that in the temporary submarket also the separation rate is larger through the destruction rate λ, it takes time for such effect to emerge. Further, market tightness is constant during the transition. As a result unemployment, initially, necessarily falls[7].

[7] Analitically this result is obtained by taking the time derivative of u_t and evaluating it at $\tau = 0$; this yields $\partial u_t(\tau)/\partial\tau|\tau = 0 < 0$. Details are in the appendix.

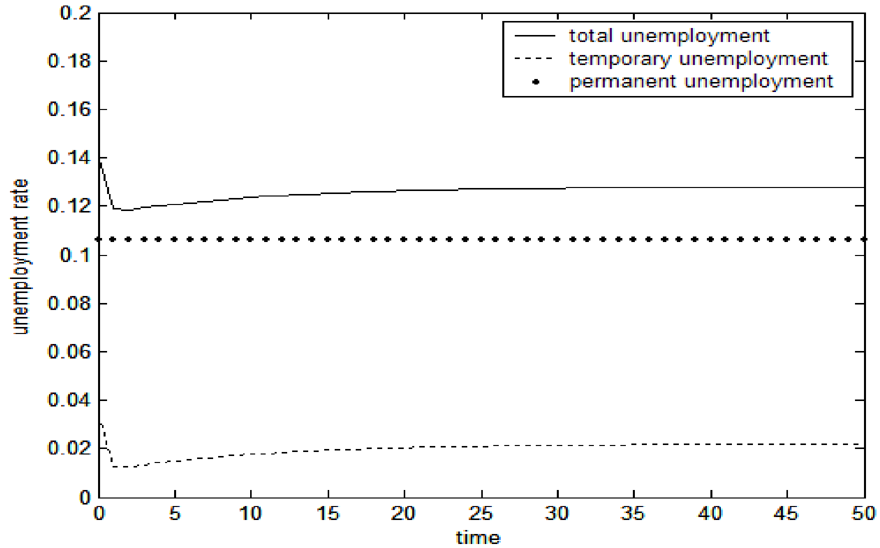

Figure 2: Dynamics of the unemployment rate

Figures (2) and (3) plot the dynamics of the unemployment and the employment rates for a given set of parameters values[8]. The downward jump represents this "honeymoon effect": on impact, the liberalization of temporary contracts has a positive effect on total employment.

The results on the long run effects are more ambiguous. Whether total unemployment is permanently lower than in the rigid regime depends on the relative strength of the job finding and job destruction rates in the two submarkets. The unemployment is permanently reduced if[9]

$$\begin{aligned} u_{p,old} &> u_p(\tau \to \infty) + u_t(\tau \to \infty) \Rightarrow \\ \frac{s}{s + h(\theta_p)} &> \frac{s[1 - F(R)]}{s + h(\theta_p)} + \frac{(s + \lambda)F(R)}{s + \lambda + h(\theta_t)} \Rightarrow \\ \lambda &< \frac{s[h(\theta_t) - h(\theta_p)]}{h(\theta_p)} \end{aligned} \tag{10}$$

i.e. if workers' turnover is not too high. This statement, however, needs to be further discussed. Using the condition for the existence of R one gets

$$\lambda < \frac{(r + s)\{h(\theta_t)w - h(\theta_p)w - b[r + s + h(\theta_p)w]\}}{b(r + s) + h(\theta_p)w}$$

When $b = 0$, from the point of view of (10) this condition is not relevant, becoming monotonically binding for increasing values of the unemployment benefit; this means that for small values of the unemployment

[8] We assumed that the matching function is a Cobb-Douglas one with unemployment elasticity α: $m_i = k_i u_i^{\alpha} v_i^{1-\alpha}$ where $\alpha = 0.5$ and $k_i = 1$. Time is expressed in years. The pure discount rate r is 0.02, worker turnover s is 0.1 and the average waiting time for a productivity shock is about six years ($\lambda = 0.15$). Productivity is either 1 or, conditional on the adverse shock, 0.6. The wage is 0.8 and the exogenous benefit b for the unemployed on permanent market is 30% of the wage. The cost of keeping open a vacancy is 0.3.

[9] The stock of unemployed workers in the old regime is discussed in the appendix

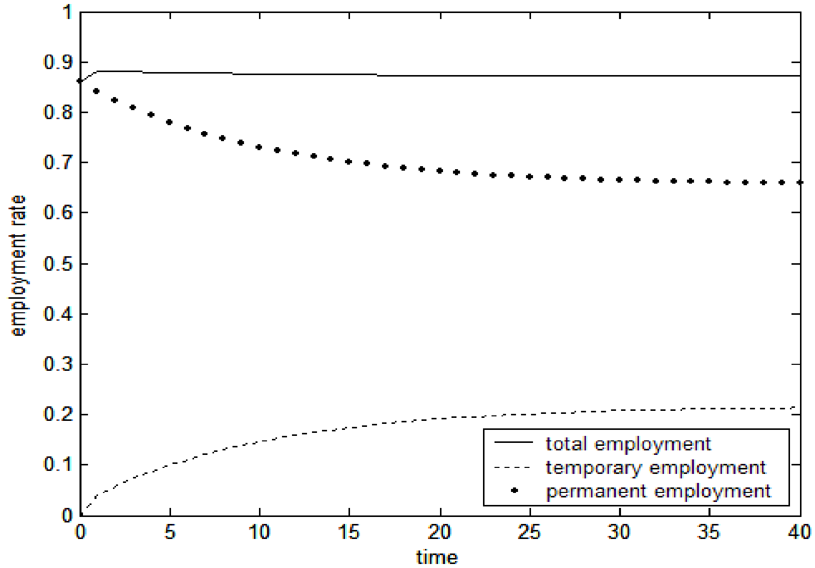

Figure 3: Dynamics of the employment rate

benefit the coexistence of permanent and temporary contracts does not prevent the labour market from a higher equilibrium unemployment rate.

5 Training

In this section we consider the possibility that firms, in the aftermath of the adverse productivity shock, may be able to jump back to the high productivity by undergoing costly training. Specifically, we assume that when the negative shock occurs firms can jump back to the high level of productivity y_h by paying a lump sum cost T in the form of training. As the wage paid to workers is held fixed, we can abstract from the issue of financing. We will show that there exist two bounds $[T_l, T_u]$ such that if $T_l < T < T_u$ only firms in the permanent submarket decide to train workers. The asset equations in the permanent market read

$$\begin{aligned} rJ_p^h &= y_h - w + s[V_p - J_p^h] + \lambda[\max(J_p^l, J_p^h - T) - J_p^h] \\ rJ_p^l &= y_l - w + s[V_p - J_p^l] \\ rV_p &= -c + q(\theta_p)[J_p^h - V_p] \end{aligned}$$

where the max operator conditional on the λ shocks highlights the training option. On the temporary market the asset equations read

$$\begin{aligned} rJ_t^h &= y_h - w + s[V_t - J_t^h] + \lambda[\max(V_t, J_t^h - T)] \\ rV_p &= -c + q(\theta_t)[J_t^h - V_t] \end{aligned}$$

