Uncertainty Shocks in a Model of Effective Demand*

Susano Basu[†] Brent Bundick[‡]

June 17, 2011

Preliminary and Incomplete

Abstract

This paper examines the role of uncertainty shocks in a one-sector, representative-agent dynamic stochastic general-equilibrium model. When prices are flexible, uncertainty shocks are not capable of producing realistic business-cycle comovements among key macro variables. However, with sticky prices (or, more generally, with countercyclical markups), uncertainty shocks can generate fluctuations that are consistent with business cycles. Monetary policy usually plays a key role in offsetting the negative impacts of uncertainty shocks. If the central bank is constrained by the zero lower bound, then monetary policy can no longer perform its usual stabilizing function and higher uncertainty has even more negative effects on the economy. We find that increased uncertainty about the future may indeed have played a significant role in worsening the Great Recession, which is consistent with statements by policymakers, economists, and the financial press.

JEL Classification: E32, E52

Keywords: Uncertainty Shocks, Monetary Policy, Sticky-Price Models

*We thank Nick Bloom, David Chapman, Julio Rotemberg and Christina Wang for helpful discussions, and seminar participants at the Federal Reserve Bank of Boston and Boston College for comments.

[†]Boston College and National Bureau of Economic Research. Email: basusd@bc.edu

[‡]Boston College. Email: bundickb@bc.edu

1 Introduction

Economists and the financial press often discuss uncertainty about the future as an important driver of economic fluctuations, and a key culprit in creating the Great Recession and the slow subsequent recovery. For example, Diamond (2010) says, "What's critical right now is not the functioning of the labor market, but the limits on the demand for labor coming from the great caution on the side of both consumers and firms because of the great uncertainty of what's going to happen next." Recent research by Bloom (2009), Bloom, Foetotto, and Jaimovich (2010), and Gilchrist, Sim, and Zakrajšek (2010) also suggests that uncertainty shocks can cause fluctuations in macroeconomic aggregates. However, these papers experience difficulty in generating business cycle comovements among output, consumption, investment, and hours worked from changes in uncertainty. If uncertainty is a contributing factor in the Great Recession and persistently slow recovery, as Diamond suggests, then increased uncertainty should reduce output and its components.

In this paper, we show why competitive, one-sector closed-economy models generally cannot generate business cycles in response to changes in uncertainty. An increase in uncertainty induces precautionary saving and, *ceteris paribus*, lower consumption. If households supply labor inelastically, then total output remains constant since the level of technology and capital stock do not change in response to the uncertainty shock. Unchanged total output and reduced consumption together imply that investment must rise. If households can adjust their labor supply and consumption and leisure are both normal goods, an increase in uncertainty also induces "precautionary labor supply," or a desire for the household to supply more labor for an given level of the real wage. As current technology and the capital stock remain unchanged, the competitive demand for labor remains unchanged as well. Thus, higher uncertainty reduces consumption but raises output, investment, and hours worked. This lack of comovement is a robust prediction of simple neoclassical models subject to uncertainty fluctuations.

We also show that non-competitive, one-sector models with sticky prices can easily overcome the comovement problem and generate simultaneous drops in output, consumption, investment, and hours worked in response to an uncertainty shock. (This model is an example of a larger class of models characterized by countercyclical markups of price over marginal cost, which is the key necessary feature for generating our results. In Section 5, we discuss our reasons for choosing this particular mechanism for generating markup cyclicality.) An increase in uncertainty induces precautionary labor supply by the representative household, which reduces the marginal costs of production. Firm markups increase when marginal costs fall and prices adjust slowly. The higher markup over marginal cost reduces the demand for consumption, and especially, investment goods. Since output is demand-determined in these models, output and employment must fall when consumption and investment both decline.

Thus, comovement is restored, and uncertainty shocks cause fluctuations that look qualitatively like a business cycle. Returning to Diamond's (2010) intuition, simple competitive business-cycle models do not exhibit movements in "the demand for labor" as a result of an uncertainty shock. However, uncertainty shocks easily cause fluctuations in the demand for labor in non-competitive, sticky price models with endogenously-varying markups. Thus, the non-competitive model captures the intuition articulated by Diamond, and this fact is the key to understanding why the two models behave so differently in response to a change in uncertainty.

To formally analyze the quantitative impact of uncertainty shocks under flexible and sticky prices, we calibrate and solve a representative-agent, dynamic stochastic general equilibrium model with nominal price rigidity. We examine uncertainty shocks to both technology and household preferences, which we interpret as cost and demand uncertainty. We calibrate our uncertainty shock processes using the Chicago Board Options Exchange Volatility Index (VIX), which measures the expected volatility of the Standard and Poor's 500 stock index over the next thirty days. Using a third-order approximation to the policy functions of our calibrated model, we show that uncertainty shocks can produce contractions in output and all its components when prices are sticky. In particular, we find increased uncertainty associated with future demand can produce significant declines in output, hours, consumption, and investment. Our model predicts that a one-standard deviation increase in the uncertainty about future demand produces a peak decline in output of approximately 0.3 percentage points.

Finally, we examine the role of monetary policy in determining the equilibrium effects of uncertainty shocks. Monetary policy usually offsets increases in uncertainty by lowering the policy rate. We show that as a consequence increases in uncertainty have larger negative impacts on the economy if the monetary authority is constrained by the zero lower bound on nominal interest rates. Our model predicts that under these circumstances an increase in uncertainty causes a much larger and more persistent decline in output and its components. The sharp increase in uncertainty during the financial crisis in late 2008 corresponds to a period when the Federal Reserve had a policy rate near zero. Thus, we believe that greater uncertainty may have plausibly contributed significantly to the large and persistent output decline starting at that time.

2 Intuition

This section formalizes the intuition from the introduction using a few key equations that characterize a large class of one-sector business cycle models. We show that the causal ordering of these equations plays an important role in understanding the impact of uncertainty shocks. These equations link total output Y_t, household consumption C_t, investment I_t, hours worked N_t, and the real wage W_t/P_t. The key equations consist of a "demand" equation:

$$Y_t = C_t + I_t, \tag{1}$$

an aggregate production function:

$$Y_t = F(K_t, Z_t N_t), \tag{2}$$

and a static first-order condition for a representative consumer to maximize utility:

$$\frac{W_t}{P_t} U_1(C_t, 1 - N_t) = U_2(C_t, 1 - N_t). \tag{3}$$

Equation (1) suggests that if an increase in uncertainty lowers consumption and investment, then it should also lower total output. Higher uncertainty induces precautionary saving by risk averse households. An increase in uncertainty also depresses investment, particularly in the presence of non-convex costs of adjustment. In a setting where output is demand-determined, economic intuition suggests that higher uncertainty should depress total output and its components.

