Do Targeted Hiring Subsidies and Profiling Techniques for Long-term Unemployed Reduce Unemployment?

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ABSTRACT: To reduce equilibrium unemployment targeted hiring subsidies and profiling techniques for long-term unemployed are often recommended. To analyze the effects of these two instruments, our model combines two search methods: the public employment service and random search, jobseekers choose between an active and a passive search strategy, while labour market policy has two options available. First, only the long-term unemployed placed by the public employment service are subsidized. Second, the subsidy is paid for each match with a long-term unemployed irrespective of the search method used. We show that under both regimes the equilibrium unemployment rate is increasing with respect to the hiring subsidy. The subsidy like the profiling measures, which improve the effectiveness of the public placement service, crowd-out the active jobseekers among the short- and the long-term unemployed and reduce total employment.

KEY-WORDS: Matching model, hiring subsidy, endogenous separation rate, active labour market policy, PES, search market

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To fight unemployment, instruments of active labour market policy (ALMP) have been in use for many years in most European countries. The quest for such policy measures continues to be central to the policy debate, particularly in continental European countries with high unemployment. To reduce these intervention costs and the short-term deadweight losses, ALMP instruments often target on the long-term unemployed or workers who are at risk of becoming long-term unemployed. In addition it is often recommended that, for instance in the European Employment Strategy (European Commission 2006) or more recently in the German "Hartz Proposals" (Hartz et al. 2002), the the public employment service (PES) increase its effectiveness. A wide variety of instruments have been to achieve this objective. Generally, profiling techniques, monitoring, and individual counselling are instruments aimed at enhancing the effectiveness of the public employment service. The implicit assumption behind these measures is that labour markets are characterized by structural coordination failures which can only be overcome by concerted interventions of the command-and-control centers of the welfare state like the PES.

To analyze the effects of targeted hiring subsidies and profiling techniques on the equilibrium rate of unemployment we use a search model of the Mortensen-Pissarides type which combines two search methods: the public employment service and random search. Workers can be either employed, short-term unemployed or long-term unemployed. Jobseekers choose between an active and a passive search strategy. As their skills depreciate during unemployment, firms must retrain new entrants, who in addition need a period of learning by doing to regain full productivity. The PES can pay a subsidy to those firms which hire a long-term unemployed and bear the training costs.

1 LITERATURE

There is growing empirical literature estimating the short-term employment effects for the participants of ALMP programs. Even if the net effects of hiring subsidies and profiling measures are positive, the effect on the aggregate rate of unemployment remains inconclusive (Heckman et al. 1999, Kluve 2006). One reason may be that there are interactions between ALMP participants and other employed or unemployed jobseekers, specifically crowding out

and general equilibrium effects. Without incorporating them into a macro framework, the micro econometric treatment effects will thus provide poor guides to policy makers.

Matching models are a powerful instrument to analyze the macroeconomic impact of ALMP. The theoretical literature of the effect of hiring subsidies and instruments which increase the efficiency of the PES on the aggregate rate of unemployment is still in its infancy and relatively small (Yashiv 2006 and Rogerson et al. 2005). Moreover, to our knowledge, the literature has paid little attention to the effects of instruments that target on particular groups of the unemployed. Furthermore this strand of the literature does not take into account that ALMP can influence the matching process itself (Brown et al. 2006, 4f). It is the objective of our paper to address these needs.

Millard and Mortensen (1997), Mortensen and Pissarides (1999, 2003) and Pissarides (2000, ch. 9) analyze the effects of non-targeted hiring subsidies on equilibrium unemployment. The labour market is characterized by a matching technology which represents the two-sided search process with its frictions arising from imperfect information, mobility costs and worker and job heterogeneities. Each new match of a jobseeker with a vacancy is eligible for a hiring subsidy. The hiring subsidy increases both the number of newly created jobs and the amount of job destruction. Therefore, its overall effect on equilibrium unemployment is ambiguous. Millard and Mortensen (1997) and Mortensen and Pissarides (1999, 2003) thus simulate the net effects of the subsidy and show that the hiring subsidy increases the aggregate unemployment rate.

Yashiv (2004) addresses the question whether hiring subsidies, employment subsidies, unemployment benefits and wage tax reductions under a given budget constraint have substantial effects on labour market outcomes and the business cycle properties when labour market frictions are present. Within a stochastic, discrete-time version of a matching model he shows that hiring subsidies reduce unemployment while employment subsidies and wage tax cuts do not. The positive effect of the hiring subsidy is partly induced as Yashiv (2006, 276) states by the setup of a model that does not allow for endogenous separations.

Brown et al. (2006) introduce self-financing hiring vouchers targeted at the long-term unemployed and unskilled workers into a dynamic model setting. Their calibrated model shows

that in contrast to wage subsidies hiring vouchers can raise employment, are “approximately welfare efficient” and reduce inequality. The main reason for this result is the following. The model setup determines that entrants or outsiders earn a wage that is negotiated between the insiders and the firm. The hiring voucher therefore does not influence the outsider wage and the initial labour costs of the firm, which pockets the entire subsidy. While the entrant’s hiring rate increases with the cash inflow from the voucher, the insider’s firing rate is unaffected.

Within a general equilibrium search model Vereshchagina (2002) evaluates the long-run implications of shifting unemployment benefits into a subsidy program targeting the long-term unemployed. In general the effect of the subsidy is ambiguous. On the one hand, the introduction of a subsidy reduces government's expenditures on unemployment benefits and weakens the distortionary effects of the unemployment insurance system by putting the unemployed back to work. On the other hand, the subsidy program puts additional economic pressure on the government budget caused by the subsidy. The higher the elasticity of labour demand, the more likely it is that the subsidy program generates a government budget surplus and positively affects welfare.

Our model differs from the above-mentioned in the following respects. First, two search methods are available, the public employment service (PES) and random search (Pissarides 1979). Second, the unemployed choose between a passive and an active search strategy. In view of the response time which the PES needs to make a first job offer to a registered unemployed, active short-term unemployed (STU) can count on no assistance from the public placement service and remain solely dependent on their own search efforts. The active LTU on the other hand can combine both methods of search. Third, we investigate two instruments of active labour market policy: profiling techniques to increase the placement effectiveness of the PES and hiring subsidies.

Hiring subsidies are only paid to firms that register their vacancy and fill it with a long-term unemployed worker or a worker who is at risk of becoming long-term unemployed. We compare two regimes. In the first regime, a hiring subsidy is paid only for LTU placed by the PES. In the second regime the PES also subsidizes matches with LTU established through random search. Fourth, the matching process comprises three subsequent stages. In the first

and second stages, respectively, the active jobseekers among the LTU randomly search for a vacancy; in the third stage the PES matches registered vacancies with registered unemployed. Fifth, most of the above mentioned models assume an exogenous job destruction rate. In these models ALMP affects primarily job creation rather than job destruction which seems counterfactual. We build on the existing theoretical matching literature by adding an endogenous job destruction rate (Mortensen and Pissarides 1994).

Our analysis is devoted to high-unemployment countries, such as the continental European societies. In these countries policy makers have every reason to believe that structural unemployment is in fact inefficiently high. Therefore, we do not address explicitly the question whether the effect of ALMP is indeed welfare improving or not.

The model generates the following results. First, equilibrium unemployment rises with the fraction of passive jobseekers and with an increasing unemployment incidence and duration. Second, hiring subsidies increase job destruction and unemployment duration of passive jobseekers, reduce the search incentives and therefore the proportion of active jobseekers among the STU and non-eligible LTU as well as job-to-job transitions. As a consequence, it decreases overall employment. Third, without a hiring subsidy the LTU, who suffer from a depreciation of their skills, must accept a wage penalty. Fourth, the subsidy increases the fraction of the LTU and their average outside wage. Finally, intuition and active policy approaches like the “Hartz-reforms” recommend increasing the effectiveness of the PES in order to overcome the coordination failures of the labour market. This intuition and the reasoning behind the policy recommendations to extend the placement capacity and the effectiveness of the PES are not confirmed by our model.

