

# **Getting Back to Basics: The Why and How of Statistical Disclosure Limitation vs. Privacy Protection**

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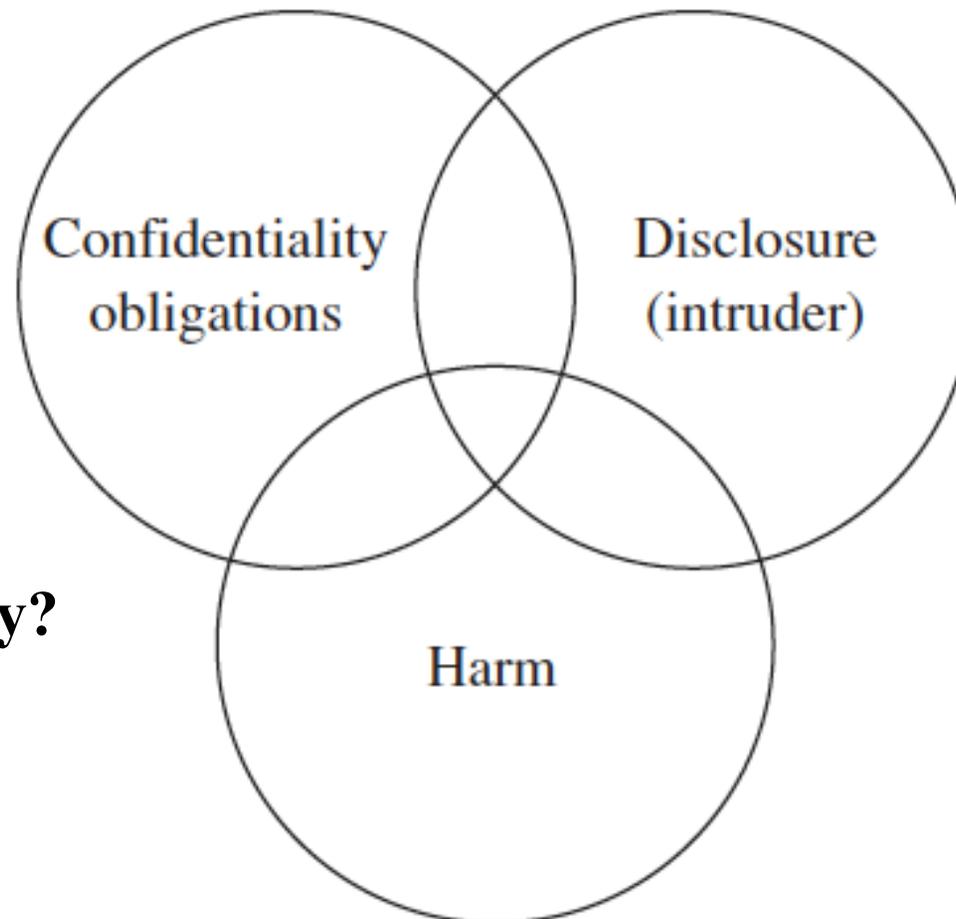
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# Disclosure Limitation, Confidentiality & Harm