However, the previous intuition is incorrect in a neoclassical model with a representative consumer and firm. In this neoclassical setting, labor demand (the partial derivative of (2) with respect to N_t) is determined by the current level of capital and technology, neither of which changes when uncertainty increases. The first-order conditions for firm labor demand derived from equation (2) and the labor supply condition in equation (3) can be combined to yield:

$$Z_t F_2(K_t, Z_t N_t) U_1(C_t, 1 - N_t) = U_2(C_t, 1 - N_t). \tag{4}$$

Equation (4) defines a positively-sloped "income expansion path" for consumption and leisure for given level of capital and technology. Thus, if higher uncertainty does indeed reduce consumption, it must increase labor supply. However, equation (2) implies that total output must rise, means that investment and consumption must move in opposite directions according to equation (1).

In a non-neoclassical setting, especially one with a time-varying markup of price over marginal cost, equations (1) and (3) continue to apply, but equation (4) must be modified such that:

$$\frac{1}{\mu_t} Z_t F_2(K_t, Z_t N_t) U_1(C_t, 1 - N_t) = U_2(C_t, 1 - N)_t \tag{5}$$

where μ_t is the markup of price over marginal cost.

In such a setting, equation (1) is causally prior to (2) and (3). From (1), output is determined by aggregate demand. Then, for given values of K and Z, (2) determines the necessary quantity of labor input. Finally, given C (determined by demand and other factors), the necessary supply of labor is made consistent with consumer optimization by having the markup taking on its required value. (Alternatively, the wage moves to the level necessary for firms to hire the required quantity of labor, and the markup ensures that the wage can move independently of the marginal product of labor.)

3 Model

This section outlines the baseline dynamic stochastic general equilibrium model that we use in our analysis of uncertainty shocks. Our model provides a specific quantitative example of the intuition of the previous section. The baseline model shares many features with the models of Ireland (2003), Ireland (2010), and Jermann (1998). The model features optimizing households and firms and a central bank that systematically adjusts the nominal interest rate to offset adverse shocks in the economy. We allow for sticky prices using the quadratic-adjustment costs specification of Rotemberg (1982). Our baseline model considers both technology shocks and household discount rate shocks. Both shocks are allowed to have time-varying second moments, which have the interpretation of cost uncertainty and demand uncertainty.

3.1 Households

In our model, the representative household maximizes expected lifetime utility from consumption C_t and leisure $1 - N_t$ subject to its intertemporal budget constraint. The household receives labor income W_t for each unit of labor N_t supplied in the representative intermediate goods-producing firm. The representative household also owns the intermediate goods firm and holds equity shares S_t and one-period risk-less bonds B_t issued by representative intermediate goods firm. Equity shares pay dividends D_t^E for each share S_t owned, and the risk-less bonds return the gross one-period risk-free interest rate R_t^R. The household divides its income from labor and its financial assets between consumption C_t and the amount of financial assets S_{t+1} and B_{t+1} to carry into next period. The discount rate of the household β is subject to shocks via the stochastic process a_t, which we interpret as demand shocks for the economy.

The representative household maximizes lifetime utility by choosing C_{t+s}, N_{t+s}, B_{t+s+1}, and S_{t+s+1} for all $s = 0, 1, 2, \ldots$ by solving the following problem:

$$\max\, E_t \left[\sum_{s=0}^{\infty} \beta^s a_{t+s} \frac{C_{t+s}^{1-\sigma} (1 - N_{t+s})^{\eta(1-\sigma)}}{1-\sigma} \right]$$

subject to the intertemporal household budget constraint each period,

$$C_t + \frac{P_t^E}{P_t} S_{t+1} + \frac{1}{R_t^R} B_{t+1} \leq \frac{W_t}{P_t} N_t + \left(\frac{D_t^E}{P_t} + \frac{P_t^E}{P_t} \right) S_t + B_t,$$

Household optimization implies the following first-order conditions:

$$a_t C_t^{-\sigma}(1-N_t)^{\eta(1-\sigma)} = \lambda_t \tag{6}$$

$$\eta \frac{C_t}{(1-N_t)} = \frac{W_t}{P_t} \tag{7}$$

$$\frac{P_t^E}{P_t} = E_t\left\{\left(\frac{\beta\lambda_{t+1}}{\lambda_t}\right)\left(\frac{D_{t+1}}{P_{t+1}} + \frac{P_{t+1}^E}{P_{t+1}}\right)\right\} \tag{8}$$

$$1 = R_t^R E_t\left\{\left(\frac{\beta\lambda_{t+1}}{\lambda_t}\right)\right\} \tag{9}$$

3.2 Final Goods Producers

The representative final goods producer uses $Y_t(i)$ units of each intermediate good produced by the intermediate goods-producing firm $i \in [0,1]$. The intermediate output is transformed into final output Y_t using the following constant returns to scale technology:

$$\left[\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}} \geq Y_t$$

Each intermediate good $Y_t(i)$ sells at nominal price $P_t(i)$ and each final good sells at nominal price P_t. The finished goods producer chooses Y_t and $Y_t(i)$ for all $i \in [0,1]$ to maximize the following expression of firm profits:

$$P_t Y_t - \int_0^1 P_t(i) Y_t(i) di$$

subject to the constant returns to scale production function. Finished goods-producer optimization results in the following first-order condition:

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\theta} Y_t$$

The market for final goods is perfectly competitive, and thus the final goods-producing firm earns zero profits in equilibrium. Using the zero-profit condition, the first-order condition for profit maximization, and the firm objective function, the aggregate price index P_t can be written as follows:

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}}$$

3.3 Intermediate Goods Producers

Intermediate goods-producing firms rent labor $N_t(i)$ from the representative household in order to produce intermediate goods $Y_t(i)$. Intermediate goods are produced in a monopolistically competitive market where producers face a quadratic cost of changing their nominal price $P_t(i)$ each period. The Intermediate-goods firms own the capital stock for the economy and issues equity shares $S_t(i)$ and one-period risk-less bonds $B_t(i)$. Firm i chooses $N_t(i), I_t(i)$, and $P_t(i)$ to maximize firm cash flows $D_t(i)/P_t(i)$ given aggregate demand Y_t and price P_t of the finished goods sector. The intermediate goods firms all have access to the same constant returns-to-scale Cobb-Douglas production function, subject to a fixed cost of production Φ.