We now formally establish under what conditions workers with a permanent job receive training. Since undergoing training transforms a low productivity job into a high productivity job, a firm with a permanent contract will undergo training if

$$J_p^h - T > J_p^l$$

Simultaneously, a firm with a temporary contract will not undergo training if

$$V > J_t^h - T$$

The first condition implies

$$\frac{y_h - w}{r+s+\lambda} + \frac{\lambda(y_l - w)}{(r+s)(r+s+\lambda)} - T > \frac{y_l - w}{r+s} \Rightarrow T < \frac{y_h - y_l}{r+s+\lambda}$$

while the condition on the temporary workers reads

$$T > \frac{y_h - w}{r+s+\lambda}$$

If the cost of training T is large enough so that the exit strategy turns out to be preferable in the temporary market, but not too large, then only firms in the permanent market are induced to train the workers

$$\frac{y_h - w}{r+s+\lambda} < T < \frac{y_h - y_l}{r+s+\lambda} \tag{11}$$

More generally, it is never the case that workers receive training only in the temporary market. Training may be viable on both markets, only in the permanent, or in none of them, depending on the level of T. When T is bounded as in condition (11) the following interesting results follow:

- The temporary market is not affected by training costs. As a consequence, the value of a filled job is the same as in the model without training.
- The value of filled jobs in the permanent market now reads

  $$J_p^h = \frac{y_h - w - \lambda T}{r+s}$$

  which is larger than in the model without training, but still lower than J_t^h.
- Free entry makes the equilibrium conditions in the temporary submarket independent on T

  $$\begin{aligned} c &= q(\theta_p)J_p^h \\ c &= q(\theta_t)J_t^h \end{aligned}$$

  This means that in equilibrium the temporary market tightness is the same as without training, while the permanent tightness has now to be higher. As a consequence, on average, in the model with training the job finding rate is higher, the arrival rate of workers for a vacancy is lower, and the steady state overall unemployment is lower.

6 On the Job Search

This section proposes a further extension of the basic model, as it allows workers (either employed or unemployed) in the temporary tier to search for a permanent job. As we keep the wage constant across submarkets, we do not need to explicitly consider wage determination, one of the (many) difficult issues to be faced when one deals with on the job search [Shimer 2003; Nagypal 2006]. Nevertheless, the matching function and the definition of market tightness need to be modified and adjusted. In what follows, the number of matches in the permanent submarket reads

$$m_p(u_p + n_t + u_t, v_p) = m_p(u_p + F(R), v_p)$$

where the pool of workers that search for a job is the sum of workers searching only in the permanent market (u_p) and the pool of workers searching in the temporary submarket ($n_t + u_t$). Since the pool of workers in the temporary submarket is the fraction of them with outside utility below R, the second expression immediately follows. As a result, market tightness in the permanent submarket is given by

$$\theta_p = \frac{v_p}{u_p + n_t + u_t} \tag{12}$$

The matching function in the temporary submarket is unchanged and is simply given by $m_t(u_t, v_t)$, with market tightness $\theta_t = v_t/u_t$.

The value functions in the permanent submarket are defined similarly to those of the baseline model (see section 2.1). The only difference is the expression for θ_p, that is defined as in (12) as a way to take into account the composition of the pool of workers searching for a permanent job. Free entry in the permanent submarket implies that

$$q(\theta_p)J_h^p = c$$

where J_h^p is given by (4).

The value functions for the temporary submarket are different, since workers leave temporary jobs at rate $s + h(\theta_p)$. When business conditions are good, the value function reads

$$rJ_t^h = y_h - w + [s + \lambda + h(\theta_p)][V_t - J_t^h]$$

while the value of a vacancy is simply given by

$$rV_t = -c + q(\theta_t)[J_t^h - V_t]$$

so that free entry implies that

$$q(\theta_t)J_t^h = c$$

where J_t^h is now given by

$$J_t^h = \frac{y_h - w}{r + s + \lambda + h(\theta_p)} \tag{13}$$

The job creation conditions are still the two key equations, but since now J_t^h depends also on θ_p they form a non linear system of two equations in two unknowns that can be solved in cascade[10]. The last variable to be determined is the reservation utility R. The value of unemployment in the temporary submarket reads

$$rU_t(z) = z + h(\theta_t)[E_t(z) - U_t(z)] + h(\theta_p)[E_p(z) - U_t(z)]$$

where it is clear that an unemployed worker with low outside utility searches both in the temporary and in the permanent submarket, and can leave the unemployment pool for both types of jobs. Unemployed workers in the permanent submarket behave as in the baseline model, and their asset value equation for the unemployment status is provided by (1). Given the expressions for $E_t(z)$ and $E_p(z)$ and after some steps of algebra (see the appendix for details), the reservation utility R reads

$$R = w - b\frac{r + s + \lambda + h(\theta_t) + h(\theta_p)}{h(\theta_t)}$$

which implies that $R < w$. Ensuring also that b is small enough[11], we can easily establish that $0 < R < w$. With respect to the base model, the value of a filled temporary job given in (13) is now lower and not necessarily higher than the value of a permanent one; however, assuming that $J_p^h < J_t^h$, the structure and functioning of this model is identical to the model without on the job search. In particular, the basic mechanism that ensure that temporary and permanent jobs coexist in equilibrium survives to this admittedly more realistic scenario. The fact that the value of a permanent job is unchanged while a temporary one is worth less than before means that firms take into account the possibility that temporary workers leave their job moving toward the permanent tier and are consequently less prone to post temporary vacancies; in equilibrium, this leads to a lower tightness in the temporary submarket where a relatively higher congestion from the point of view of the workers emerges.

7 The empirical analysis

This section tests one of the main empirical implications of the model, namely the fact that the job finding rate for people searching on the temporary market is higher, using Italian administrative data. We draw a sample of non employment spell of male Italian workers attached to the labour market, and we use survival analysis with multiple destinations. Our results suggest that the exit rate into temporary jobs is higher than that to permanent workers, and the result appears particularly strong among the recipients of unemployment benefits.

[10] Starting from job creation in the permanent submarket one gets θ_p; using this result with job creation in the temporary submarket also θ_t is obtained.

[11] Technically the equilibrium of the model must be such that

$$b < \frac{h(\theta_t)}{r + s + \lambda + h(\theta_t) + h(\theta_p)}$$

Before turning to the empirical analysis, we first briefly review the empirical literature on temporary workers. A large bulk of the evidence looks at the transition rate of temporary contracts into permanent contracts. The transition from fixed-term to permanent contracts has been analyzed by Booth et al. [2002] for the U.K., Güell and Petrongolo [2000] for Spain, and Holmlund and Storrie [2002] for Sweden. Another bulk of the literature focuses on the stepping stone channel played by temporary contracts. Temporary contracts are used by firms to screen applicants and serve as a gateway toward a permanent job; papers with these focus are Booth et al. [2002] and De Graaf-Zijl et al. [2004]. These dimensions of temporary contract, albeit important, are not the key focus of the paper.

While we did not find direct evidence of our predictions, namely the fact that the job finding rate into the temporary submarket is larger than the job finding rate into permanent jobs, indirect evidence appears consistent with this basic implications. Güell [2000] estimates a hazard rate from unemployment for people that were previously employed with a temporary and a permanent job. She finds that the hazard rate for workers that had a temporary job is consistently larger. This is certainly coherent with the mechanism of the paper. Blanchard and Landier [2002] estimate transition rate of youth French unemployed into permanent and temporary jobs. The transition rate in the late nineties is more than 20 percent for temporary contracts and around 15 percent (or even less) from unemployment into regular jobs. Bover and Gomez [2004] use the Spanish Labor Force Survey and find that the transition rate into temporary jobs in the late Eighties and early Nineties was up to ten times larger the one into permanent jobs. Van Ours and Vodopivec [2006] find a positive correlation between the duration of unemployment and the probability to find a permanent job in Slovenian administrative data. Our dedicated empirical analysis about Italy clearly supports the theoretical prediction.