**Where is privacy?**

# Privacy vs. Confidentiality

**Privacy**

**Don't ask.**

**Confidentiality**

**Don't tell.**

**Prewitt, 2011**

# **Do you agree/disagree with:**

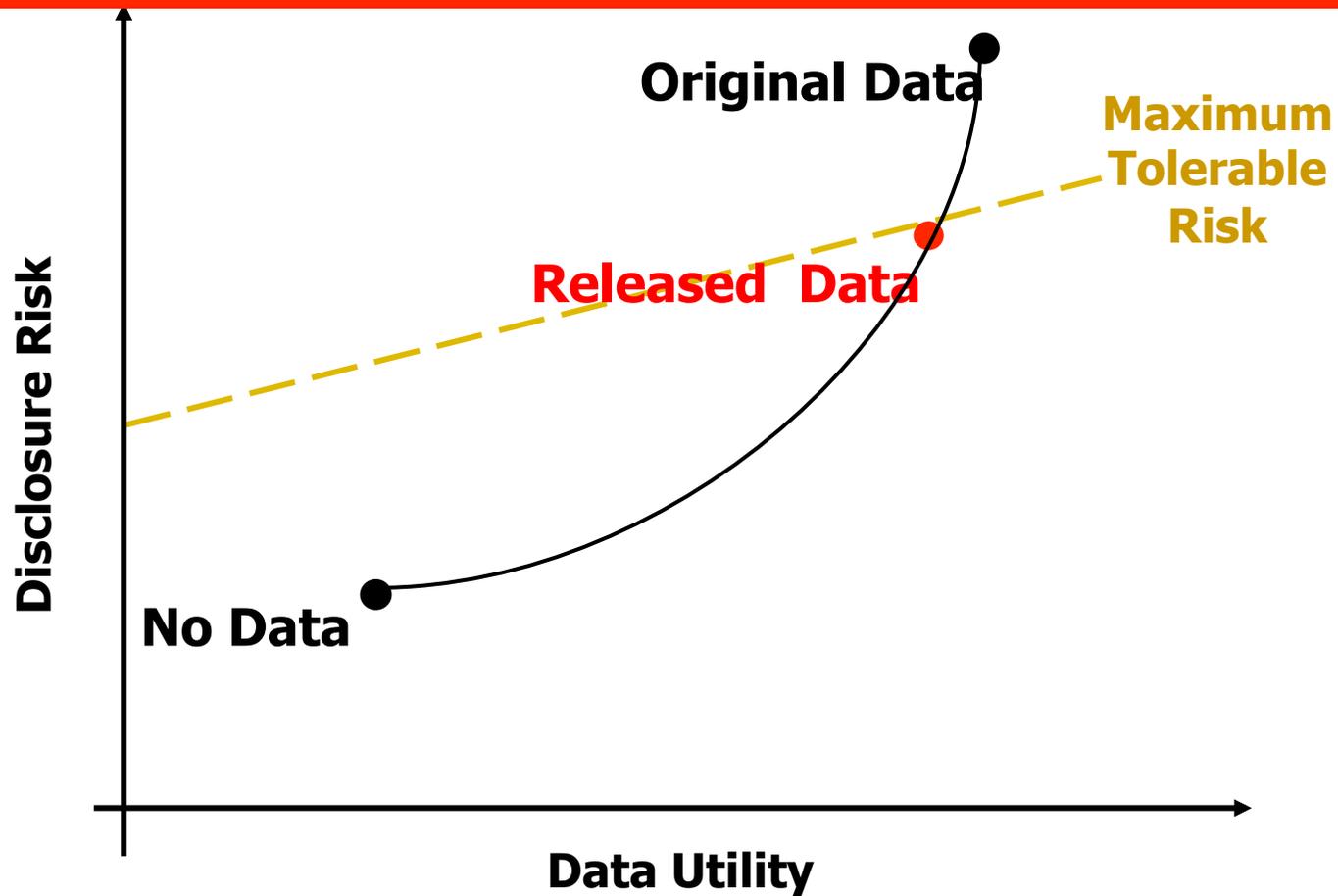
- **The Census Bureau's promise of confidentiality cannot be trusted.**
- **My answers to the census could be used against me.**
- **The census is an invasion of my privacy.**

**Prewitt, 2011**

# Outline

- **Some Statistical ideas on confidentiality and privacy protection.**
- **Differential Privacy (DP) in a focused statistical problem:**
  - Protecting contingency table data.
- **Extensions to DP.**
- **Record Linkage as alternative to DP:**
  - **A partially baked idea!**

# R-U Confidentiality Map



# Usability, Transparency, & Duality in Privacy Protection

- **Usability:** extent to which released data are free from systematic distortions that impair inference.
- **Transparency:** extent to which methodology provides direct or implicit information on bias and variability resulting from disclosure limitation mask.
- **Duality:** extent to which methods aim at both disclosure limitation and making the maximal amount of data available for analysis.