This paper is structured as follows: Section 2.1 derives the equilibrium rate of unemployment. Section 2.2 introduces the asset equations of filled jobs and employed workers. Section 2.3 deals with job creation. Section 2.4 covers the asset equations of the unemployed and wage negotiations. Section 2.5 derives the equilibrium values of the jobs filled, the dispersions of the outside wages of the LTU, and the job destruction condition are derived. Section 3 presents the simulation of the model and Section 4 concludes. A graphical presentation of the simulation results can be found in Appendices I and II and the market values of the vacancies, the model equations and proofs of the propositions in Appendices III-V.[1]

2 THE MODEL

Section 2.1 introduces the search channels, the search strategies and the three stages of the search process. Moreover, we analyze the number of job-to-job transitions, the duration of unemployment of active and passive jobseekers, and derive the steady-state unemployment rate.

2.1 SEARCH CHANNELS, WORKER FLOWS AND HIRING SUBSIDIES

The time of the model is discrete. Job creation takes place at the beginning and job destruction at the end of a period. A continuum of vacancies is confronted with two different types of applicants. The first type are the short-term unemployed (STU) who have lost their job at the end of the previous period. The second type have been unemployed for at least one period and are either threatened by long-term unemployment or already belong to the pool of long-term unemployed (LTU). For simplicity we do not introduce intermediate durations of unemployment and concentrate on the simple distinction between short-term and long-term unemployed instead.

Methods and search strategies. There are two search methods, the PES and random search, and two search strategies, active job search on the private search market and passive search through the PES placement service. Workers choose either the active or the passive search strategy or a combination of both. The search strategy of the vacancies is not specialized, since they are simultaneously posted on the private search market and are registered with the PES.

To claim unemployment benefits and to use the job placement service, jobseekers register with the PES. The PES verifies eligibility, registers and advises, and refers the worker to the placement service. The placement service then looks for available jobs and either makes a job offer or not, depending on the number of vacancies and unemployed waiting for placement. How much time passes between the first day of an unemployment spell and the first job offered by the PES? This time interval consists of two phases. First, the time which passes until the unemployed decides to ask the PES for support; second, the *response time* of the PES. Despite extensive search we have not found data and empirical research on the distribution and the structural and cyclical components of the *response time* of the PES. The German Federal Employment Office estimates an average *response time* of three months for 2005. We as-

sume that the decision time of the worker and the average *response time* of the PES together last at least one period which is equal to a quarter of a year in the calibrated version of our model.

Hence, STU who decide on the active search strategy receive no support from the PES and are solely dependent on their own search efforts. Moreover, STU opting for the passive strategy and leaving all search activities up to the PES cannot expect a job offer until the second period of the unemployment spell at the earliest. Given the decision and *response time* the PES can place only LTU workers, workers who are at risk of long-term unemployment or who are already long-term unemployed. Passive LTU leave the job search up to the PES. Active jobseekers among the LTU use both search channels simultaneously. Hence, in equilibrium their transition probability is higher than that of the passive LTU. But using the search market causes private search costs so that only a part of the unemployed decide to pursue an active strategy.

Search process (see Figure 1). The search process consists of three stages. In the first, only the S_I active jobseekers among the STU are on the market. They possess the best information about current labour market conditions and, therefore, their applications are more targeted and arrive earlier than the placements of the PES or the applications of the active LTU. In the second stage, advertised vacancies meet the *S* active jobseekers among the LTU. In the last stage of the matching process the PES arranges matches between registered vacancies and the registered unemployed.

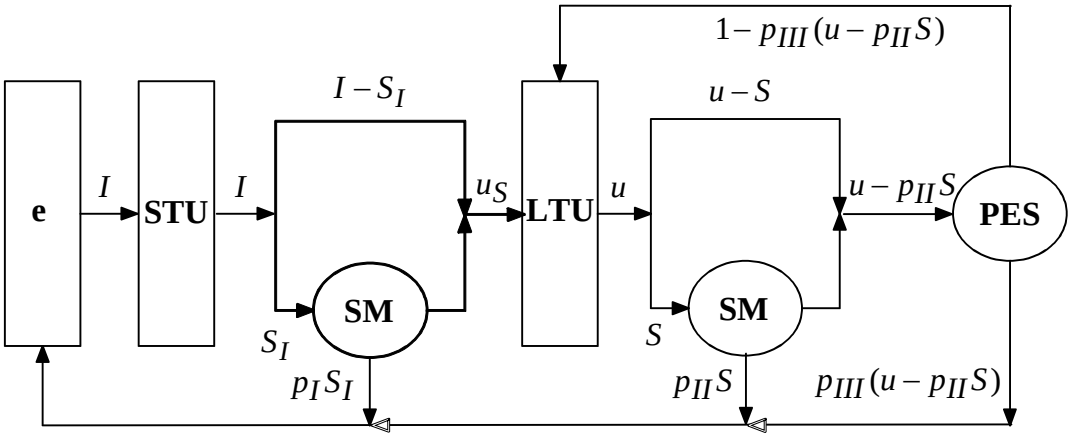

Figure 1: Search Process, with $p_{II} \equiv p_S(1-q_I)F(T_S)$ and $p_{III} \equiv P(1-q_I)(1-q_S)F(T_P)$

Transition probabilities. The labour force is normalised to one. Of the $1-u$ employed, $I \equiv \lambda G(R)(1-u)$ lose their job at the end of a period. $\lambda G(R)$ is the unemployment incidence

where λ is the probability of an idiosyncratic productivity shock, $G(x)$ with support $0 \le \alpha \le x \le 1$ is the c.d.f. of the multiplicative shock *x* and *R* is the endogenous reservation productivity. yx, with $y > 0$, is the flow output of a job. If a match draws productivity *x* with $R \le x \le 1$, the job is continued. If $x < R$, the match dissolves, the job becomes vacant, and the worker unemployed.

Of the *I* workers who lose their job, S_I decide for the active search strategy and immediately at the beginning of the next period start to search randomly for a new job. The other $I - S_I \ge 0$ workers prefer the passive strategy. With the beginning of the subsequent period, they are threatened with long-term unemployment and belong to the group of LTU. The matching technology of the search market generates the transition probability p_I that a given jobseeker among the STU will meet a vacancy. As the STU have a marginal product which is at least as high as that of the LTU and do not cause training costs it does not pay for a firm to wait for the subsequent stage of the matching process. Therefore each match of a STU worker with an advertised vacancy results in an employment contract. The number of STU, u_S, is given by:

$$I - p_I S_I = u_S \tag{1}$$

Of the pool *u* of LTU, $u - S \ge 0$ workers choose the passive search strategy and wait for placement by the PES. Their transition probability into employment is $P(1-q_I)(1-q_S)F(T_P)$, where *P* denotes the probability of a contact with a vacancy found by the PES, q_I and q_S are the probabilities that the vacancy is already filled either by one of the S_I jobseekers among the STU or by one of the *S* jobseekers among the LTU, so that $P(1-q_I)(1-q_S)$ is the probability of a contact with a vacancy which is not yet occupied.

Each match with a LTU worker generates match specific training costs $t \ge 0$, of which *ex ante* only the c.d.f. $F(t)$ with support $0 \le t < \infty$ and the endogenous reservation costs T_P are known. The reservation costs T_P are the training costs up to which firms are ready to sign a job contract. Therefore, $F(T_P)$ denotes the probability that the firm faces training costs $t \le T_P$ and continues the job. A match with a LTU worker with training costs $t > T_P$ is immediately dissolved; the job remains vacant and the worker unemployed.