Each intermediate goods-producing firm solves the following problem:

$$\max E_t \sum_{s=0}^{\infty} \beta^s \left(\frac{\lambda_{t+s}}{\lambda_t}\right) \left[\frac{D_{t+s}(i)}{P_{t+s}}\right]$$

subject to the production function:

$$\left[\frac{P_t(i)}{P_t}\right]^{-\theta} Y_t \leq K_t(i)^{\alpha} \left[Z_t N_t(i)\right]^{1-\alpha} - \Phi$$

where

$$\frac{D_t(i)}{P_t} = \left[\frac{P_t(i)}{P_t}\right]^{-\theta} Y_t - \frac{W_t}{P_t} N_t(i) - I_t(i) - \frac{\phi_P}{2}\left[\frac{P_t(i)}{\Pi P_{t-1}(i)} - 1\right]^2 Y_t$$

The first-order conditions for the firm i are as follows:

$$\frac{W_t}{P_t} N_t(i) = (1-\alpha)\Xi_t K_t(i)^{\alpha} \left[Z_t N_t(i)\right]^{1-\alpha} \tag{10}$$

$$\frac{R_t^K}{P_t} K_t(i) = \alpha \Xi_t K_t(i)^{\alpha} \left[Z_t N_t(i)\right]^{1-\alpha} \tag{11}$$

$$\begin{aligned} \phi_P \left[\frac{P_t(i)}{\Pi P_{t-1}(i)} - 1\right]\left[\frac{P_t}{\Pi P_{t-1}(i)}\right] = (1-\theta)\left[\frac{P_t(i)}{P_t}\right]^{-\theta} + \theta \Xi_t \left[\frac{P_t(i)}{P_t}\right]^{-\theta-1} \\ + \beta \phi_P E_t \left\{\frac{\lambda_{t+1}}{\lambda_t}\frac{Y_{t+1}}{Y_t}\left[\frac{P_{t+1}(i)}{\Pi P_t(i)} - 1\right]\left[\frac{P_{t+1}(i)}{\Pi P_t(i)}\frac{P_t}{P_t(i)}\right]\right\} \end{aligned} \tag{12}$$

$$1 = E_t \left\{\left(\frac{\beta \lambda_{t+1}}{\lambda_t}\right)\left(\frac{R_{t+1}^K + q_{t+1}(1-\delta)}{q_t}\right)\right\} \tag{13}$$

$$\begin{aligned} \lambda_t = & \lambda_t q_t \left[1 - \frac{\phi_I}{2}\left(\frac{I_t(i)}{I_{t-1}(i)} - 1\right)^2 - \phi_I \left(\frac{I_t(i)}{I_{t-1}(i)} - 1\right)\left(\frac{I_t(i)}{I_{t-1}(i)}\right)\right] \\ & + \beta E_t \left\{\lambda_{t+1} q_{t+1}\left[\phi_I \left(\frac{I_{t+1}(i)}{I_t(i)} - 1\right)\left(\frac{I_{t+1}(i)}{I_t(i)}\right)^2\right]\right\} \end{aligned} \tag{14}$$

where Ξ_t is the marginal cost of producing one additional unit of intermediate good i, and q_t is the price of capital.

Each intermediate goods firm finances a percentage ν of its capital stock each period with one-period risk-less bonds. The bonds pay the one-period real risk-free interest rate. Thus, the quantity of bonds $B_t = \nu K_t$. Total firm cash flows are divided between payments to bond holders and equity holders as follows:

$$\frac{D_t^E(i)}{P_t} = \frac{D_t(i)}{P_t} - \nu\left(K_t(i) - \frac{1}{R_t^R}K_{t+1}(i)\right). \tag{15}$$

The Modigliani & Miller (1963) theorem holds in our model and thus leverage does not affect firm value or optimal firm decisions. Leverage simply makes the payouts and price of equity more volatile. In equilibrium, our leverage ratio of debt to total firm value is approximately 50%.

3.4 Monetary Policy

We assume a cashless economy where the monetary authority sets the nominal interest rate to stabilize inflation and output growth. Monetary policy adjusts the nominal interest rate in accordance with the following rule:

$$\ln(R_t) = \rho_r \ln(R_{t-1}) + (1-\rho_t)(\ln(R) + \rho_\Pi \ln(\Pi_t/\Pi) + \rho_y \ln(Y_t/Y_{t-1})) \tag{16}$$

3.5 Equilibrium

In the symmetric equilibrium, all intermediate goods firms choose the same price $P_t(i) = P_t$, employ the same amount of labor $N_t(i) = N_t$, and choose to hold the same amount of capital $K_t(i) = K_t$. Thus, all firms have the same cash flows and payout structure between bonds and equity. Thus, we can define inflation as $\Pi_t = P_{t+1}/P_t$, and define the markup over marginal cost as $\mu_t = 1/\Xi_t$. Thus, we can model our intermediate-goods firms with a single representative intermediate goods-producing firm.

3.6 Shock Processes

In our baseline model, we are interested in capturing the effects of independent changes in the level and volatility of both the technology process and the preference shock process. The technology and preference shock processes are parameterized as follows, which allows us to examine both first and second moment shocks separately:

$$\ln(Z_t) = \rho_z \ln(Z_t) + \sigma_t^z \varepsilon_t^z, \quad \varepsilon_t^z \sim N(0,1)$$

$$\ln(\sigma_t^z) = (1-\rho_{\sigma^z})\ln(\sigma^z) + \rho_{\sigma^z}\ln(\sigma_{t-1}^z) + \sigma^{\sigma^z}\varepsilon_t^{\sigma^z} \quad \varepsilon_t^{\sigma^z} \sim N(0,1).$$

$$\ln(a_t) = \rho_a \ln(a_t) + \sigma_t^a \varepsilon_t^a, \quad \varepsilon_t^a \sim N(0,1)$$

$$\ln(\sigma_t^a) = (1-\rho_{\sigma^a})\ln(\sigma^a) + \rho_{\sigma^a}\ln(\sigma_{t-1}^a) + \sigma^{\sigma^a}\varepsilon_t^{\sigma^a} \quad \varepsilon_t^{\sigma^a} \sim N(0,1).$$

3.7 Solution Method

Our primary focus of this paper is to examine the effects of increases in the second moments of the shock processes. Using a standard first-order or log-linear approximation to all the equilibrium conditions of our model would not allow us to examine second moment shocks since the approximated policy functions are invariant of the volatility of the shock process. Alternatively, second moment shocks would only enter as cross-products with the other state variables in a 2nd-order approximation to the policy functions. In a 3rd-order approximation, however, second moment shocks enter independently in the approximated policy functions. Thus, a 3rd-order approximation allows us to compute an impulse response to an increase in the volatility of technology or discount rate shocks, while holding constant the levels of those variables.