# General Methods for Protection

- **Removing obvious identifiers/near-identifiers**
  - Names, geography, birthdate, etc.
- **Data transformations:**
  - **Matrix masking**  $X \rightarrow AXB + C$ 
    - e.g., noise addition
  - **Data suppression**
    - Deleting cases / sampling
    - Cell suppression
- **Synthetic data**

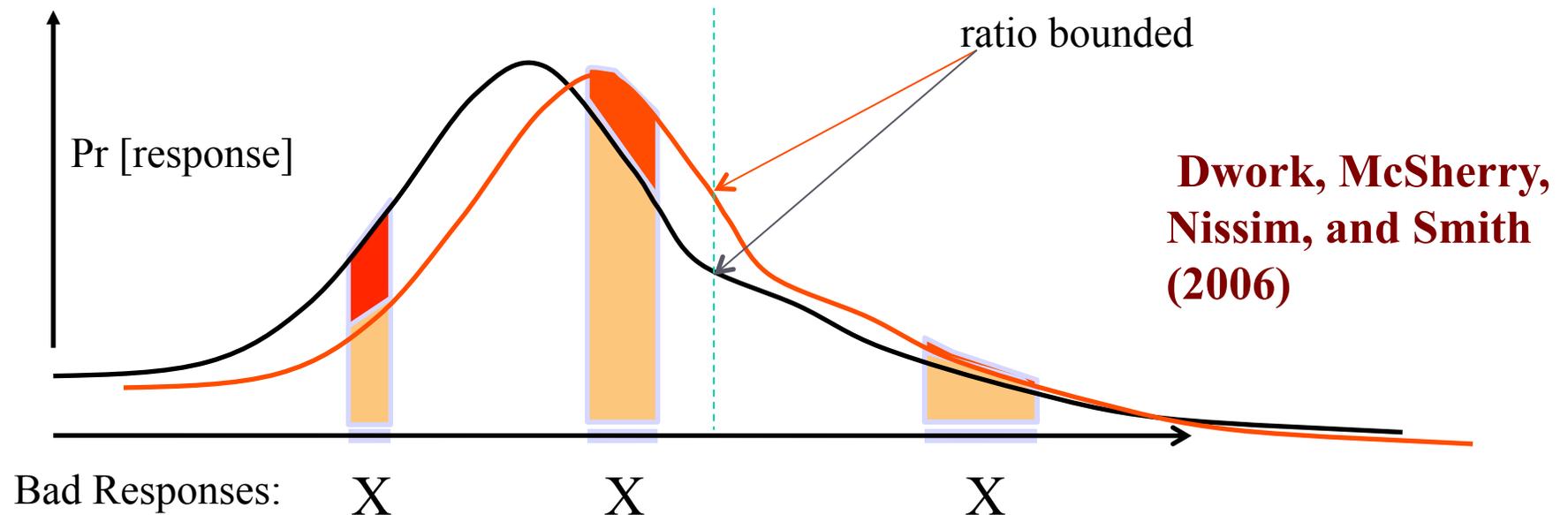
# Inferential Utility

- **Want to achieve “Statistical reversibility” of data transformation:**
  - **Need (a) released data and (b) likelihood function including full information on transformation applied.**
  - **For noise addition this may involve using “measurement error model.”**
- **Contrast with Naïve DP perspective and agency view of “just using” released data.**

# Enter $\epsilon$ -Differential Privacy

Randomized function  $\mathcal{K}$  gives  $\epsilon$ -differential privacy if for all neighboring  $D_1$  and  $D_2$ , and all  $C \in \text{range}(\mathcal{K})$ :

$$e^{-\epsilon} \leq \Pr[\mathcal{K}(D_1) \in C] / \Pr[\mathcal{K}(D_2) \in C] \leq e^{\epsilon}$$



# Differential Privacy

- **Standard “DP mechanism” is addition of Laplace noise, with parameter  $\epsilon$ .**
  - **The more data or statistics you protect the larger the noise required.**
- **Refinements such as “exponential mechanism,” and perturbing an estimating equation, exploit a Lipschitz condition, and require less noise.**

# Differential Privacy

- **DP offers strong privacy “guarantees,” through all possible violations, but...**
  - **Strong privacy “guarantees” may destroy utility of the data.**
  - **Does not recognize iterative and possibly nature of statistical data analysis.**
- **Research users want data sets to analyze, not DP-protected coefficients.**