7.1 The data and the sampling strategy

Our sample is from WHIP, a longitudinal dataset of individual work histories built up by LABORatorio R. Revelli using the Italian Social Security Administration (INPS) archives[12]. WHIP is a large random sample[13] of the population of all the employees of the private sector, of craftsmen, traders, collaborators and the professionals without an autonomous security fund. WHIP also accounts for supported unemployment spells (when either the unemployment benefit or the collective dismissal benefit are present) and retirement, but is not able to distinguish among unsupported unemployment, non-participation and unobservable employment spells (mainly in the public sector or in the black economy). All such careers are observed monthly from January 1985 to December 1999.

Our goal is draw from WHIP a sample of non-employment duration for individuals strongly attached to the labour market. As a way to avoid left censoring problems, we draw all the separations occurred from

[12] See www.laboratoriorevelli.it/whip

[13] All the individuals whose date of birth is March, June, September or December, 10th are sampled; the sampling rate is about 1:90.

January to November 1998. We basically work with a random sample of non-employment spells started in 1998.

The most subtle issues in the data sampling is how to select workers that are strongly attached to the labour market, in line with the type of workers of our theoretical perspective. We thus focus on prime age (between 20 and 40 years of age) non-seasonal male employees workers from the Northern labour market. Job to job transition, that are traditionally defined as those that lead to a new job in less than a month, are also excluded. Finally, when more than one separation per individual is observed, only the last one is included in the sample.

Some descriptive statistics confirm that our exclusions make sense: Berton and Pacelli [2007] show that women have longer re-entry times after an involuntary separation, and that this duration is particularly large after a maternity leave. The focus on Northern Italy is linked to the large share of shadow employment in the South. In this respect, within WHIP the share of prime aged individuals that appear just once in the entire WHIP is [**twice**] as large in the South.

Through direct access to raw data the final censoring point is December 2002. Our sample includes 4095 individuals (3560 uncensored unemployment spells) or 75727 month-person observations.

7.2 Some descriptive statistics

Table ?? shows that our prime-aged sample is young on average (67% was at most 30 years old at separation) and works mainly in northern regions. Most of those workers saparated from a full time permanent job, thus confirming that open end contracts do not prevent workers from losing their jobs [Berton et al. 2007]; the share of full time fixed term arrangements, however, is all but negligible.

7.3 The econometric model and its specification

We consider a survival model of non-employment with different exits. We model non-employment as a single duration process that is terminated by one out of M exhaustive and mutually exclusive possible destinations. We let T be the duration of stay and $\{D_m\}$ a set of M dummies taking the value of one if state m is entered and zero otherwise. Following Lancaster [1990], the transition intensity into state m is defined as

$$\theta_m(t) = \lim_{dt \to 0} \frac{\Pr(t \leq T \leq t + dt, D_m = 1 | T \geq t)}{dt}$$

Assuming that the time-process is intrinsically discrete, the hazard function to *any* destination reads

$$\theta(t) = \sum_{m=1}^{M} \theta_m(t)$$

Following Allison [1982], transition intensities are modeled as

$$\theta_m(t) = \frac{\exp(\beta'_m X)}{1 + \sum_{m=1}^{M} \exp(\beta'_m X)}$$

where X is a set of covariates. One can show that the resulting likelihood function takes the same form of the likelihood of a multinomial logit regression applied to a dataset reorganised into a person-period form [Jenkins 1995; 2005]. The matrix X includes a set of covariates and a set of time-dummies $\{D_t\}$ such that $D_t = 1[T = t]$; this structure is intended to identify the duration dependence shape without assuming any a-priori parametric functional form. In our specification, the set of covariates are age, unitary wage, actual experience, local unemployment rate, work area, firm size and industry with reference to the last employment spell before the separation

Our prediction is that the conditional transition rate of exit to a temporary job is higher than to a permanent one. More formally, if we take the transition to a permanent job as the reference exit in the multinomial logit regression, the exponentiated coefficients of the time-dummies are larger than one

$$\exp(\beta_{m=temporary}[D_t]) > 1$$

In our specification we have $M = 3$: full time permanent, full time temporary, other contracts (self-employment, collaborators, temporary work agency jobs, trainees, apprentices, part timers and supported contracts). We will focus on the comparison between full time permanent and full time temporary, which only differs in the time duration of the contract.

7.4 Results

Our key results are reported in Table 1, where we report the time-dummies exponentiated coefficients when the reference exit state is permanent employment. Point estimates on the whole sample (columns 2 and 3) are larger than one, but only a small number of them is significant. Things are different when the time-dummies are interacted with the unemployment benefit[14]. *Workers that after the separation receive the unemployment benefit clearly experience a higher probability of exit towards a temporary job*, whatever the unemployment duration: all the exponentiated coefficients are significant and larger than one (columns 5 and 6). On the contrary, workers without the unemployment benefit behave more similarly to the whole sample (columns 7 and 8); they are actually the vast majority (82% of the sampled individuals).

The larger effect found among the recipients of unemployment benefits should not be too surprising. First of all, Table 2 shows that this result is not due to sample selection bias. In addition, the recipients of the unemployment benefit are subject to administrative controls and in some way "forced" to accept the first suitable job offer they receive. In this sense they display a stronger attachment to the (formal) labor market and their behaviour is more in line with the assumptions of our theoretical model.

The effects of the other covariates are described in Table 3. It reports the exponentiated coefficients for exit to temporary jobs, other jobs and for persistence in non-employment, using the exit to permanent jobs

[14] A part for the intercations with the covariates, this is equivalent to estimate two instead of only one set of baseline exit rates: one for the workers who receive the unemployment benefit after the separation, and the other for those who do not get it.

Table 1: Time - dummies exponentiated coefficients

All				With UB		Without UB	
$Time - dummies$	$Coefficient$	Pr > z	$Time - dummies$	$Coefficient$	Pr > z	$Coefficient$	Pr > z
Month 2	1.049	0.864	Months 2 - 3	2.715	0.011	1.332	0.309
Month 3	1.477	0.191	Months 4 - 6	2.704	0.001	1.357	0.269
Month 4	1.197	0.529	Months 7 - 9	3.087	0.000	1.676	0.070
Month 5	1.527	0.138	Months 10 - 12	3.574	0.001	1.622	0.086
Month 6	1.331	0.344	Months 13 - 18	3.590	0.001	1.244	0.447
Month 7	1.522	0.173	Months 19 - 24	4.186	0.002	1.441	0.214
Month 8	2.299	0.004	Months 25 - 30	3.232	0.034	1.970	0.050
Month 9	1.142	0.692	Over 30	2.169	0.117	-	-
Month 10	1.101	0.763					
Month 11	2.293	0.006					
Month 12	1.532	0.213					
Months 13 - 15	1.400	0.234					
Months 16 - 18	1.184	0.571					
Months 19 - 24	1.447	0.156					
Months 25 - 30	1.854	0.044					
Source: analysis on WHIP data							

Table 2: Recipents vs. non-recipients: average characterisitcs

Characteristics at separation		Recipients	Non-recipients
Mean age (years)		30.2	27.7
Mean actual experience (months)		60	46.4
Monthly wage (1998 Euros)		1123.5	947.7
Local unemployment rate (%)		6.9	6.9
Work area (%)	North-west	33.1	40.8
	North-east	34.8	33.1
	Centre	32.1	26.1
Firm size (workers)		484	469.8
Labor contract (%)	Full time open end	63.7	59.0
	Full time fixed term	19.0	13.1
	Full time CFL	5.8	7.0
	Part time open end	5.4	6.5
	Ful time apprent.	1.2	9.6
	Other	4.9	4.8
Source: analysis on WHIP data			

as the reference case.