To solve the baseline model, we use the Perturbation AIM algorithm and software developed by Swanson, Anderson, and Levin (2006), which is available on Eric Swanson's webpage. Perturbation AIM uses Mathematica to compute the rational expectations solution to the model using nth-order Taylor series approximation around the nonstochastic steady state of the model. Similarly to the findings of Fernàndez-Villaverde, Guerròn-Quintana, Rubio-Ramìrez, and Uribe (2010), we find that a 3rd-order approximation to the policy functions is sufficient to capture the dynamics of the model, and we find little gain to using an approximation higher than 3rd-order.

4 Calibration and Baseline Results

4.1 Calibration

Table 1 lists the calibrated parameters of the model. We calibrate the model using standard parameters for one-sector models of fluctuations. Since our model shares many features with the estimated models of Ireland (2003) and Ireland (2010), we calibrate our model to match the estimated parameters of reported by those papers. We calibrate our investment adjustment costs parameter to match the value of Christiano, Eichenbaum, and Evans (2005). We calibrate the steady-state volatilities for the technology and preference shocks, σ^a and σ^z, in line with the findings of Ireland (2003) and Ireland (2010). We discuss our calibration of the uncertainty shocks in depth in section 6. In the following analysis, we compare the results from our baseline sticky-price calibration ($\phi_P = 160$) with a flexible-price calibration ($\phi_P = 0$), leaving all other parameters unchanged.

4.2 Uncertainty Shocks & Business Cycle Comovements

Holding the calibrated parameters fixed, we analyze the effects of an exogenous increase in uncertainty associated with both technology and household demand. Figure 3 plots impulse responses of the model to an technology uncertainty shock and Figure 4 plots the responses to a demand uncertainty shock. The results are consistent with the intuition of section 2 and the labor market diagrams in Figures 1 and 2. Uncertainty with either technology or household demand both enter equation (4) or equation (5) through the marginal utility of wealth. An uncertainty shock associated with either stochastic process induces wealth effects on the household and induces precautionary labor supply. Thus, the responses and time paths for the endogenous variables look qualitatively similar for both types of uncertainty shocks.

Households want to consume less and save more when uncertainty increases in the economy. In order to save more, households optimally wish to both reduce consumption and increase hours worked. Under flexible prices and constant markups, equilibrium labor supply and consumption follow the path that households desire when they face higher uncertainty. On impact of the uncertainty shock, the level of capital is predetermined, the level of the shock is held constant, and thus labor demand (for a given wage) is unchanged. Consequently, under flexible prices, the outward shift in labor supply combined with unchanged labor demand increases hours worked and output. After the impact period, households continue to save, accumulate more capital, consume less, and work more hours. Throughout the life of the uncertainty shock, consumption and investment move in opposite directions, which is inconsistent with basic business cycle comovements.

Under sticky prices, households also want to consume less and save more when the economy is hit by an uncertainty shock associated with technology or household demand. On impact, households increase their labor supply and reduce consumption to accumulate more assets. With sticky prices, however, increased labor supply decreases the marginal costs of production of the intermediate goods firms. A reduction in marginal cost with slowly-adjusting prices increases firm markups. An increase in markups lowers labor demand, which lowers the demand for household labor and lowers the real wage earned by the representative household. The decrease in labor demand also lowers the demand for capital by firms. Thus, consumers find it optimal to help smooth consumption by disinvesting in the capital stock. In equilibrium, these effects combine to produce significant and hump-shaped falls in output, consumption, investment, hours worked, and the real wage, which are consistent with business cycle facts. Thus, the desire by households to work more can actually lead to lower labor input and output in equilibrium.

For completeness, we also plot the responses to the first-moment technology and household preference shocks in Figure 4 and 5. As a check on our calibration strategy, we compare the impulse responses of the model to a first-moment technology shock to the empirical impulse responses to the same shock estimated by Basu, Fernald, and Kimball (2006). We find that the calibration of our baseline model produces impulse responses to first-moment technology shocks that are consistent with the estimates of Basu, Fernald, and Kimball (2006), which provides some evidence that our calibration is reasonable.

5 Discussion and Connections

TO BE ADDED

6 Quantitative and Timely Considerations

6.1 Uncertainty Shock Calibration

The intuition laid out in Sections 1 and 2, and the previous qualitative results suggest that uncertainty shocks can produce declines in output and its components when prices adjust slowly. This section uses the previous sticky price model to determine if uncertainty shocks are quantitatively important for business cycle fluctatutions. A related issue is determining the proper calibration of our shock processes for the uncertainty shocks associated with technology and household demand. The transmission of uncertainty to the macroeconomy in our model crucially depends on the calibration of the size and persistence of the uncertainty shock processes. However, aggregate uncertainty shocks are an *ex ante* and difficult to observe concept, which may be difficult to measure using *ex post* economic data. To ensure our calibration of an unobservable process is reasonable, we want our model and uncertainty shock processes to be consistent with a well-known and observable measure of aggregate uncertainty.

We choose the Chicago Board Options Exchange Volatility Index (VIX) as our observable measure of aggregate uncertainty due to its significant in financial markets, ease of observability, and the ability to generate a model counterpart. The VIX is a forward-looking indicator of the expected volatility of the Standard and Poor's 500 stock index. To match the frequency our of model, we aggregate a monthly series to quarterly frequency by averaging over the three months in each quarter. The top panel of Figure 7 plots our quarterly VIX series. Using our VIX data series, denoted V_t^D, we estimate the simple reduced-form autoregressive time series model:

$$\ln(V_t^D) = (1-\rho_V)\ln(V^D) + \rho_V \ln(V_{t-1}^D) + \sigma^{V^D}\varepsilon_t^{V^D}, \quad \varepsilon_t^{V^D} \sim N(0,1). \tag{17}$$

The ordinary least squares regression results are $V^D = 20.4\%$, $\rho_V = 0.83$, and $\sigma^{V^D} = 0.19$ with an $R^2 = 0.68$. Using our reduced-form model, we can also compute the VIX-implied uncertainty shocks as the series of standardized regression residuals $\varepsilon_t^{V^D}$. A typical one-standard deviation VIX-implied uncertainty shock increases the VIX by 19 percentage points. Compared to its sample average of

20.4%, a one-standard deviation uncertainty shock raises the level of the VIX to 24.27%. The bottom plot of Figure 7 shows the time series of the VIX-implied uncertainty shocks. We use this reduced-form time-series model for the quarterly VIX series to ensure that our calibration for our technology and demand uncertainty shocks is reasonable.