# Differential Privacy

- **DP is fundamentally a *frequentist* notion:**
  - **Privacy resides in the method that generates the altered data, as well as extremal aspects of data themselves.**
  - **Has the flavor on minimax approaches.**

# Protecting Contingency Tables

Barak et al. (2007)

- **Want to release a set of altered MSS marginals.**
  - Use Fourier coefficient basis for noise addition.
  - This produces non-integer and inconsistent marginals.
  - Consistency of marginals doesn't guarantee existence of a table satisfying released marginals.
  - Barak et al. find “nearby” set of consistent integer marginals which preserve DP property.
- **Assessment by Fienberg, Rinaldo and Yang (2010, 2011) show that the approach obliterates the data for large sparse tables.**

# Lessons Learned

- **As  $\epsilon$  increases, amount of noise added decreases**
  - **Deviance between DP generated tables and real MLEs gets smaller.**
  - **If we add a lot of noise, it has strong privacy guarantees but the statistical inference becomes infeasible.**
  - **When we add little noise, the statistical inference is better but no privacy guarantees.**
- **DP struggles with releasing useful information associated with large sparse contingency tables.**

# Implications

- **Need to incorporate RU ideas into DP formulation for data releases to have real utility:**
  - **Learn how to draw inferences from privacy-protected releases.**
  - **Focus on model search processes, not simply reporting one set of summary statistics.**
- **Move from frequentist to Bayesian formulation.**

# Extensions to DP: I

- $(\epsilon, \delta)$ -DP (Dwork, et al. 2006)

- A randomized algorithm  $\mathcal{K}$  gives  $(\epsilon, \delta)$ -DP if for all  $S \subseteq \text{Range}(\mathcal{K})$ ,

$$\Pr[\mathcal{K}(D_1) \in S] \leq e^\epsilon \Pr[\mathcal{K}(D_2) \in S] + \delta,$$

where the probabilities are over the coin flips of the algorithm  $\mathcal{K}$ .

# Extensions to DP: II

- **$(\epsilon, \delta)$ -Probabilistic DP** (Machanavajjhala et al., 2008)

$$\Pr[\mathcal{K}(\mathbf{D}) \in \text{Disc}(\mathbf{D}, \epsilon)] < \delta.$$

- **Claim:**  $(\epsilon, \delta)$ -PDP lies strictly between  $(\epsilon, \delta)$ -DP and  $\epsilon$ -DP. **True?**
- How do we compute  **$\text{Disc}(\mathbf{D}, \epsilon)$** ? **With respect to a prior, w.r.t. the joint distribution of the data and the prior (Abowd and Vilhuber, 2008), w.r.t. the randomizing function?**

# Extensions to DP: III

**$(\epsilon, \delta)$ -Random DP** (Hall, Rinaldo, Wasserman, 2011)

$$\Pr_{\mathcal{D}}\{e^{-\epsilon} \leq \Pr[\mathcal{K}(\mathbf{D}_1) \in C] / \Pr[\mathcal{K}(\mathbf{D}_2) \in C] \leq e^{\epsilon}\} \geq 1 - \delta$$

- Key here is that data are treated as random and deviations from DP are with respect to distribution of data.
- $\mathbf{D}_2$  adds a randomly drawn new data element to database  $\mathbf{D}_1$ .
- Get composition property w.r.t.  $\epsilon$  and much better utility w.r.t. risk function.

# Related DP Issues

- Should the bound on

$$|\Pr[\mathcal{K}(\mathbf{D}_1) \in C] / \Pr[\mathcal{K}(\mathbf{D}_2) \in C]|$$

be constant,  $\epsilon$ , or depend on  $\mathbf{D}$ ?

- Should perturbations of the data always involve adding continuous noise?
  - What about restricted swapping for count data?

# Statistical View of Record Linkage (Hall & Fienberg)

There exist two sets of observable records:

$$A = \{a_1 \dots a_n\} \quad B = \{b_1 \dots b_m\}$$

Data are via  
model depending  
on Q

$$P_{\theta}(A, B; Q)$$

Goal of record  
linkage is to  
estimate the  
parameter Q

$$Q \in \{0, 1\}^{n \times m} \quad q_{i,j} = \begin{cases} 1 & a_i, b_j \text{ link} \\ 0 & \text{o/w} \end{cases}$$

There is an **unknown** matrix that contains  
**the true record linkage** information.