Exits to part time jobs are included in the residual category "other jobs"; as long as part time is a labor supply decision[15], a coefficient larger than one in the residual category is thus not surprising. Having a larger experience increases the probability of getting a permanent job, while the local unemployment rate pushes towards flexible jobs or long-run non-employment. Individuals who were employed in a small firm display a higher probability of exit towards different contracts, while those working in the construction sector are more likely to end up with a permanent job. This last result is somewhat puzzling, since constructions in Italy are strongly linked to the informal labor market. The explanation could be that employers in this sector post either permanent or informal vacancies, and the latter are simply not observable through our administrative data. Flexible workers (both temporary in a broad sense, trainees and apprentices) display a higher probability of exit to a temporary or atypical jobs. This finding is coherent with Berton et al. [2007] and Picchio [2006]: temporary workers in Italy seem to experience a quite high persistence in the

[15] This is the case for 70% of the Italian part time workers; ISTAT [2005].

Table 3: The effect of the covariates (exponentiated coefficients)

	Exit to fixed term jobs		Exit to other jobs		No exit (censored obs.)	
	exp(β)	Pr > z	exp(β)	Pr > z	exp(β)	Pr > z
Part time	0.940	0.773	1.716	0.000	1.363	0.004
Age (years in 1998)	0.996	0.697	0.950	0.000	0.999	0.885
Montlhy wage	1.000	0.983	1.000	0.331	1.000	0.666
Previous exp. (months)	0.995	0.000	0.999	0.507	0.996	0.000
Collaborator in 1997	2.058	0.147	1.528	0.293	1.345	0.359
Collaborator in 1998	1.587	0.430	1.429	0.438	1.205	0.617
Local unemp. rate in 1998	1.050	0.037	1.047	0.005	1.048	0.000
North east	1.449	0.006	1.322	0.002	1.257	0.000
Centre	1.271	0.098	1.158	0.146	1.128	0.100
Small firm (less than 20)	0.779	0.026	1.238	0.006	1.036	0.520
Constructions	0.537	0.000	0.726	0.002	0.762	0.000
Services	0.984	0.896	0.927	0.379	0.933	0.272
White collars	0.991	0.956	1.596	0.000	1.365	0.001
Managers	0.664	0.711	0.390	0.249	1.712	0.166
Fixed term contract	2.664	0.000	1.262	0.036	1.214	0.018
Trainees or apprentices	1.867	0.000	2.424	0.000	1.551	0.000
Sickness benefit	1.030	0.854	0.948	0.639	0.979	0.789
Collective dismissal benefit	1.657	0.551	5.786	0.000	1.599	0.269
Severance payment	1.674	0.219	1.292	0.402	1.288	0.269
Ordinary unemp. benefit	1.849	0.001	1.218	0.130	0.947	0.559
Reduced unemp. benefit	2.338	0.000	0.993	0.958	0.832	0.062
Source: analysis on WHIP data						

labor contract. The unemployment benefit increases the transition intensity both to temporary end to other non-typical jobs.

Our approach does not explicitly take into account unobserved heterogeneity. To the best of our knowledge, in our econometric framework it is not possible to control for more than random effects. In any event, some considerations are in order. First, including a set of time-dummies to describe the duration dependence structure is not only robust to possible misspecification but also to unobserved heterogeneity [Dolton and Van Der Klaauw 1999]. Second, the main source of unobserved heterogeneity is individual ability; since education, its usual proxy, is positively correlated with the probability of finding a permanent job and negatively with the duration of unemployment [ISTAT 2006], we expect the exit rate to temporary employment to be underestimated.

8 Other Implications

The model features several other empirical implications. At the micro level, the most important implication is the effect of training on temporary/permanent workers. At the macro level, the model implies a transitional dynamics on employment in the aftermath of the liberalization of temporary contracts. We briefly discuss these in turn.

Section 5 derives a strong implication on the difference in training incidence between temporary and permanent workers. In the extreme form presented in the paper, it suggests that only permanent workers are trained. The empirical evidence largely supports this implication. Arulampalam and Booth [1998] investigate the relationship between employment flexibility and training using UK data, and find that workers on temporary contracts are less likely to receive work – related training. The Oecd [2002] reports that the diffusion of temporary contracts has been associated to a reduction in training incidence. Brunello et al. [2006] estimate the probability both of taking any training and of receiving employer - sponsored training as a function of educational attainment, gender, tenure, marital status, age (divided in four classes), public/private sector employment, part time/full time status, type of contract (fixed term, casual job and other, with permanent job as the reference), country, industry, firm size and occupation. Controlling for all these effects, as well as country fixed effect, they find temporary workers have a 4 percentage penalty rate in the probability of receiving training. All these results are coherent with the relationship between training and temporary contracts implied by this paper.

The transitional effects of the liberalization of temporary contracts have been studied by Boeri and Garibaldi [2006]. They show that most countries that experienced a gradual liberalization of temporary contracts experienced also employment gains. Such honeymoon effect is clearly present in the mechanism analyzed in this paper.

9 Concluding Remarks

The liberalization of fixed term contracts in Europe has led to a two tiers regime, with a growing share of jobs covered by temporary contracts. The paper proposed and solved a matching model with direct search in which temporary and permanent jobs coexist in a long run equilibrium. When temporary contracts are allowed, firms are willing to open permanent jobs in as much as their job filling rate is faster than that of temporary jobs. The prediction that the job offer arrival rate for temporary workers is higher is supported by our analysis of Italian administrative data. The model is also consistent with the observation that workers hired on a permanent contract receive more training.

A Existence

The coexistence of the two submarkets in equilibrium depends on the existence of a positive reservation outside utility strictly lower than the wage. We show this result in two steps.

- Existence of R. Since both $U_p(z)$ and $U_t(z)$ are linear and monotonically increasing in z, R does exist (and moreover is unique) if and only if $U_t(z=0) < U_p(z=0)$ and $\frac{\partial U_p(z)}{\partial z} > \frac{\partial U_t(z)}{\partial z}$. Using equation (8) and (9), the condition on the slopes says that

$$\frac{r+s}{r+s+h(\theta_p)} > \frac{r+s+\lambda}{r+s+\lambda+h(\theta_t)}$$

and the one on the intercepts reads

$$\frac{h(\theta_t)w}{r+s+\lambda+h(\theta_t)} > \frac{h(\theta_p)w+(r+s)b}{h(\theta_t)(w-b)} \tag{14}$$

- Existence of the two submarkets. The existence of a reservation outside option is a necessary but not sufficient condition for the coexistence of temporary and permanent contracts in equilibrium. We already know, in fact, that if $R \geq w$ all workers search for a temporary job. We need then that

$$\begin{aligned} R &< w \Rightarrow w - b\frac{(r+s)[r+s+\lambda+h(\theta_t)]}{(r+s)h(\theta_t)-(r+s+\lambda)h(\theta_p)} < w \Rightarrow \\ (r+s)h(\theta_t) &> (r+s+\lambda)h(\theta_p) \Rightarrow \\ \frac{r+s}{r+s+\lambda} &> \frac{h(\theta_p)}{h(\theta_t)} \end{aligned}$$

and we can conclude that if (14) holds then R exists and is lower than the wage.