The transition probability for the S active jobseekers among the LTU is $(1-q_I)[p_S F(T_S)+(1-p_S)P(1-q_S)F(T_P)]$, see Figure 1. First, the probability of locating an unfilled vacancy through random search is $p_S(1-q_I)$, where p_S is the contact probability and $1-q_I$ the probability that the vacancy is not yet filled. Even if random search is not successful, the active jobseekers among the LTU might still be placed by the PES. The probability for this event after random search has failed is $P(1-q_I)(1-q_S)$. Second, the jobseeker must draw training costs that are below the reservation costs of the respective search channel. If active labour market policy discriminates between the two methods of search and subsidizes only placements by the PES then reservation costs depend on the search channel. T_S is the reservation cost of the search market and T_P the reservation cost of the matching process organised by the PES.

Summarising the flows into employment which result from the above transition probabilities and taking into account that u_S denotes the inflow into the pool u of long-term unemployed yields the steady state condition:

$$u_S = P(1-q_I)(1-q_S)F(T_P)u + p_S(1-q_I)[F(T_S)-P(1-q_S)F(T_P)]S\,. \qquad (2)$$

To determine the sign of the expression in square brackets note that the LTU prefer the active search strategy only if the transition probability from combining the two methods of search is higher than that of the passive search strategy alone. Hence the S LTU opt for the active search strategy only because $F(T_S) > P(1-q_S)F(T_P)$.

Matching function. The function $m(x,v)$ represents the matching technology of the search market, where m is the number of contacts per period for a given measure of jobseekers x and advertised vacancies v. The matching function has constant returns to scale and is strictly concave and monotone in both arguments. Immediately at the beginning of a period, $m(S_I,v)$ of the v advertised vacancies are filled by the inflow from the S_I active jobseekers among the STU. For a given vacancy posted at the beginning of a period, the probability of a match with a STU is $q(\theta_I) \equiv m(1/\theta_I,1) = m(S_I,v)/v$, with $\theta_I = v/S_I$ denotes the tightness of the labour market in the first stage of the matching process. The transition probability of a given active jobseeker among the STU is $p(\theta_I) = \theta_I q(\theta_I)$. For convenience we write $q_I = q(\theta_I)$ and $p_I = p(\theta_I)$.

The S active jobseekers among the LTU workers face the same v advertised vacancies. $m(S,v)$ represents the measure of contacts, and $q(\theta_S) \equiv m(1/\theta_S, 1) = m(S,v)/v$ is the contact probability of a given vacancy with an active LTU, where $\theta_S = v/S$ is the tightness of the labour market in the second stage of the matching process. The contact probability for a given jobseeker is $p(\theta_S) = \theta_S q(\theta_S)$, and we write $q_S = q(\theta_S)$ and $p_S = p(\theta_S)$.

As all vacancies are advertised as well as registered, v is also an argument in the matching function $M(u,v)$ of the PES which has the same properties as $m(x,v)$. M is the measure of contacts per period which are arranged by the PES between the v vacancies and the stock u of LTU. For a given vacancy, therefore, $Q(\Theta) \equiv M(1/\Theta, 1) = M(u,v)/v$ is the contact probability with a LTU worker via the PES with $\Theta = v/u$ denoting the tightness between both registers. Finally, $P(\Theta) = \Theta Q(\Theta)$ is the contact probability of a LTU worker with a vacancy.

LEMMA 1 [UNEMPLOYMENT DURATION]. T*he length of an unemployment spell depends on the search strategy and is in general different from the duration of a job search.*

(i) *If the unemployed leaves the job search up to the PES, then he will be out of work for at least one period. The average length of time required for the registration and processing of his personal data and for a successful job search is given by* $D_P = 1 + d_P$, *where* $d_P = 1/P(1-q_I)(1-q_S)F(T_P)$ *is the average duration of a job search by the PES.*

(ii) *An unemployed who combines the passive search strategy in the first period of his unemployment spell with the active search strategy in all subsequent periods faces a duration of unemployment* $D_S = 1 + d_S$, *where* $d_S = 1/(1-q_I)\left[p_S F(T_S) + (1-p_S)P(1-q_S)F(T_P)\right]$ *is the average duration of job search of an active LTU worker.*

(iii) *An unemployed who opts for the active search strategy from the beginning of his unemployment spell faces an expected duration of job search of* $(1-p_I)D_S$ *periods.*

Inserting equation (1) into equation (2), using $I = \lambda G(R)(1-u)$ and taking into account the above definitions of the tightness in the three labour market segments, we obtain the following equation for equilibrium unemployment in the steady state

$$u = \frac{\lambda G(R)}{\lambda G(R) + \sigma_I p_I + \sigma_S / d_S + (1-\sigma_S)/d_P}, \tag{3}$$

where $\sigma_S = S/u \le 1$ is the share of active LTU among the unemployed and $\sigma_I = S_I/u$ is the ratio of active jobseekers among the STU to the pool of unemployed u. Contrary to the share σ_S the ratio σ_I is not bounded from above. The unemployment rate (3) increases with the job destruction rate λG, the duration of job search of the active or the passive search strategy

d_S and d_P, respectively, and the fraction of job-to-job transitions to the number of unemployed, $\sigma_I p_I$; while (3) decreases with the share of active jobseekers among the LTU worker σ_S. The sign of the impact of σ_S on *u* is due to the fact that $d_S < d_P$.

Hiring subsidy. The PES is fully integrated and provides the following services. First, it pays unemployment benefits. Second, it matches registered vacancies with registered jobseekers, and third it pursues ALMP. In this last function, the PES pays a hiring subsidy to firms that enter into an employment contract with a LTU worker. The hiring subsidy is paid when the match partners sign the contract and observe the training costs $t \geq 0$. The subsidy compensates for the match specific training costs but only up to a given amount *H*. Training expenditures can be monitored by the PES without costs. Since the support of the distribution of training costs is not bounded from above, the PES establishes in fact an upper limit *H* on the hiring subsidy so that the training costs *t* of all matches with $t \leq H$ are fully refunded, whereas matches with $t - H > 0$ have to finance the balance out of their match rent.

2.2 Filled Jobs and Employed Workers

If a firm and a jobseeker meet, the firm tests the skills of the applicant and observes the match specific training costs *t*. If *t* exceeds the reservation costs, firm and worker separate immediately. Otherwise, they negotiate the employment contract and begin production.

An employment contract $[w_i, w(x), R]$ has three components. The first is the outside wage w_i which is paid to the worker at the end of the initial period, the training period. It depends on his status *i* as a jobseeker, where $i = I, P, S$. For STU worker we set $i = I$ and for a LTU worker who has opted either for the passive or the active search strategy (which combines the two methods of search) we set, respectively, $i = P$ or $i = S$. The second component of the contract is the match specific inside wage represented by the wage function $w: [R,1] \to \Re$. At the end of a period the productivity *x* of the subsequent period is revealed. If $x \in [R,1]$, the match is continued and the worker earns the bargained inside-wage $w(x)$.[2] The third component denotes the reservation productivity *R* at which the firm will destroy the job.

Continuation periods. After the training period all jobs have the same productivity *y*. Shocks hit a match with probability $\lambda \geq 0$, are match specific, and manifest themself in the

multiplicative productivity component *x*, which is a random variable with c.d.f. $G(x)$ defined on $x \in [\alpha,1]$. Within each period only one shock can occur. Furthermore, shocks are iid.