Using a third-order approximation, we compute a model-implied VIX index as the expected conditional volatility of the return on the equity of the representative intermediate-goods producing firm. Formally, we define our model-implied VIX V_t^M as follows:

$$V_t^M = 100 * \sqrt{4 * Var_t \left(R_{t+1}^E\right)}, \tag{18}$$

where $Var_t R_{t+1}^E$ is the quarterly conditional variance of the equity return. We annualize the quarterly conditional variance, and then transform the annual volatility units into percentage points. Using hat-notation to denote percentage deviations from the steady-state, we can write the model-implied VIX as follows using our third-order approximation:

$$\hat{V}_t^M = \ldots + \eta^{\sigma^a} \hat{\sigma}_{t-1}^a + \eta^{\varepsilon^a} \varepsilon_t^{\sigma^a} + \eta^{\sigma^Z} \hat{\sigma}_{t-1}^Z + \eta^{\varepsilon^Z} \varepsilon_t^{\sigma^Z}, \tag{19}$$

where we use the ellipsis as a place holder for the other state variables (a_{t-1}, I_{t-1}, K_{t-1}, R_{t-1}, Y_{t-1}, Z_{t-1}, ε_t^a, ε_t^Z) and their respective coefficients. Thus, conditional on the values of the state variables, our model-implied VIX has an AR(1) representation in each of the two types of uncertainty shocks. We choose our calibrated parameters of our uncertainty shock process such that a one-standard deviation uncertainty shock to either technology or household demand generate a conditional AR(1) representation that matches our reduced-form model for the VIX in the data. For example, a one-standard deviation shock to technology or household demand produces a 19 percentage point increase in the model-implied VIX and has a first-order autoregressive term of 0.83.

Figure 3 and 4 show the impact of our calibrated uncertainty shock process on the endogenous variables of the sticky price model. Section 4.2 shows that the responses are qualitatively similar for both technology and household demand uncertainty shocks. In this section, we analyze the quantitative differences between technology and household demand uncertainty shocks. The middle plot in the bottom row of both Figures 3 and 4 show that both uncertainty shocks produce a similar law of motion in the model-implied VIX, which approximately matches the reduced-form VIX model. The bottom right plot of each figure shows that the percentage increase in the volatility of the exogenous shocks to generate the same movement in the model-implied VIX differs between technology and household demand shocks. Household preference shocks require an 88 percent increase in volatility to produce the same movement in the VIX as a 42 percent increase in the volatility of technology.

In addition, the quantitative transmission of uncertainty to the macroeconomy differs greatly between the technology and household demand uncertainty shocks. A one-standard deviation technology uncertainty shock generates a peak drop in output of 0.02 percentage points. The size of this peak drop is output is very small in comparison to the movements of a first-moment technology shock in Figure 5. However, a one-standard deviation household demand uncertainty shock produces a 0.3 percentage point peak drop in output. Household demand uncertainty shocks can cause fluctuations in output and its components that can be quantitatively significant.

7 Conclusions and Extensions

TO BE ADDED

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Table 1: Baseline Calibration

Parameter	Description	Calibrated Value
α	Capital's Share in Production	0.333
β	Household Discount Factor	0.9987
δ	Depreciation Rate	0.025
η	Household Labor Supply	2.90
ϕ_I	Adjustment Cost to Changing Investment	2.5
ϕ_P	Adjustment Cost to Changing Prices	160.0
Π	Steady State Inflation Rate	1.0062
ρ_r	Central Bank Interest Rate Smoothing Coefficient	0.50
ρ_Π	Central Bank Reaction Coefficient on Inflation	1.50
ρ_y	Central Bank Reaction Coefficient on Output Growth	0.50
σ	Parameter Affecting Household Risk Aversion	2.0
θ	Elasticity of Substitution Intermediate Goods	6.0
ρ_a	First Moment Preference Shock Persistence	0.90
ρ_{σ^a}	Second Moment Preference Shock Persistence	0.83
σ_a	Steady-State Volatility of Preference Shock	0.03
σ_{σ^a}	Volatility of Second Moment Preference Shocks	0.88
ρ_z	First Moment Technology Shock Persistence	0.99
ρ_{σ^z}	Second Moment Technology Shock Persistence	0.83
σ_z	Steady-State Volatility of Technology	0.01
σ_{σ^z}	Volatility of Second Moment Technology Shocks	0.42