B Dynamics

In the rigid market all the workforce is either employed with a permanent contract or unemployed

$$u_p + n_p = 1$$

The differential equations describing the dynamics of these two components therefore does not depend on the outside utility and read

$$\begin{aligned} \dot{u}_p(\tau) &= sn_p(\tau) - h(\theta_p)u_p(\tau) \\ \dot{n}_p(\tau) &= h(\theta_p)u_p(\tau) - sn_p(\tau) \end{aligned}$$

It's easy to see that when the old regime reaches its steady state the stocks amount to

$$u_p = \frac{s}{s+h(\theta_p)}$$

and

$$n_p = \frac{h(\theta_p)}{s+h(\theta_p)}$$

As we pointed out above, in order to fully describe the dynamic behavior of employed and unemployed workers in both submarkets we need to separately consider people with outside option below and above the reservation value R. In every moment in time the distribution of the formers reads

$$F(z) = n_p(z,\tau) + n_t(z,\tau) + u_t(z,\tau), \quad z \leq R \tag{15}$$

When $\tau = 0$ the stock of workers who start searching in the new submarket is given by the fraction of unemployed workers of the previous regime whose outside option is lower than R

$$u_t(z, \tau = 0) = \frac{sF(z)}{s + h(\theta_p)}, \quad z \leq R \tag{16}$$

Since right after the introduction of the new regime nobody works with a temporary contract ($n_t(z, \tau = 0) = 0$), the initial condition for permanently employed workers with $z \leq R$ can be obtained through (15)

$$\begin{aligned}
n_p(z, \tau &= 0) = F(z) - u_t(z, \tau = 0) - n_t(z, \tau = 0) \Rightarrow \\
n_p(z, \tau &= 0) = F(z) - \frac{sF(z)}{s + h(\theta_p)} - 0 = \\
&= \frac{h(\theta_p)F(z)}{s + h(\theta_p)}, \quad z \leq R
\end{aligned} \tag{17}$$

We are now in a position to describe the dynamic behavior of n_p, n_t and u_t provided $z \leq R$.

- In the rigid market a fraction of workers was employed with a permanent contract even if endowed with a low outside option. From $\tau = 0$ onwards, once they are fired (what happens at rate s), they start searching in the temporary submarket with no possibility to come back to the permanent tier

$$\begin{aligned}
\dot{n}_p(z,\tau) &= -sn_p(z,\tau) \Rightarrow \\
\dot{n}_p(z,\tau) + sn_p(z,\tau) &= 0 \Rightarrow \\
\int e^{s\tau}[\dot{n}_p(z,\tau) + sn_p(z,\tau)]d\tau &= b_1 \Rightarrow \\
n_p(z,\tau)e^{s\tau} + b_0 &= b_1 \Rightarrow n_p(z,\tau) = Be^{-s\tau}, \quad z \leq R
\end{aligned}$$

  where b_0 and b_1 are constants of integration. Using (17) and solving for B

$$n_p(z, 0) = B = \frac{h(\theta_p)F(z)}{s + h(\theta_p)}, \quad z \leq R$$

  therefore

$$n_p(z,\tau) = \frac{h(\theta_p)F(z)}{s + h(\theta_p)}e^{-s\tau}, \quad z \leq R \tag{18}$$

  i.e. the initial stock of permanent workers with a low outside option decreases at rate s down to zero; in fact

$$\lim_{\tau\to\infty} n_p(z \leq R, \tau) = 0$$

- Since in the rigid market only permanent contracts were allowed, right after the shock the stock of temporary workers is an empty set, but immediately firms post temporary vacancies and fill them at rate $h(\theta_t)$ by hiring from the stock of "temporary" unemployed. Temporary matches are then destroyed at rate $s+\lambda$

$$\dot{n}_t(z,\tau) = h(\theta_t)u_t(z,\tau) - (s+\lambda)n_t(z,\tau)$$

using (15) and (18) one gets

$$\begin{aligned}
\dot{n}_t(z,\tau) &= h(\theta_t)[F(z) - n_p(z,\tau) - n_t(z,\tau)] - (s+\lambda)n_t(z,\tau) \Rightarrow \\
\dot{n}_t(z,\tau) + [h(\theta_t)+s+\lambda]n_t(z,\tau) &= h(\theta_t)\left[F(z) - \frac{h(\theta_p)F(z)}{s+h(\theta_p)}e^{-s\tau}\right] \Rightarrow \\
e^{[h(\theta_t)+s+\lambda]\tau}\left\{\dot{n}_t(z,\tau) + [h(\theta_t)+s+\lambda]n_t(z,\tau)\right\} &= e^{[h(\theta_t)+s+\lambda]\tau}h(\theta_t)\left[F(z) - \frac{h(\theta_p)F(z)}{s+h(\theta_p)}e^{-s\tau}\right] \Rightarrow \\
n_t(z,\tau)e^{[h(\theta_t)+s+\lambda]\tau} + b_0 &= \int h(\theta_t)F(z)e^{[h(\theta_t)+s+\lambda]\tau}d\tau - \int \frac{h(\theta_t)h(\theta_p)F(z)}{s+h(\theta_p)}e^{[h(\theta_t)+\lambda]\tau}d\tau \Rightarrow \\
n_t(z,\tau)e^{[h(\theta_t)+s+\lambda]\tau} &= b_0 + \frac{h(\theta_t)F(z)}{h(\theta_t)+s+\lambda}e^{[h(\theta_t)+s+\lambda]\tau} + b_1 - \frac{h(\theta_t)h(\theta_p)F(z)}{[s+h(\theta_p)][h(\theta_t)+\lambda]}e^{[h(\theta_t)+\lambda]\tau} + b_2 \Rightarrow \\
n_t(z,\tau) &= Be^{-[h(\theta_t)+s+\lambda]\tau} + \frac{h(\theta_t)F(z)}{h(\theta_t)+s+\lambda} - \frac{h(\theta_t)h(\theta_p)F(z)}{[s+h(\theta_p)][h(\theta_t)+\lambda]}e^{-s\tau} \quad z \leq R
\end{aligned}$$

A unique solution for the dynamics of $n_t(z,\tau)$ is obtained imposing the initial condition $n_t(z,0) = 0$, solving for B and substituting the expression below into the previous equation

$$B = \frac{h(\theta_t)h(\theta_p)F(z)}{[s+h(\theta_p)][h(\theta_t)+\lambda]} - \frac{h(\theta_t)F(z)}{h(\theta_t)+s+\lambda}, \quad z \leq R$$

The stock of temporary workers therefore grows from zero to

$$\lim_{\tau\to\infty} n_t(z,\tau) = \frac{h(\theta_t)F(z)}{h(\theta_t)+s+\lambda}, \quad z \leq R$$

- The dynamic behavior of u_t is not simply the reverse of n_t. The stock of workers looking for a temporary job grows also because people with $z \leq R$ eventually loose their permanent job at rate s and move to the temporary tier

$$\dot{u}_t(z,\tau) = sn_p(z,\tau) + (s+\lambda)n_t(z,\tau) - h(\theta_t)u_t(z,\tau), \quad z \leq R$$

Again, using (15) with (18)

$$\begin{aligned}
\dot{u}_t(z,\tau) &= sn_p(z,\tau) + (s+\lambda)[F(z) - u_t(z,\tau) - n_p(z,\tau)] - h(\theta_t)u_t(z,\tau) \Rightarrow \\
\dot{u}_t(z,\tau) + [h(\theta_t)+s+\lambda]u_t(z,\tau) &= (s+\lambda)F(z) - \frac{\lambda h(\theta_p)F(z)}{s+h(\theta_p)}e^{-s\tau} \Rightarrow \\
e^{[h(\theta_t)+s+\lambda]\tau}\left\{\dot{u}_t(z,\tau) + [h(\theta_t)+s+\lambda]u_t(z,\tau)\right\} &= e^{[h(\theta_t)+s+\lambda]\tau}\left[(s+\lambda)F(z) - \frac{\lambda h(\theta_p)F(z)}{s+h(\theta_p)}e^{-s\tau}\right] \Rightarrow \\
u_t(z,\tau)e^{[h(\theta_t)+s+\lambda]\tau} + b_0 &= \int (s+\lambda)F(z)e^{[h(\theta_t)+s+\lambda]\tau}d\tau - \int \frac{\lambda h(\theta_p)F(z)}{s+h(\theta_p)}e^{[h(\theta_t)+\lambda]\tau}d\tau \\
u_t(z,\tau) &= Be^{-[h(\theta_t)+s+\lambda]\tau} + \frac{(s+\lambda)F(z)}{h(\theta_t)+s+\lambda} - \frac{\lambda h(\theta_p)F(z)}{[s+h(\theta_p)]\,[h(\theta_t)+\lambda]}e^{-s\tau}, \quad z \leq R
\end{aligned}$$