Let $\Pi(x)$ be the present value of a filled job after the manifestation of a shock $x \in [\alpha,1]$. Since $\Pi(x)$ is a continuously increasing function of *x* a reservation threshold *R* exists, for which $\Pi(R)=0$. Only jobs with $x \geq R$ will be continued. The steady state equation for the present value $\Pi(x)$ of a filled job is

$$\Pi(x) = \rho\left\{yx - w(x) + \lambda\int_R^1 \Pi(h)dG(h) + (1-\lambda)\Pi(x)\right\}. \tag{4}$$

Flow and stock variables are discounted at the rate ρ, where $0 < \rho = 1/(1+r) < 1$ with the interest rate $r > 0$. With probability λ the job is hit by a shock and changes into state *h*. If $R \leq h \leq 1$ the match is continued and the probability weighted continuation value becomes $\Pi(h)dG(h)$. With probability $1-\lambda$ the match specific productivity does not change.

The present value of the workers human capital $W(x)$ is

$$W(x) = \rho\left\{w(x) + \lambda\left[\int_R^1 W(h)dG(h) + G(R)U_I\right] + (1-\lambda)W(x)\right\}. \tag{5}$$

With probability λ a shock occurs and the match draws the productivity *h*. If $h \geq R$, the value of the worker is $W(h)$ and the match continues. If, on the other hand, $h < R$, which occurs with probability $G(R)$, the job is destroyed, the worker becomes a STU and the value of his human capital is U_I.

Training period. Firms choose the initial productivity when they set up the match and negotiate the outside wage. For STU the initial productivity is set at $x = 1$. Moreover, the STU cause no training costs and the initial value Π_I of a job filled by a STU worker is

$$\Pi_I = \rho\left\{y - w_I + \lambda\int_R^1 \Pi(h)dG(h) + (1-\lambda)\Pi(1)\right\}, \tag{6}$$

with w_I denoting the negotiated outside wage. If the match is not hit by a shock, the worker's productivity remains at $x = 1$, and the filled job has the value $\Pi(1)$. The human capital of a STU worker is

$$W_I = \rho\left\{w_I + \lambda\left[\int_R^1 W(h)dG(h) + G(R)U_I\right] + (1-\lambda)W(1)\right\}, \tag{7}$$

The LTU find a vacancy either through random search or via the PES. When wage negotiations start, jobs filled by the STU are already productive. Moreover, the skills of the LTU worker depreciate during unemployment such that they need a period of practice to regain full productivity. The depreciation rate is $1-z \geq 0$, and their initial productivity *yz*. The allocation of the training costs and the hiring subsidy is subject to negotiation, but the outside wage $w_i(t)$ and the initial value of the job $\Pi_i(t)$ depend only on *t* if *t* exceeds the subsidy limit *H*.

For the sake of brevity, we present the asset equations only for the case where the training costs exceed the subsidy *H*. The indicator variable τ takes on the value one if the PES also subsidises the matches formed by random search, while $\tau = 0$ if the subsidy is paid only to matches arranged by the PES. Considering the status of the jobseeker $i = P, S$, the present value of a job filled with a LTU worker is given by

$$\Pi_i(t) = \rho\left\{yz - w_i(t) + \lambda\int_R^1 \Pi(h)dG(h) + (1-\lambda)\Pi(1)\right\}, \tag{8}$$

where in (8) and also in (9) below $H \leq t \leq T_P$ for $i = P$ and $\tau H \leq t \leq T_S$ for $i = S$.

Taking into account the negotiated outside wage $w_i(t)$ the corresponding present value of the worker's human capital during the training period is

$$W_i(t) = \rho\left\{w_i(t) + \lambda\left[\int_R^1 W(h)dG(h) + G(R)U_I\right] + (1-\lambda)W(1)\right\}. \tag{9}$$

2.3 JOB CREATION

Vacancies are advertised and registered.[3] Entrance into the labour market is free for all vacancies, but open only at the beginning of a period. The inflow of vacancies therefore persists until the present value of a vacancy is driven to zero, $V = 0$. Considering the infinitely elastic supply of vacancies, the *job creation* condition is $0 = -k + q_I\Pi_I + (1-q_I)V_I$, where *k* denotes the flow search costs. If there is no contact with a STU worker in the first stage of the matching process, an event which has the probability $1-q_I$, the vacancy takes on the value of its outside option $V_I > 0$, which is explained below.

There are three reasons for the existence of an outside option $V_I > 0$. First, vacancies are not specialised. Second, the matching process consists of three stages. A vacancy that is not

filled during the first has the option to meet a LTU worker who is actively searching for a job or placed by the PES in the second or third stage of the matching process, respectively. V_I is the value of this option. Third, the supply of vacancies is perfectly inelastic in the last two stages of the matching process, such that $V_I > 0$.

The above *job creation* condition can also be interpreted as follows. Due to search costs, each successful match generates a rent, which is distributed between worker and firm through the wage. $\Pi_I - V_I$ is the firm's contribution to the rent of a match with a STU worker. The price which the firm pays for participating in all three stages of the matching process is *k*, the *implicit* price for participating in the *first* stage of the matching process is lower and equal to $k - V_I$. Thus, the *job creation* condition states that the flow of vacancies into the labour market lasts until the implicit search cost a firm has to incur to take part in the first stage of the matching process equals its share of the match rent: $(k - V_I)/q_I = \Pi_I - V_I$.

The option value V_I of a vacancy in the first stage of the matching process, when the search costs *k* are sunk, is

$$V_I = q_S V_S + [1 - q_S F(T_S)] Q V_P, \tag{10}$$

where q_S denotes the probability that the vacancy will be filled by an active LTU. V_S is the conditional expected value of a job filled by such a worker. If the vacancy does not meet an active LTU or if the training costs of the applicant exceed T_S, a composite event with the probability $1 - q_S F(T_S)$, then the vacancy still has the third option to meet a LTU placed by the PES. The probability of a match with a LTU worker placed by the PES is *Q*, and the conditional option value of the job is V_P.[4]

Reservation costs. The hiring subsidy of the PES refunds the training costs up to the limit *H*. Matches with a worker placed by the PES and with training costs *t* higher than *H* must finance the balance $t - H > 0$ out of the match rent. The allocation of the remaining training costs is part of the contract negotiations, and the value of the filled job, $\Pi_P(t)$, therefore depends on *t*. As will be shown, $\Pi_P(t)$ is an *increasing* function of *t*, while the net value of the job, $\Pi_P(t) + H - t$, is a contraction and fulfils the reservation property with respect to *t*. Hence, reservation costs T_P exist, with

$$T_P = \Pi_P(T_P) + H\,. \tag{11}$$

Match partners whose training costs are lower than T_P sign an employment contract while with $t > T_P$ they separate immediately.

A vacancy filled by a LTU who is actively searching has the value $\Pi_S(t)$ if the match draws training costs *t*, with $t - \tau H \geq 0$. Remember that $\tau = 1$ if the PES subsidises both search methods, while $\tau = 0$ if the subsidy is paid only to matches arranged by the PES. In view of the third stage of the matching process, QV_P is the value of the outside option of the firm. Therefore, the job will only be filled if its net value is at least as high as the value of the option to meet a LTU worker placed by the PES, $\Pi_S(t) + \tau H - t \geq QV_P$. Since the net value of the job is a contraction and has the reservation property, reservation costs T_S also exist for the random search method

$$T_S = \Pi_S(T_S) + \tau H - QV_P\,. \tag{12}$$

2.4 Jobseeker and Wage Negotiations

The unemployed must decide between the active or the passive search strategy. During unemployment the worker receives the income *b*, which represents the unemployment benefit or imputed value of leisure. Given the exogenous unemployment benefit *b* the worker chooses the search strategy which maximises the present-discounted value of his human capital U_I

$$U_I = \max\{\rho(b+U),\quad -c_I + p_I W_I + (1-p_I)\rho(b+U)\}, \tag{13}$$

The Bellman equation (13) contains two alternatives. The first gives the value of the passive the second of the active search strategy. If the worker prefers the passive strategy he receives the unemployment benefit *b*, but does not receive support from the PES and his human capital takes on the value *U*.[5] In order to determine *U*, note that in the subsequent period the worker is still unemployed and has to decide again whether to wait for a placement via the PES or to search for a vacancy on the market. In the first case, the value of his human capital is U_P, in the second, it is U_S. The worker will opt for the search strategy that maximises the present value of his human capital so that

$$U = \max\{U_P, U_S\}. \tag{14}$$

In case the STU worker decides in favour of the second alternative in (13) he would choose to search randomly and would have to bear search costs $c_I > 0$. With probability p_I, he will locate a vacancy, and his value is W_I. With probability $1 - p_I$ his search fails, he receives the unemployment benefit *b* and takes on the value *U*. Next, we have to develop the present values of a passive and an active LTU worker U_P and U_S, respectively.