Figure 1: Flexible Price Model Intuition

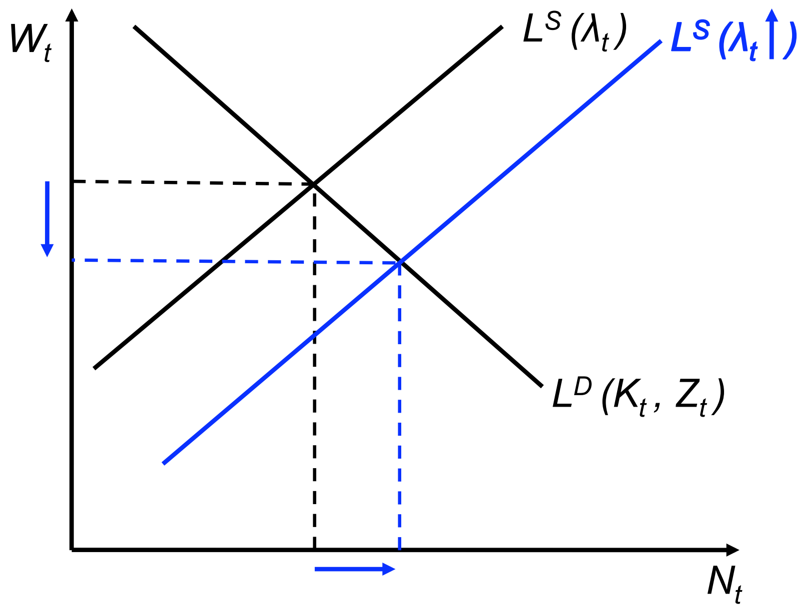


Figure 2: Sticky Price Model Intuition

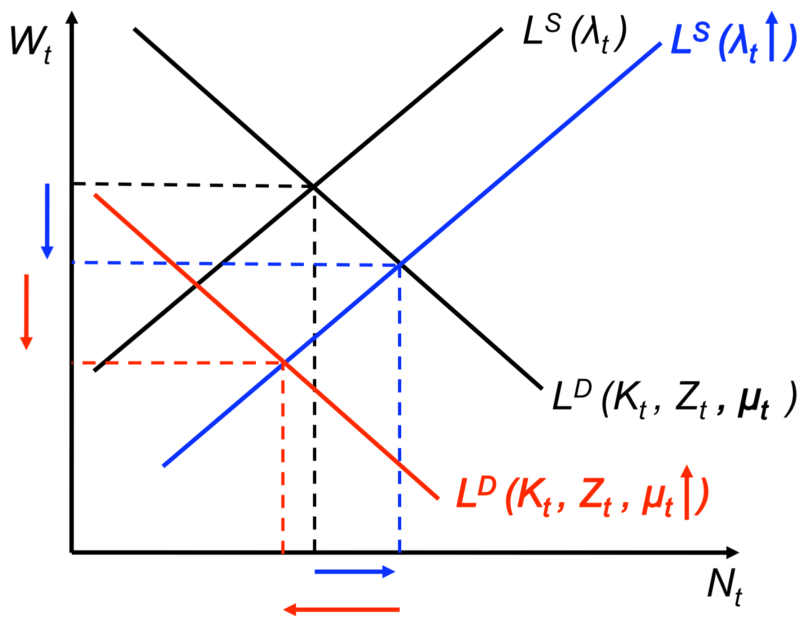
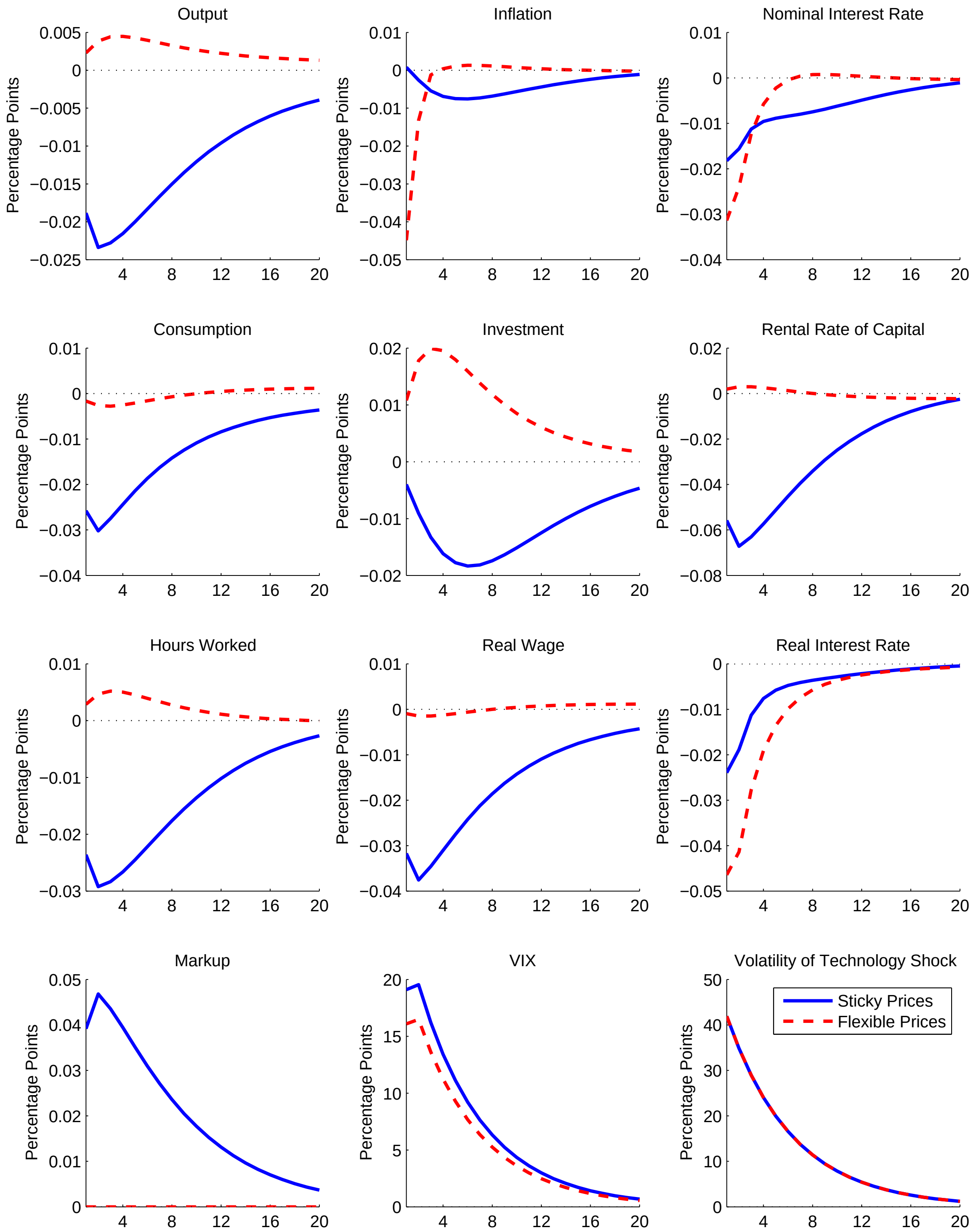
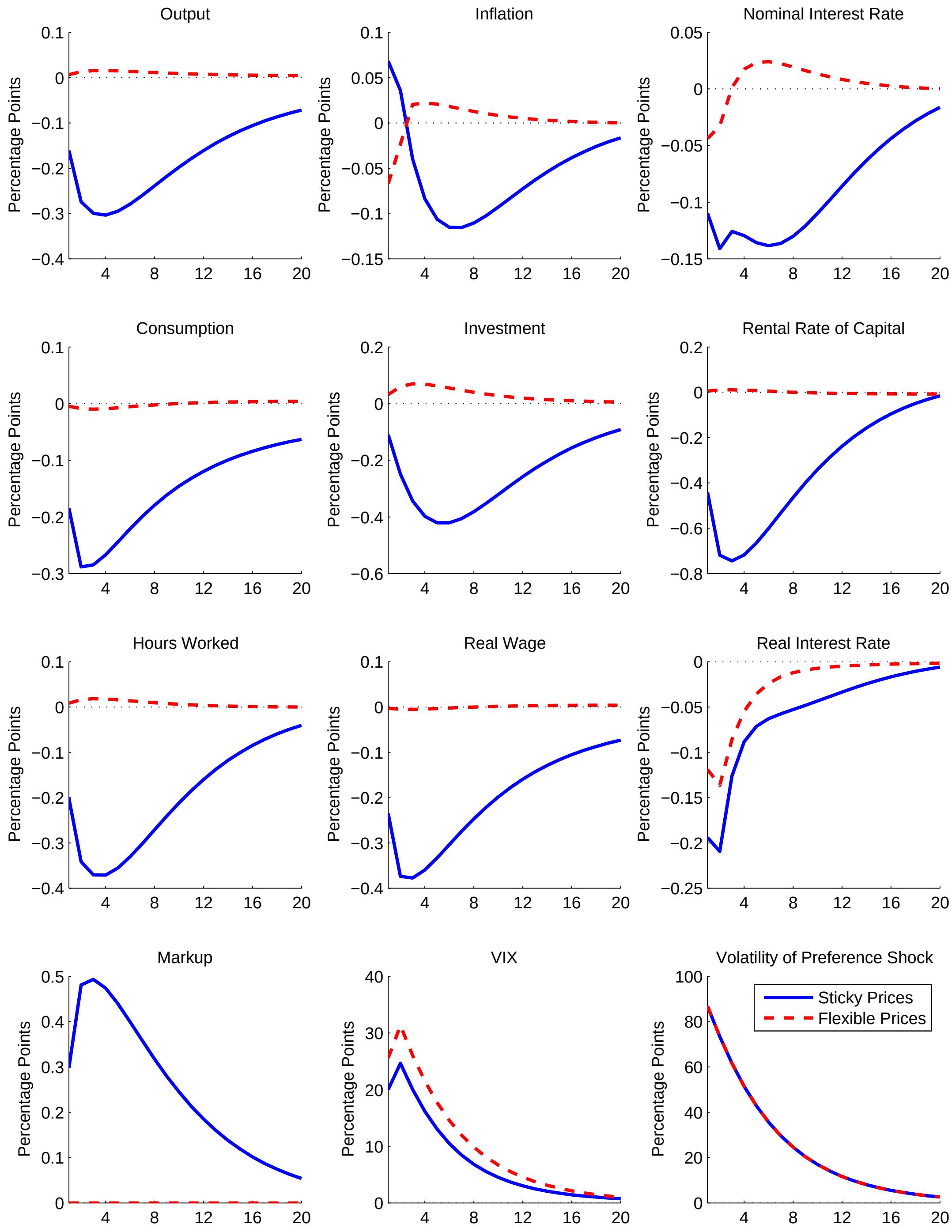