Imposing the initial condition (16) and solving for B one gets a unique solution for the dynamics of $u_t(z,\tau)$

$$B = F(z)\left\{\frac{s}{s+h(\theta_p)} + \frac{\lambda h(\theta_p)}{[s+h(\theta_p)]\,[h(\theta_t)+\lambda]} - \frac{s+\lambda}{h(\theta_t)+s+\lambda}\right\}, \quad z \leq R$$

The stock of unemployed workers on the temporary market therefore goes from the initial level

$$u_t(z,\tau=0) = \frac{sF(z)}{s+h(\theta_p)}, \quad z \leq R$$

to its steady state value

$$\lim_{\tau\to\infty} u_t(z,\tau) = \frac{(s+\lambda)F(z)}{s+\lambda+h(\theta_t)}, \quad z \leq R$$

Let us now turn to the stock of workers with $z > R$. People with large outside utility never move from the permanent tier; in every moment in time they are either employed or unemployed with a permanent contract

$$1 - F(z) = u_p(z,\tau) + n_p(z,\tau), \quad z > R \tag{19}$$

The initial stock of unemployed workers searching for a permanent job is given by the proportion of unemployed workers in the old regime with $z > R$

$$u_p(z,\tau=0) = \frac{s[1-F(z)]}{s+h(\theta_p)}, \quad z > R \tag{20}$$

Using (19) one gets the initial condition for $n_p(z > R,\tau)$

$$n_p(z,\tau=0) = \frac{h(\theta_p)[1-F(z)]}{s+h(\theta_p)}, \quad z > R$$

- The stock of unemployed with $z > R$ increases when permanently employed workers with large outside option leave their jobs and decreases when they find a new one

  $$\dot{u}_p(z,\tau) = sn_p(z,\tau) - h(\theta_p)u_p(z,\tau), \quad z > R$$

  using (19)

  $$\begin{aligned}
  \dot{u}_p(z,\tau) + [s+h(\theta_p)]u_p(z,\tau) &= s[1-F(z)] \Rightarrow \\
  e^{[s+h(\theta_p)]\tau}\left\{\dot{u}_p(z,\tau) + [s+h(\theta_p)]u_p(z,\tau)\right\} &= s[1-F(z)]e^{[s+h(\theta_p)]\tau} \Rightarrow \\
  u_p(z,\tau)e^{[s+h(\theta_p)]\tau} + b_o &= s[1-F(z)]\int e^{[s+h(\theta_p)]\tau}d\tau \Rightarrow \\
  u_p(z,\tau) &= Be^{-[s+h(\theta_p)]\tau} + \frac{s[1-F(z)]}{s+h(\theta_p)}, \quad z > R
  \end{aligned}$$

  As usual, a unique solution is obtained through the imposition of the initial condition in (20); solving by B one gets

  $$\begin{aligned}
  B &= \frac{s[1-F(z)]}{s+h(\theta_p)} - \frac{s[1-F(z)]}{s+h(\theta_p)} = 0 \Rightarrow \\
  u_p(z,\tau) &= \frac{s[1-F(z)]}{s+h(\theta_p)}, \quad z > R
  \end{aligned}$$

  The stock of unemployed workers in the permanent market does not depend on time: its level is constant during the transition to the new steady state.

- The dynamics of $n_p(z > R, \tau)$ is its exact reverse

$$\dot{n}_p(z,\tau) = h(\theta_p)u_p(z,\tau) - sn_p(z,\tau), \quad z > R$$

Using (19)

$$\begin{aligned}
\dot{n}_p(z,\tau) &= h(\theta_p)[1 - F(z) - n_p(z,\tau)] - sn_p(z,\tau) \Rightarrow \\
\dot{n}_p(z,\tau) + [h(\theta_p) + s]n_p(z,\tau) &= h(\theta_p)[1 - F(z)] \Rightarrow \\
e^{[h(\theta_p)+s]\tau}\left\{\dot{n}_p(z,\tau) + [h(\theta_p) + s]n_p(z,\tau)\right\} &= h(\theta_p)[1 - F(z)]e^{[h(\theta_p)+s]\tau} \Rightarrow \\
n_p(z,\tau)e^{[h(\theta_p)+s]\tau} + b_0 &= h(\theta_p)[1 - F(z)]\int e^{[h(\theta_p)+s]\tau}d\tau \Rightarrow \\
n_p(z,\tau) &= Be^{-[h(\theta_p)+s]\tau} + \frac{h(\theta_p)[1 - F(z)]}{h(\theta_p) + s}, \quad z > R
\end{aligned}$$

The imposition of the initial condition for $\tau = 0$ yields the unique value of B

$$\frac{h(\theta_p)[1 - F(z)]}{s + h(\theta_p)} = B + \frac{h(\theta_p)[1 - F(z)]}{h(\theta_p) + s} \Rightarrow B = 0 \Rightarrow n_p(z > R,\tau) = \frac{h(\theta_p)[1 - F(z)]}{h(\theta_p) + s}, \quad z > R$$

So also the dynamic equation of $n_p(z > R, \tau)$ does not depend on time; nonetheless we have to keep in mind that the full dynamics for n_p depends also on workers with $z \leq R$.

We are now in a position to describe the whole dynamics of the system. $n_t(z,\tau)$ and $u_t(z,\tau)$ are fully determined by workers with $z \leq R$, while $u_p(z,\tau)$ by the ones with $z > R$; $n_p(z,\tau)$ depends on both

$$\begin{aligned}
n_t(z,\tau) = n_t(z \leq R,\tau) = &\left\{\frac{h(\theta_t)h(\theta_p)F(z)}{[s + h(\theta_p)][h(\theta_t) + \lambda]} - \frac{h(\theta_t)F(z)}{h(\theta_t) + s + \lambda}\right\}e^{-[h(\theta_t)+s+\lambda]\tau} + \\
&+ \frac{h(\theta_t)F(z)}{h(\theta_t) + s + \lambda} - \frac{h(\theta_t)h(\theta_p)F(z)}{[s + h(\theta_p)][h(\theta_t) + \lambda]}e^{-s\tau}
\end{aligned}$$

$$\begin{aligned}
u_t(z,\tau) = u_t(z \leq R,\tau) = F(z) &\left\{\frac{s}{s + h(\theta_p)} + \frac{\lambda h(\theta_p)}{[s + h(\theta_p)]\,[h(\theta_t) + \lambda]} - \frac{s + \lambda}{h(\theta_t) + s + \lambda}\right\}e^{-[h(\theta_t)+s+\lambda]\tau} + \\
&+ \frac{(s + \lambda)F(z)}{h(\theta_t) + s + \lambda} - \frac{\lambda h(\theta_p)F(z)}{[s + h(\theta_p)]\,[h(\theta_t) + \lambda]}e^{-s\tau}
\end{aligned}$$

$$\begin{aligned}
u_p(z,\tau) &= u_p(z > R,\tau) = \frac{s[1 - F(z)]}{s + h(\theta_p)} \\
n_p(z,\tau) &= n_p(z \leq R,\tau) + n_p(z > R,\tau) = \frac{h(\theta_p)F(z)}{s + h(\theta_p)}e^{-s\tau} + \frac{h(\theta_p)[1 - F(z)]}{h(\theta_p) + s}
\end{aligned}$$

Taking $\lim_{\tau\to\infty}$ and using $z = R$ one gets the expressions for the two tiers steady state.