# “Privacy” Overview

**Goal:** To release a **sanitized database** that includes potentially sensitive data elements, while maintaining individual privacy.

## Police Records

Name	Address	Criminal?
Robert	123 Fake St	N
Dave	456 Fake St	Y



## Sanitized Police Records

Name	Zip Code	Criminal?
REDACTED	15232	N
REDACTED	15232	Y

In general, we must **sanitize** the data somehow.

## Adversary's Data

Name	City	...
Robert	Pittsburgh	...
...	...	...



Envision an adversary attempting to **infer the sensitive information** via **record linkage**.

# Setting/Assumptions

The columns of the data partition into the sensitive attributes, and the quasi-identifiers:

Name	Address	Criminal?
Robert	123 Fake St	N
Dave	456 Fake St	Y

“Quasi-identifiers” aka “key variables”      “Sensitive attribute”

complete record

$$a_i = (a'_i, s_i)$$

*sensitive attributes*

quasi-identifiers

The goal is to release a set of sanitized records:

$$b_i = (b'_i, \tilde{s}_i)$$

# “Privacy” and Record Linkage

- Suppose adversary knows exact values for quasi-identifiers for subset of records in private database:

Complete database

$$A = \{a_1 \dots a_n\}$$

Adversary's database

$$A' = \{a'_{i_1} \dots a'_{i_m}\}$$

$$P_\theta(A, B; Q)$$

Choose a permutation  $Q$  uniformly at random, and a model  $P$ , then draw  $B|A;Q$

$$B = \{b_1 \dots b_n\}$$

Sanitized database

Adversary faces record linkage problem, where model is specified by the data owner.

# Fully Bayesian “Privacy”?

- Suppose that the choice of model  $P$  is made public knowledge:
- Then the “correct” way to do inference about  $S$  is to maintain uncertainty about the record linkage:

$$\pi(S | B) \propto \sum_{Q_i \in Q} P_{\theta}((A', S), B; Q_i) \pi(S)$$

(sum over all possible linkage structures)

- **A possible criterion for privacy protection would be to require the “statistical distance” between the posterior and prior is small for all prior distributions:  $D_H(\pi(\cdot), \pi(\cdot | B)) \leq \tau$**
- Adversaries and legitimate statisticians are treated the same.
- Choice of  $D_H$  and  $\tau$  gives tradeoff between utility and privacy.

# Fully Bayesian “Privacy”?

- **Some Context:**
  - *k-anonymity, l-diversity, t-closeness* may be viewed as successively improving approximations to this idea, but they also unnecessarily restrict the model class.  
 **$P(A,B;Q)$  concentrated on  $\{B: B \text{ is } k\text{-anonymized}\}$**
- **“Protect” sensitive values?**
  - We could output exact identifiers, allow adversary perfect record linkage, but apply double exponential or other kind of perturbations to sensitive attributes.
  - Expanded options to explore.
- **We need to understand the formal properties.**

# Relationship to DP

- **Differential privacy from BP perspective:**
  - Adversary has  $n-1$  complete records and belief about  $n$ th record doesn't change much when seeing data.
  - DP criterion implies Hellinger distance ( $f$ -information).
  - In BP approach, use  $n-1$  quasi-identifiers, and point mass prior on  $n$  true sensitive values.
    - Adversary's prior on  $n$ th sensitive value doesn't change much re inferring quasi-identifiers for  $n$ th record.
    - Choice of distance function, e.g., KL-information.
    - BP scheme doesn't protect the identifiers.

# Summary

- **Some Statistical ideas on confidentiality and privacy protection.**
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- **Extensions to DP.**
- **Record Linkage as alternative to DP:**
  - **A partially baked idea!**

**End**

- **My CMU privacy collaborators:**
  - **Rob Hall, Jiashin Jin, Alessandro Rinaldo, Xiaolin Yang, Larry Wasserman**
- **Joint CMU/PSU/Cornell collaboration**