Figure 3: Impulse Responses to Second Moment Technology Shock



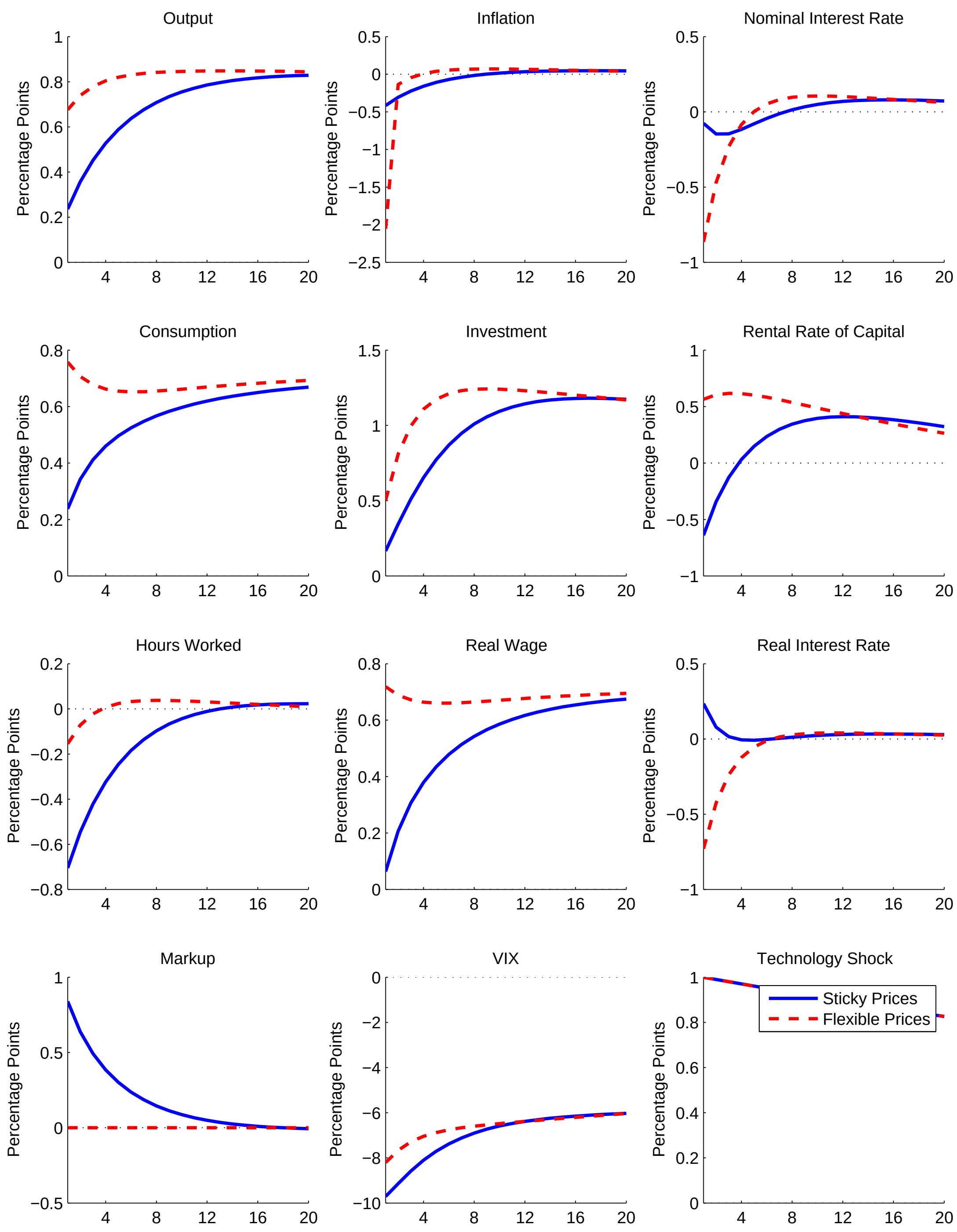
Note: Impulse responses are plotted as percentage deviations from their ergodic mean.