C The "honeymoon effect"

In order to prove the existence of what we called the "honeymoon effect" of the introduction of temporary jobs we take the time derivative of the equation describing the dynamics of total unemployment and evaluate it at $\tau = 0$;

more precisely, since permanent unemployment does not display any dynamics (see the subsection above), we will focus on the behavior of temporary unemployment. If the liberalization of temporary contracts leads to an immediate reduction of total unemployment, the time derivative of temporary unemployment evaluated at $\tau = 0$ must be negative. From section 9.2 we know that

$$u_t(\tau) = F(R)\left\{\frac{s}{s+h(\theta_p)} + \frac{\lambda h(\theta_p)}{[s+h(\theta_p)]\,[h(\theta_t)+\lambda]} - \frac{s+\lambda}{h(\theta_t)+s+\lambda}\right\} e^{-[h(\theta_t)+s+\lambda]\tau} + \\ + \frac{(s+\lambda)F(R)}{h(\theta_t)+s+\lambda} - \frac{\lambda h(\theta_p)F(R)}{[s+h(\theta_p)]\,[h(\theta_t)+\lambda]}e^{-s\tau} \Rightarrow$$

$$\frac{\partial u_t(\tau)}{\partial \tau} = -[s+\lambda+h(\theta_t)]F(z)\left\{\frac{s}{s+h(\theta_p)} + \frac{\lambda h(\theta_p)}{[s+h(\theta_p)][\lambda+h(\theta_t)]} - \frac{s+\lambda}{s+\lambda+h(\theta_t)}\right\}\cdot \\ \cdot e^{-[s+\lambda+h(\theta_t)]\tau} + \frac{s\lambda h(\theta_p)F(z)}{[s+h(\theta_p)][\lambda+h(\theta_t)]}e^{-s\tau}$$

imposing $\tau = 0$

$$\frac{\partial u_t(\tau)}{\partial \tau | \tau = 0} = (s+\lambda)F(z) - \frac{s[s+\lambda+h(\theta_t)]F(z)}{s+h(\theta_p)} - \frac{\lambda h(\theta_p)F(z)[s+\lambda+h(\theta_t)]}{[s+h(\theta_p)][\lambda+h(\theta_t)]} + \\ + \frac{s\lambda h(\theta_p)F(z)}{[s+h(\theta_p)][\lambda+h(\theta_t)]} =$$

Omitting the common denominator, which is not relevant for the sign of the expression above, one gets

$$[s+h(\theta_p)][\lambda+h(\theta_t)](s+\lambda)F(z) - s[\lambda+h(\theta_t)][s+\lambda+h(\theta_t)]F(z) + \\ - \lambda h(\theta_p)F(z)[s+\lambda+h(\theta_t)] + s\lambda h(\theta_p)F(z) =$$

$$= h(\theta_p)[\lambda+h(\theta_t)](s+\lambda)F(z) - s[\lambda+h(\theta_t)]h(\theta_t)F(z) + \\ - \lambda h(\theta_p)F(z)[s+\lambda+h(\theta_t)] + s\lambda h(\theta_p)F(z) =$$

$$\begin{aligned} &= [\lambda+h(\theta_t)]F(z)[h(\theta_p)(s+\lambda) - sh(\theta_t) - \lambda h(\theta_p)] = \\ &= [\lambda+h(\theta_t)]F(z)\left\{s[h(\theta_p)-h(\theta_t)]\right\} < 0 \end{aligned}$$

D Search on the job

The proof of the existence of the equilibrium in the model with on the job search follows the lines of section 9.1: we need to find the conditions for the existence of a positive reservation outside utility that is strictly lower than the wage. Once θ_t and θ_p are determined by sequentially solving the job creation conditions system (see section 6), both U_t and U_p are linear functions of z; a positive R therefore exists when the intercept of U_t is larger than the intercept of U_p and its slope is smaller[16]. We will then prove that under the same conditions not only R is positive, but is also strictly lower than w.

[16] In principle, the existence of a positive R would be shown also under the opposite conditions, i.e. a higher intercept and a smaller slope for U_p; however, as a few steps of algebra will make clear, the slope of U_p is always larger than the one of U_t.

The value functions for the supply side of the permanent submarket look as in section 2.1

$$\begin{aligned} rE_p(z) &= w + s[U_p(z) - E_p(z)] \\ rU_p(z) &= z + b + h(\theta_p)[E_p(z) - U_p(z)] \end{aligned}$$

so that the value of unemployment for a permanent worker reads

$$U_p(z) = \frac{(z+b)(r+s) + h(\theta_p)w}{r[r+s+h(\theta_p)]}$$

In the temporary submarket the asset equations are a bit more complicated, since workers leave their temporary jobs not only because of natural turnover, but also when a permanent vacancy becomes available

$$\begin{aligned} rE_t(z) &= w + h(\theta_p)[E_p(z) - E_t(z)] + (s+\lambda)[U_t(z) - E_t(z)] \\ rU_t(z) &= z + h(\theta_t)[E_t(z) - U_t(z)] + h(\theta_p)[E_p(z) - U_t(z)] \end{aligned}$$

Using $E_t(z)$, $E_p(z)$ and $U_p(z)$ one gets the expression for $U_t(z)$

$$\begin{aligned} U_t(z) = \frac{[r+s+\lambda+h(\theta_p)]z}{[r+h(\theta_p)][r+s+\lambda+h(\theta_t)+h(\theta_p)]} &+ \frac{\{(r+s)h(\theta_t) + h(\theta_t)h(\theta_p) + h(\theta_p)[r+s+\lambda+h(\theta_p)]\}\, w}{(r+s)[r+h(\theta_p)][r+s+\lambda+h(\theta_t)+h(\theta_p)]} + \\ &+ \frac{h(\theta_p)s[(z+b)(r+s)+h(\theta_p)w]}{r(r+s)[r+h(\theta_p)][r+s+h(\theta_p)]} \end{aligned}$$

We are now ready to go through the steps of the proof.