The present value of the human capital of a passive LTU is

$$\begin{aligned} U_P = P\{(1-q_I)(1-q_S)\left[F(H)W_P + \int_H^{T_P} W_P(t)dF(t) + [1-F(T_P)]\rho(b+U)\right] \\ + [q_I + (1-q_I)q_S]\rho(b+U)\} + (1-P)\rho(b+U) \end{aligned} \tag{15}$$

If the passive LTU is matched and if the vacancy for which he applies is not yet filled, the probability for this composite event is $P(1-q_I)(1-q_S)$, the value of the worker is W_P provided that the subsidy compensates fully for the training costs, that is if $t - H \leq 0$, an event which has the probability $F(H)$. Otherwise, if the training costs exceed *H* but are lower than the reservation costs T_P, the integral in (15) denotes the expected value of the employed worker. If the training costs exceed T_P, firm and applicant separate, and the present value of the worker is $\rho(b+U)$ as in the last two cases where the vacancy for which the worker applies is already filled, an event with probability $q_I + (1-q_I)q_S$, or the LTU is not offered a vacancy by the PES, an event which has the probability $1 - P$.

If the LTU worker decides for the active search strategy, he will incur search costs $c_S > 0$. Considering the contact probability p_S generated by random search, his present discounted value U_S is

$$\begin{aligned} U_S = -c_S + p_S\left\{(1-q_I)\left[F(\tau H)W_S + \int_{\tau H}^{T_S} W_S(t)dF(t) + [1-F(T_S)]U_P\right] + q_I U_P\right\} \\ + (1-p_S)U_P \end{aligned} \tag{16}$$

If the job search fails, either because the LTU is confronted with a vacancy already filled or because he incurs training costs that exceed T_S or because he does not find a vacancy, his value is equal to the value of the passive strategy U_P because placement via the PES is the final option which concludes the matching process.

Wage negotiations. Job search takes time and incurs search costs. Therefore, each match generates a monopoly rent that is distributed between firm and worker through the wage. The

distribution rules are obtained according to the generalised Nash solution to a bargaining problem, with $\beta \in (0,1)$ denoting the bargaining strength of the jobseeker.

If a STU meets a vacancy, the outside wage w_I for the initial period of the match is derived from the sharing rule

$$W_I - U_I = \frac{\beta}{1-\beta}(\Pi_I - V_I) \tag{17}$$

If the vacancy meets a LTU, the sharing rule depends on whether the PES refunds the training costs, or whether the agents have to negotiate the allocation of the balance $t - H \geq 0$. For wage negotiations with a LTU who is randomly searching, the sharing rule is

$$W_S(t) - U_S = \begin{cases} \dfrac{\beta}{1-\beta}\left[\Pi_S - QV_P\right], \text{ for } 0 \leq t \leq \tau H \\ \\ \dfrac{\beta}{1-\beta}\left[(\Pi_S(t) + \tau H - t) - QV_P\right], \text{ for } \tau H \leq t \leq T_S \end{cases} \tag{18}$$

where $W_S(t) - U_S$ is the jobseeker's share of the rent, and QV_P is the reservation value of the vacancy given the third stage of the matching process.

The sharing rule for workers placed by the PES is

$$W_P(t) - U_P = \begin{cases} \dfrac{\beta}{1-\beta}\Pi_P, \text{ for } 0 \leq t \leq H \\ \\ \dfrac{\beta}{1-\beta}\left[\Pi_P(t) + H - t\right], \text{ for } H \leq t \leq T_P \end{cases} \tag{19}$$

where $\Pi_P(t) + H - t$ is the firm's share of the rent if $H \leq t \leq T_P$.

Taking into account the idiosyncratic shock $x \in [R,1]$, the value of a STU, U_I, and the fact that in equilibrium the asset price of a vacancy at the initial stage of the search process is $V = 0$, the sharing rule implemented by negotiations with an insider is

$$W(x) - U_I = \frac{\beta}{1-\beta}\Pi(x). \tag{20}$$

Considering the asset pricing equations (4) – (9) and the sharing rules (17) – (20), we obtain

LEMMA 2 [BARGAINED WAGES]. *Given the reservation income rU_I of a STU and the asset values U_S and U_P of a LTU who prefers the active or passive search strategy respectively, the agents negotiate the following inside and outside wages.*

(i) *The bargained inside wage at a match specific productivity $x \in [R,1]$ is*

$$w(x) = rU_I + \beta(yx - rU_I). \tag{21}$$

(ii) *A STU worker who makes a job-to-job transition and produces, in the initial period, with productivity $x = 1$ receives the outside wage*

$$w_I = w(1) - \beta V_I \rho^{-1}, \tag{22}$$

where $w(1)$ is the inside wage (21) for $x = 1$, and $\rho^{-1} = 1 + r$.

(iii) *If the PES refunds the training costs, a LTU worker with human capital U_P placed by the PES receives the outside wage w_P with*

$$w_P = w(1) - \beta(1 - z)y + (1 - \beta)(U_P - U_I)\rho^{-1}, \text{ for } t \le H, \tag{23}$$

where $1 - z \ge 0$ is the skill depreciation rate during unemployment and yz is the flow output in the training period.

If $H \le t \le T_P$, the outside wage $w_P(t)$ in the training period is

$$w_P(t) = w_P - \beta(t - H)\rho^{-1}. \tag{24}$$

(iv) *If $t \le \tau H$ a LTU worker with human capital U_S who finds a job through random search receives the outside wage w_S:*

$$w_S = w(1) - \beta(1 - z)y + (1 - \beta)(U_S - U_I)\rho^{-1} - \beta Q V_P \rho^{-1}, \text{ for } t \le \tau H. \tag{25}$$

If the training costs exceed τH the bargained wage is

$$w_S(t) = w_S - \beta(t - \tau H)\rho^{-1}, \text{ for } \tau H \le t \le T_S. \tag{26}$$

As equation (21) shows, the inside wage equals the reservation income of the worker plus a share of the current match rent that depends on his bargaining strength β. As (17) makes clear, the value of the outside option V_I reduces the rent of a match with a STU, and, as a consequence, reduces the share of the current rent (22) a STU can appropriate in the contract negotiation. The time of the model is discrete. While the reservation value of the vacancy refers to the beginning of the period, wages are paid at the end; V_I, therefore, is discounted in (22) to the end of the period.

The higher the skill depreciation factor $1-z$ of a LTU, the lower the bargained outside wages, as equations (23) and (25) show. Moreover, training costs higher than *H* are partially passed on to the worker, so that the outside wages (24) and (26), respectively, fall with *t*.

Finally, the outside wages (23) and (25) depend on the balance of the present values of a LTU and a STU, $U_i - U_I$, $i = P, S$, and hence on their search strategies. To determine the signs and the magnitudes of the rents $U_i - U_I$, $i = P, S$, we first have to explain which search strategies the LTU and the STU use in equilibrium.