Figure 4: Impulse Responses to Second Moment Preference Shock



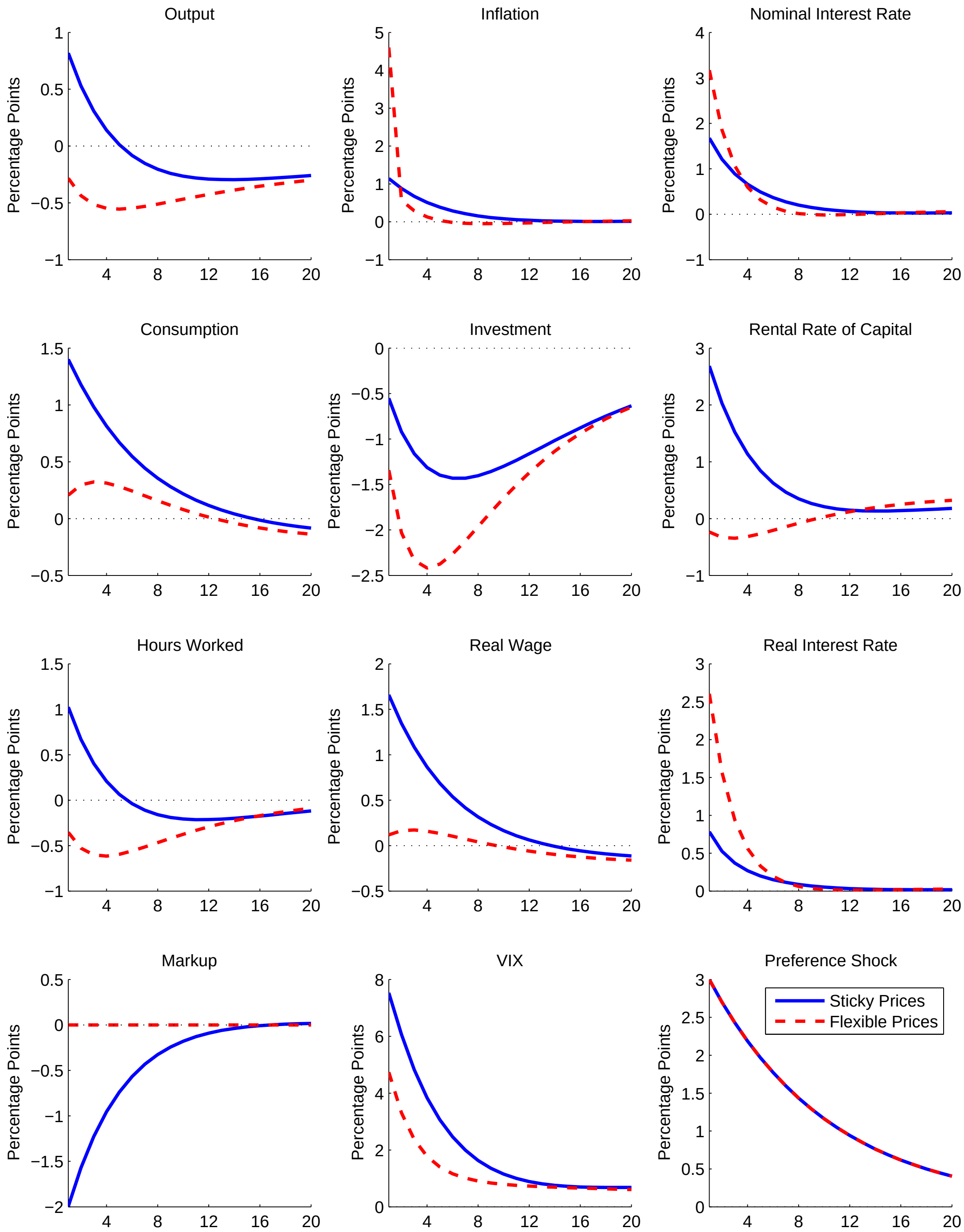
Note: Impulse responses are plotted as percentage deviations from their ergodic mean.

Figure 5: Impulse Responses to First Moment Technology Shock



Note: Impulse responses are plotted as percentage deviations from their ergodic mean.

Figure 6: Impulse Responses to First Moment Preference Shock



Note: Impulse responses are plotted as percentage deviations from their ergodic mean.

Figure 7: VIX and VIX-Implied Uncertainty Shocks

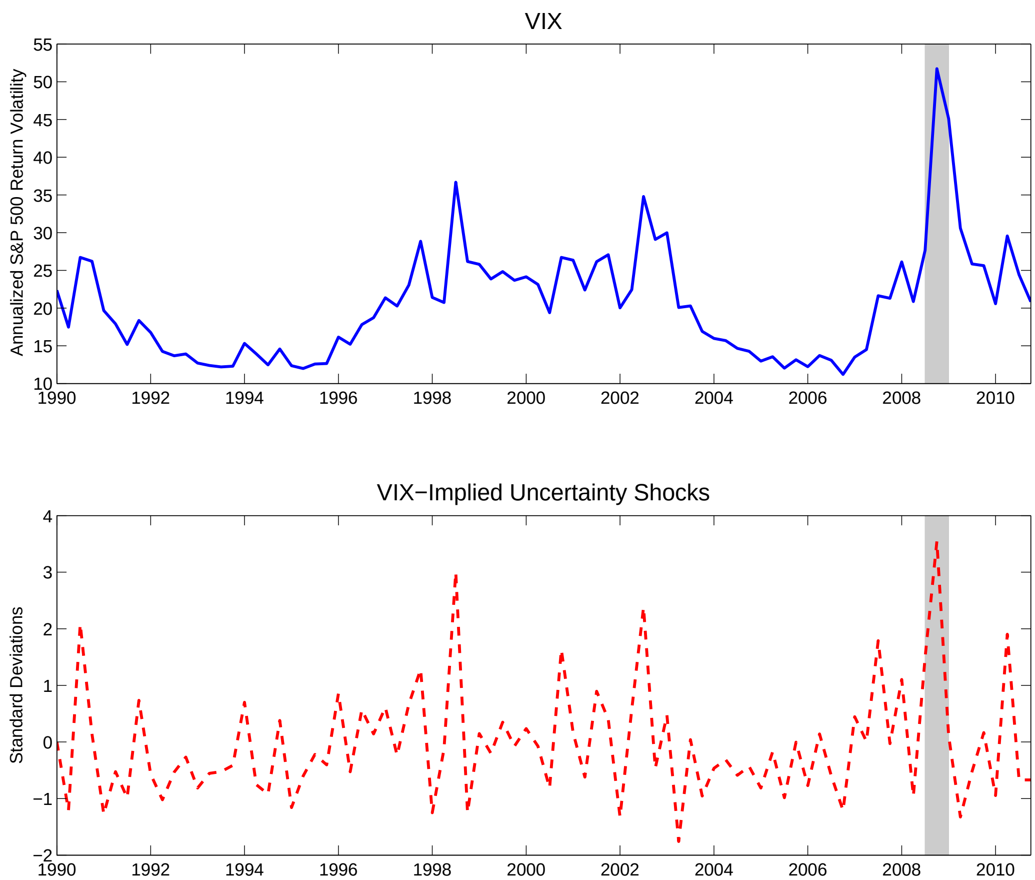


Figure 8: Impulse Responses to Second Moment Preference Shock at ZLB