- Condition on the slopes: $\partial U_p/\partial z > \partial U_t/\partial z$

$$\frac{(r+s)}{r[r+s+h(\theta_p)]} > \frac{r+s+\lambda+h(\theta_p)}{[r+h(\theta_p)][r+s+\lambda+h(\theta_t)+h(\theta_p)]} + \frac{sh(\theta_p)}{r[r+h(\theta_p)][r+s+h(\theta_p)]}$$

Using and omitting the common denominator (which is not relevant for the sign) one gets

$$\begin{aligned} &(r+s)[r+h(\theta_p)][r+s+\lambda+h(\theta_t)+h(\theta_p)] - r[r+s+\lambda+h(\theta_p)][r+s+h(\theta_p)] + \\ &\qquad - sh(\theta_p)[r+s+\lambda+h(\theta_t)+h(\theta_p)] > 0 \Rightarrow \end{aligned}$$

$$\begin{aligned} [r^2 + rh(\theta_p) + rs][\lambda + h(\theta_t)] - r\lambda[r+s+h(\theta_p)] &> 0 \Rightarrow \\ r^2 h(\theta_t) + rh(\theta_t)h(\theta_p) + rsh(\theta_t) &> 0 \quad \textit{always} \end{aligned}$$

- Condition on the intercepts: $U_p(0) < U_t(0)$

$$\begin{aligned} \frac{b(r+s)+h(\theta_p)w}{r[r+s+h(\theta_p)]} < \frac{\{(r+s)h(\theta_t) + h(\theta_p)h(\theta_t) + h(\theta_p)[r+s+\lambda+h(\theta_p)]\}\, w}{(r+s)[r+h(\theta_p)][r+s+\lambda+h(\theta_p)+h(\theta_t)]} &+ \\ + \frac{h(\theta_p)sb(r+s) + h(\theta_p)sh(\theta_p)w}{r(r+s)[r+h(\theta_p)][r+s+h(\theta_p)]} \end{aligned}$$

Multiplying both sides by the common denominator the expression reads

$$\begin{aligned}(r+s)[r+h(\theta_p)][r+s+\lambda+h(\theta_p)+h(\theta_t)][b(r+s)+h(\theta_p)w]+\\ -r[r+s+h(\theta_p)]\left\{(r+s)h(\theta_t)+h(\theta_p)h(\theta_t)+h(\theta_p)[r+s+\lambda+h(\theta_p)]\right\}w+\\ -[r+s+\lambda+h(\theta_p)+h(\theta_t)][h(\theta_p)sb(r+s)+h(\theta_p)sh(\theta_p)w]<0;\end{aligned}$$

$$\begin{aligned}[r^2+rh(\theta_p)+rs][r+s+\lambda+h(\theta_p)+h(\theta_t)]b(r+s)+\\ +(r+s)[r+h(\theta_p)][r+s+\lambda+h(\theta_p)+h(\theta_t)]h(\theta_p)w+\\ -rw[r+s+h(\theta_p)](r+s)h(\theta_t)-rw[r+s+h(\theta_p)]h(\theta_t)h(\theta_p)+\\ -rw[r+s+h(\theta_p)]h(\theta_p)[r+s+\lambda+h(\theta_p)]-[r+s+\lambda+h(\theta_p)+h(\theta_t)][h(\theta_p)swh(\theta_p)]<0;\end{aligned}$$

$$\begin{aligned}[r^2+rh(\theta_p)+rs][r+s+\lambda+h(\theta_p)+h(\theta_t)]b(r+s)+\\ +[r^2+rh(\theta_p)+rs][r+s+\lambda+h(\theta_p)+h(\theta_t)]h(\theta_p)w-rw[r+s+h(\theta_p)](r+s)h(\theta_t)+\\ -rw[r+s+h(\theta_p)][h(\theta_t)h(\theta_p)]-rw[r+s+h(\theta_p)]h(\theta_p)[r+s+\lambda+h(\theta_p)]<0;\end{aligned}$$

$$\begin{aligned}[r^2+rh(\theta_p)+rs][r+s+\lambda+h(\theta_p)+h(\theta_t)]b(r+s)+\\ -w[r^3h(\theta_t)+2r^2sh(\theta_t)+rs^2h(\theta_t)+rsh(\theta_p)h(\theta_t)+r^2h(\theta_p)h(\theta_t)]<0;\end{aligned}$$

$$[r^2+rh(\theta_p)+rs][r+s+\lambda+h(\theta_p)+h(\theta_t)]b(r+s)-wrh(\theta_t)[(r+s)^2+h(\theta_p)(r+s)]<0;$$

$$[r+s+\lambda+h(\theta_p)+h(\theta_t)]b<wh(\theta_t)\Rightarrow$$

$$b<\frac{wh(\theta_t)}{[r+s+\lambda+h(\theta_p)+h(\theta_t)]} \tag{21}$$

that is the condition for the existence of a positive reservation outside option.

By equating $U_p(z)$ to $U_t(z)$ and solving for $z=R$, we are now in a position to determine its exact value:

$$\begin{aligned}\frac{(R+b)(r+s)+h(\theta_p)w}{r[r+s+h(\theta_p)]}=\frac{[r+s+\lambda+h(\theta_p)]R}{[r+h(\theta_p)][r+s+\lambda+h(\theta_t)+h(\theta_p)]}+\\ +\frac{\left\{(r+s)h(\theta_t)+h(\theta_t)h(\theta_p)+h(\theta_p)[r+s+\lambda+h(\theta_p)]\right\}w}{(r+s)[r+h(\theta_p)][r+s+\lambda+h(\theta_t)+h(\theta_p)]}+\frac{h(\theta_p)s[(R+b)(r+s)+h(\theta_p)w]}{r(r+s)[r+h(\theta_p)][r+s+h(\theta_p)]};\end{aligned}$$

Collecting terms with R and multiplying both sides by the common denominator one gets

$$\begin{aligned}(r+s)\left\{\begin{array}{c}(r+s)[r+h(\theta_p)][r+s+\lambda+h(\theta_p)+h(\theta_t)]-r[r+s+h(\theta_p)][r+s+\lambda+h(\theta_p)]+\\ -sh(\theta_p)[r+s+\lambda+h(\theta_p)+h(\theta_t)]\end{array}\right\}R=\\ =wr[r+s+h(\theta_p)]\left\{(r+s)h(\theta_t)+h(\theta_t)h(\theta_p)+h(\theta_p)[r+s+\lambda+h(\theta_p)]\right\}+\\ +[r+s+\lambda+h(\theta_p)+h(\theta_t)][h(\theta_p)sb(r+s)+h(\theta_p)sh(\theta_p)w]+\\ -[b(r+s)+h(\theta_p)w](r+s)[r+h(\theta_p)][r+s+\lambda+h(\theta_p)+h(\theta_t)];\end{aligned}$$

For simplicity we separately consider the two sides of the equation; starting from the rhs

$$w\left\{\begin{array}{c} r[r+s+h(\theta_p)](r+s)h(\theta_t)+r[r+s+h(\theta_p)]h(\theta_p)h(\theta_t)+r[r+s+h(\theta_p)]h(\theta_p)[r+s+\lambda+h(\theta_p)]+ \\ -h(\theta_p)[r+s+\lambda+h(\theta_t)+h(\theta_p)][r^2+rs+rh(\theta_p)] \end{array}\right\}+$$
$$-b(r+s)[r+s+\lambda+h(\theta_p)+h(\theta_t)][r^2+rs+rh(\theta_p)]=$$

$$w[r^2+rs+rh(\theta_p)](r+s)h(\theta_t)-b(r+s)[r^2+rs+rh(\theta_p)][r+s+\lambda+h(\theta_p)+h(\theta_t)]=$$
$$=[r^2+rs+rh(\theta_p)](r+s)\left\{h(\theta_t)w-b[r+s+\lambda+h(\theta_p)+h(\theta_t)]\right\}$$

The lhs in turn reads

$$(r+s)R\left\{[r^2+rs+rh(\theta_p)][r+s+\lambda+h(\theta_p)+h(\theta_t)]-[r^2+rs+rh(\theta_p)][r+s+\lambda+h(\theta_p)]\right\}=$$
$$=h(\theta_t)(r+s)[r^2+rs+rh(\theta_p)]$$

so that

$$\begin{aligned} Rh(\theta_t) &= h(\theta_t)w-b[r+s+\lambda+h(\theta_p)+h(\theta_t)] \Rightarrow \\ R &= w-b\frac{r+s+\lambda+h(\theta_p)+h(\theta_t)}{h(\theta_t)} \end{aligned}$$

which implies that $R<w$; moreover, under condition (21), $0<R<w$.

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