Choice of the search strategy. If $U_I > \rho(b+U)$, then all STU workers immediately search for a new job. The number of active jobseekers S_I among the STU rises, the tightness θ_I of the search market in the first stage of the matching process decreases, and the transition rate p_I falls. The adjustment process comes to an end either because the gains from the private job search are driven to zero, as $U_I = \rho(b+U)$, or because the total inflow of unemployed searches randomly for a job, so that $S_I = I$. In the following, we investigate the first case and assume that in equilibrium the gains from the search vanish so that $U_I = (\rho + U)$ and $S_I \leq I$.

The LTU choose the active search strategy if $U_S > U_P$. The number of active jobseekers *S* increases, the tightness θ_S of the labour market in the second stage of the matching process decreases, and the contact probability p_S diminishes until either all workers in the unemployment pool *u* search actively for a job, so that $S = u$, or the gains from private job search vanish, so that $U_S = U_P$ and $S \leq u$. In the following, we investigate the second case.

With $U_S = U_P$, the LTU are indifferent between the search strategies, and from the wage equations (23) and (25) it follows for the outside wage of an active jobseeker among the LTU: $w_S = w_P - \beta Q V_P \rho^{-1}$, for $t \leq \tau H$. Moreover, with $U_S = U_P$, it suffices to determine the sign and the magnitude of the rent $U_P - U_I$. If in equilibrium the STU are indifferent between the active or the passive search strategies, as we assume, then the differential rent $U_P - U_I$ can be derived from the asset equation (15), the sharing rule (19), and equation (A1) for the option value V_P of a vacancy that, in view of the third stage of the matching process, expects to meet a LTU placed by the PES (see Appendix V):

$$U_P - U_I = \frac{\beta}{1-\beta} \frac{P(1-q_I)(1-q_S)V_P}{\left[1 - P(1-q_I)(1-q_S)F(T_P)\right]}. \tag{27}$$

As we assume that the STU are indifferent between both search strategies in equilibrium, the differential rent (27) is strictly positive. The reason for this is the *response time* the PES needs to arrange a first contact between a registered STU and a registered vacancy. The PES is de facto only available to the LTU, the STU must wait at least one period, until the first PES job offer arrives. During this time the STU have to rely on their own search efforts.

The differential rent (27) increases together with the probability P for a contact via the PES, the reservation costs T_P, the probability $(1-q_I)(1-q_S)$ of finding a job that is not yet filled by one of the active jobseekers, and with the option value V_P.

2.5 Wage Dispersion and Job Destruction

With the wage equations from Lemma 2, the asset equations from Section 2.2, and the condition of the reservation productivity, $\Pi(R) = 0$, we can now derive the value of a filled job.

Lemma 3 [Filled Jobs]. (i) *The continuation value of a filled job producing with the idiosyncratic productivity* $x \in [R,1]$ *is*

$$\Pi(x) = (1-\beta) y \frac{x-R}{\lambda + r}. \tag{28}$$

(ii) *Taking into account the reservation value* V_I*, a job filled by a STU worker has the present value*

$$\Pi_I = \Pi(1) + \beta V_I, \tag{29}$$

where $\Pi(1)$ *is the continuation value (28) for the match productivity* $x = 1$.

(iii) *A job filled by a LTU who is placed and whose training costs are refunded by the PES has the value*

$$\Pi_P = \Pi(1) - \rho(1-\beta)(1-z)y - (1-\beta)(U_P - U_I), \text{ for } t \le H. \tag{30}$$

A job filled by a subsidised LTU whose training costs exceed H has the present value

$$\Pi_P(t) = \Pi_P + \beta(t-H), \text{ for } H \le t \le T_P. \tag{31}$$

(iv) *Since the LTU are indifferent between the two search strategies, taking into account the reservation value* QV_P*, a job filled by a worker who is actively searching has the asset price*

$$\Pi_S = \Pi_P + \beta Q V_P, \text{ for } t \le \tau H. \tag{32}$$

For training costs t with $\tau H \le t \le T_S$ *we finally obtain*

$$\Pi_S(t) = \Pi_S + \beta(t - \tau H). \tag{33}$$

From the value equations for the filled jobs, we can derive the reservation costs T_P and T_S.

LEMMA 4 [RESERVATION COSTS]. (i) *The reservation costs T_P which are applied to the LTU who are placed by the PES follow from (31) together with $\Pi_P(T_P) + H - T_P = 0$:*

$$T_P = \frac{\Pi_P}{1-\beta} + H. \tag{34}$$

From the asset pricing equations (31) - (33) and $T_S = \Pi_S(T_S) + \tau H - QV_P$ we can derive the reservation costs for active LTU who opt for the method of random search

$$T_S = T_P - (1-\tau)H - QV_P. \tag{35}$$

(ii) *As a consequence of the fact that $T_P - T_S = (1-\tau)H + QV_P > 0$, the percentage of LTU who are 'non-placeable' via the search market, is always higher than the percentage of LTU who cannot be placed via the PES: $1 - F(T_S) > 1 - F(T_P)$.*[6]

The dispersions of the outside wages of the LTU during the training period depend on the method of search and the distribution of the training costs.

LEMMA 5 [WAGE DISPERSIONS]. (i) *The dispersions of the outside wages of the LTU are defined on the closed intervalls $[w_P(T_P), w_P]$ and $[w_S(T_S), w_S]$, where $w_i(T_i)$ is the lowest and w_i is the highest wage of the respective wage dispersion, $i = P, S$. From Lemma 2 and Lemma 4, taking into account that in equilibrium $U_S = U_P$, it follows that $w_P(T_P) = w_S(T_S)$ and $w_P - w_S = \beta Q V_P \rho^{-1} > 0$.*

(ii) *The average wages of the normalized dispersions are given by $\overline{w}_P = [F(H)w_P + \int_H^{T_P} w_P(t)\,dF(t)] / F(T_P)$ and $\overline{w}_S = [F(\tau H)w_S + \int_{\tau H}^{T_S} w_S(t)\,dF(t)] / F(T_S)$. If the training costs are exponentially distributed, then $\overline{w}_P > \overline{w}_S$.*

Job destruction. The *job destruction* condition can be derived by evaluating the asset equation (4) at the reservation threshold $x = R$. Taking into account the wage equation (21) we obtain:

$$R = \frac{rU_I}{y} - \frac{\lambda}{\lambda + r} \int_R^1 (h - R)\,dG(h). \tag{36}$$

In order to close the model, we still have to determine the reservation income of a STU, rU_I, and the transition probabilities of the method of random search.

In equilibrium the STU and the LTU, by assumption, are indifferent between the active or the passive search strategy so that $U_I = \rho(b+U)$ and $U_P = U_S = U$. With these conditions, the reservation income of a STU is equal to the sum of the unemployment benefit and the differential rent $U_P - U_I$:

$$rU_I = b + (U_P - U_I). \tag{37}$$

Taking into account that jobseekers in equilibrium are indifferent between the two search strategies, we finally obtain the transition probabilities generated by the search market, $p(\theta_I)$ and $p(\theta_S)$, as follows.

LEMMA 6 [RANDOM SEARCH]. (i) *From the Bellman equation (13) and* $U_I = \rho(b+U)$ *it follows that, in equilibrium, the expected search costs of a STU worker who is randomly searching are equal to his share of the match rent,* $c_I / p_I = W_I - U_I$. *From this, together with the sharing rule (17) and the asset equation (29), we obtain*

$$\frac{c_I}{p(\theta_I)} = \frac{\beta}{1-\beta}[\Pi(1) - (1-\beta)V_I]. \tag{38}$$

(ii) *Using the assumption* $U_P = U_S = U$ *and the asset equation (16), it follows that, in equilibrium, the expected search costs of a LTU worker who is randomly searching are equal to his expected share in the match rent:* $c_S/(1-q_I)p_S = F(\tau_H)(W_S - U_S) + \int_{\tau_H}^{T_S}(W_S(t) - U_S)dF(t)$. *From this equilibrium condition we obtain with respect to the sharing rule (18) and the option value (A2)* (see Appendix III)

$$\frac{c_S}{[1-q(\theta_I)]p(\theta_S)} = \frac{\beta}{1-\beta}[V_S - QV_P]. \tag{39}$$

Equilibrium. The equilibrium of the search model consists of solutions $[\Pi(1), \Pi_P, \Theta, \theta_I, \theta_S, R, T_P, T_S, u]$ to the model equations (A5) – (A12) in Appendix IV and the equilibrium unemployment (3). The comparative static effects of the hiring subsidy are indeterminate as a consequence of the multiplicity of the channels through which the hiring subsidy works. We therefore have carried out a series of numerical experiments.

3 SIMULATION

Parameters and matching functions. The choice of the baseline parameters, as shown in Table1, is made with respect to the following three criteria. First, parameters are chosen so that

the steady-state unemployment rate and the incidence of long-term unemployment without labour market policy are somewhat below the European level of the past few years. The reason for this decision is that most European countries extensively use hiring subsidies and profiling techniques by now so that the unemployment rates have already been affected by these instruments. In 2005, e.g., the average EU-15 unemployment rate was 7.9 % (OECD 2006). Because the time period in our model is assumed to be one quarter, our empirical reference point is the average incidence of job seekers who are threatened with long-term unemployment or who have already been long-term unemployed (3 months or more) which was 76.5 % in the EU-15 in 2005 (OECD 2006). Second, we use the parameters choosen by Mortensen and Pissarides (2003). Third, we have to take into account that in equilibrium the numbers of active jobseekers, S_I and *S*, have to be 'interior solutions' of the model. A sensitivity analysis is conducted on all parameters. Results vary in size but the general conclusions are robust.

Table 1: Baseline parameters of the model

Output y	100	Bargaining power β	0.50
Skill depreciation rate 1 - z	0.40	Real interest rate r	0.02
Unemployment benefit b	60	Probability of a shock λ	0.10
Recruiting costs of a vacancy k	30	Elasticity of the job matches SM ϕ	1/5
Search cost STU c_I	40	Elasticity of the job matches PES Φ	4/5
Search cost LTU c_S	25	Total factor productivity SM d	0.30
Mean training costs $1/\delta$	15	Total factor productivity PES ef	0.30

We assume the following baseline parameter values, see Table 1. The bargaining power of the workers is $\beta = 0.50$, the marginal product of a job at full productivity is $y = 100$. The rate of skill depreciation is 40 percent, such that during the training period a LTU worker produces an output of $yz = 60$. The UI benefits, $b = 60$, match the continental European replacement rates which vary between 40 % (Italy) and 90 % (Denmark) of the last net-income of the unemployed (OECD 2004, 21); the real interest rate *r* is 2 %; the probability of a productivity shock λ is 10 %; the search costs are $c_I = 40$ and $c_S = 25$ and amount to roughly 40 % of the initial productivity of both types of jobseekers; the recruiting costs of a vacancy amount to $k = 30$.

The distribution function $G(x)$ of the productivity shocks is assumed to be uniform on $[\alpha,1]$, with the lower support $\alpha = 0.65$. Training costs $t \geq 0$ are exponentially distributed with mean $1/\delta = 15$. The matching functions of the PES and the search market are of the Cobb

Douglas type (Petrongolo and Pissarides 2001). For a given vacancy the probabilities of a contact with a jobseeker are for the PES $Q(\Theta) = ef * (1/\Theta)^{1-\Phi}$ and for the search market $q(\theta) = d * (1/\theta)^{1-\phi}$. The values of the "total factor productivities" of the basic scenario are $ef = d = 0.30$; for the elasticities of the job matches *M* and *m* with respect to vacancies we use $\Phi = 4/5$ and $\phi = 1/5$ respectively. Thus, among the arguments of the matching technology of the PES, the vacancies dominate, while in the search market the active jobseekers are the dominating input factor.[7]

Indicators. The period of analysis is one quarter of a year. The following indicators, summarized in Table 2, are used to evaluate the simulations:

Table 2: Indicators

u	Quarterly unemployment rate in percent;
λG	Quarterly unemployment incidence in percent;
$\sigma_I p_I = S_I p_I / u$	Ratio of the STU making job-to-job transitions;
$\sigma_S = S/u * 100$	Fraction of active jobseekers among the LTU;
$LTU = (1 - u_S/u) * 100$	Fraction of the LTU;
d_S , d_P	Unemployment duration of active and passive LTU in quarters;
$100 - wIP$ with $wIP = \overline{w}_P / w_I * 100$	Average wage penalty, a LTU placed by the PES must accept due to skill loss and training costs, s. Lemma 4;
$PES = (p_{III}(u - p_S S)/u_S) * 100$	Placement rate of the PES, defined as the share of successful matches of passive LTU arranges by the PES to the inflow u_S of STU to the pool of LTU, with $p_{III} = P(1-q_I)(1-q_S)F(T_P)$

The results of the simulations with the upper limit *H* for the hiring subsidy are shown graphically in Appendices I – II. We distinguish between a policy design which supports only placements by the PES (regime $\tau = 0$) and a design which gives equal support to both search methods (regime $\tau = 1$). Appendix I shows the results for both regimes ($\tau = 0$ and $\tau = 1$). Appendix II depicts the results for $\tau = 0$ at varying matching productivities of the public placement service ($ef = 0.25$, $ef = 0.30$ and $ef = 0.35$).

In the Mortensen-Pissarides model (1999, 2003) the hiring subsidy lowers the costs of job creation, so that on the one hand creation is stimulated and the unemployment duration falls. On the other hand the unemployment incidence increases. Because of the increasing tightness the opportunity costs of a filled job rise and the match partners separate faster. The second effect outweighs the first so that total employment decreases.

One innovation of our model is that four factors have an affect on equilibrium unemployment, see equation (3) and Table 2. First, the incidence of unemployment and the job destruction rate, second the ratio of job-to-job transitions, third the share of active jobseekers among the LTU, and finally the length of the unemployment spells of active and passive LTU.

Result 1. The figures in Appendices I and II show that consistent with Mortensen and Pissarides (1999, 2003) the hiring subsidy *H* increases the equilibrium rate of unemployment *u*. In regime $\tau = 0$, where only PES placements are subsidised, *u* increases from 7.4 % ($H = 0$) to 8.4 % ($H = 30$). In comparison the average EU unemployment rate was 7.9 % in 2005 (OECD 2006).

There are four channels through which the hiring subsidy impacts the aggregate rate of unemployment. In the regime $\tau = 0$ firms and workers can only pocket the hiring subsidy if they are matched by the PES. Therefore the subsidy increases the opportunity costs of a start-up in the first and second stages of the search process. The consequences are: First, the fraction of active job-seekers among the LTU, σ_S and, second, the ratio of job-to-job transitions, $\sigma_I p_I$, fall; third, the fractions of those STU and LTU who prefer to wait for a placement by the PES increase. As a result the incidence of LTU increases from 72 % ($H = 0$) to 73.6 % ($H = 30$). In comparison, in 2005 the average incidence of job seekers threatened with long-term unemployment or being long-term unemployed (3 months or more) was 76.5 % in the EU.

Fourth, the hiring subsidy reduces the duration of unemployment but only the duration of the active job-seekers, d_S, while the average spell length d_P of an unemployed worker who decides on the passive search strategy increases in the regime $\tau = 0$. The reason for these four effects is that the growing number of passive job-seekers is concentrated in the third stage of the matching process. As a result the tightness between the registers of the PES declines, the *response time* $1+1/P(\Theta)$ of the PES rises and the probability of a successful match arranged by the PES falls. It is not surprising that in the regime $\tau = 0$ the duration of the unemployment spell of the active job-seeker falls because, on the one hand, the supply of vacancies rises due to the subsidy and, on the other hand, the number of active job-seekers falls.

Result 2. Although the subsidy raises the fraction of active job-seekers among the LTU in regime $\tau = 1$, the symmetrical labour market policy lowers overall employment. The reasons

are: First the subsidy increases the opportunity costs of the filled jobs even more than in the policy regime $\tau = 0$, as start-ups under $\tau = 1$ can pocket the money not only in the third but also in the second stage of the search process. Consequently the unemployment incidence increases with the subsidy and is even higher than in the regime $\tau = 0$. Second the policy leads to a crowding-out of active job-seekers among the STU and reduces the job-to-job transitions below the level reached in the regime $\tau = 0$. Third, the fraction of LTU workers declines, but not enough to overcome the growing concentration of active job-seekers among the LTU. That leads contrary to the regime $\tau = 0$ to an increasing duration of job search of the active job-seekers d_S. Fourth, as in the regime $\tau = 0$ the unemployment duration d_P of the passive LTU rises. Nevertheless, as a consequence of the lower fraction of LTU workers and the higher and increasing fraction of active LTU, the equilibrium rate of unemployment does not increase as much as it does in the regime $\tau = 0$.

Result 3. In regime $\tau = 0$ the subsidy raises the placement rate of the PES, while it lowers the rate in regime $\tau = 1$. The main reasons for these two effects are that in regime $\tau = 0$ the subsidy is only paid for LTU workers placed by the PES, whereas in regime $\tau = 1$ the symmetric labour market policy increases the fraction of active job-seekers among the LTU worker and therefore the fraction of vacancies which have already been filled at the time of the arrival of the LTU placed by the PES.

Result 4. Without hiring subsidy ($H = 0$) the LTU placed by the PES must accept on average a 5.6 % wage penalty compared to a STU worker when making a job-to-job transition. The subsidy ($H = 30$) turns this penalty into a wage advantage of 3.5 % (4.0 %) for the LTU under the regime $\tau = 0$ ($\tau = 1$). The reason is that the subsidy refunds the training costs and thus raises the surplus of a match with a LTU worker. Through the wage negotiations the workers appropriate a share of the additional surplus and their outside wage increases.

In comparison: based on the first seven rounds of the British Household Panel Survey, Arulampalam (2001) estimates that, after an unemployment spell, a worker must accept a wage penalty of 5.7 % compared to the wage he would have been offered had he made a job-to-job transition.[8]

Result 5. The more effective the placement service of the PES, measured by the total factor productivity *ef* of the PES matching function, the higher the equilibrium unemployment is (see Appendix II). Nevertheless the placement rate of the PES, and thus the success, which the PES will claim for its organization reforms, increases with total factor productivity.

The reasons for the rising structural unemployment are: First, the more effective job placement service raises the opportunity costs of the occupied jobs and therefore the incidence of unemployment. While a job with $ef = 0.30$ has a mean durability of $1/\lambda G(R) * 100 = 36$ quarters or 9.0 years, the durability falls to 8.2 years for $ef = 0.35$. Second, the fraction of active job-seekers among the STU and thus the ratio of the STU making job-to-job transitions strongly declines with increasing *ef*. Third, on the one hand the duration of the unemployment spells for both types of job-seekers is reduced. For the passive strategy the duration falls from 5.2 to 4.8 quarters, for the active from about 1.9 to 1.6 quarters ($H = 0$). The fact that d_P falls is obviously due to the higher productivity of the PES. On the other hand the decline of d_S results from the reduction in the number of the active job-seekers among the LTU worker. This improves the chances of the remaining job-seekers who stick to their search strategy.

Even though the higher productivity of the PES lowers the unemployment duration of both search strategies the negative effects outweigh the positive effect of the lower unemployment duration. The job destruction rate will increase and the fraction of active STU and LTU workers will decrease, so that the improvement of the effectiveness of the PES will lead to a higher aggregate rate of unemployment.

4 SUMMARY

One innovation of our model is that apart from the endogenous job destruction rate the equilibrium rate of unemployment reflects the ratio of job-to-job transitions, the share of active jobseekers among the short-term (STU) and the long term unemployed (LTU) and the length of the unemployment spells of active and passive jobseekers among the LTU. The type-specific durations of unemployment depend for its part on the effectiveness of the placement service of the PES, the rate of skill depreciation during unemployment and the distribution of the match specific training costs, which a firm employing LTU workers must spend to train its entrants.

In our model jobseekers have two methods of search available, the placement service of the PES and random search. They can choose between an active and a passive search strategy, while the matching process consists of three stages. In the first only the active jobseekers among the STU search randomly for a vacancy. The STU have lost their job at the end of the previous period and, therefore, of all jobseekers the STU possess the best information about current labour market conditions. Their applications are more targeted and reach the firms earlier than the applications of other unemployed. In the second stage the active jobseekers among the LTU apply, and finally, in the third stage, also those LTU who are sent by the PES.

The PES refunds the training costs with a hiring subsidy. Two regimes are compared. One regime under which only the matches arranged by the PES are subsidised, and a second regime which subsidises all matches with a LTU worker, irrespective of the method of search. The effects of the hiring subsidy depend on the target group. For STU, who are only indirectly affected, hiring subsidies are always counterproductive and reduce the incentives to search actively for a vacancy. But for eligible LTU the subsidy has a stimulating effect on job creation. If all LTU are supported, the subsidy can even reduce the incidence of LTU. Nevertheless under both regimes the unemployment rate increases with the hiring subsidy. The main reasons are the rising job destruction rate, the declining fraction of active jobseekers among the STU and of the job-to-job transitions, and the increasing duration of unemployment of the passive jobseekers.

Certainly, the PES can increase its placement success by improving the effectiveness of its matching service. But the job destruction rate will rise and the fraction of active jobseekers among the STU will fall so that the organizational reform will lead to higher equilibrium unemployment, although the unemployment duration for both groups of jobseekers, the passive and the active, is reduced.

The economic policy consequences of the model are clear: the effects of a hiring subsidy and profiling techniques to increase the effectiveness of the placement service depend on the target group. For the short-term unemployed not only indirect effects of the hiring subsidy but also of the job placement activities of the PES are counterproductive. On the other hand these instruments have a stimulating effect on job creation for target groups who in the equilibrium

without policy have no incentive to actively search for a job. But policy makers have to take into account that despite their stimulating effects the instruments of active labour market policy reduce aggregate employment.

NOTES

1 Appendices III-V are mainly provided for the purpose of the referees and are available upon request.

2 Mortensen and Pissarides (1999) and Pissarides (2000) present a discussion of objections against the plausibility of this assumption and the two-tier wage structure that results from the possibility of renegotiation.

3 Specialisation of one of the two search methods may occur because of the heterogeneity of the jobseekers or the jobs or because of increasing search costs. We assume that the search cost function of a vacancy with respect to the two search methods is sub-additive, so that, considering the asset value of a vacancy, it is advantageous for firms to offer vacancies through both channels.

4 Appendix AIII contains the asset equations for V_S and V_P.

5 The endogenisation of *b* (Mortensen and Pissarides 2003) lowers the search incentive and thus strengthens the comparative static effects of *H*, which are shown in Section 3.

6 With Lemma 3 and Lemma 4 the option values of a vacancy V_P and V_S are only functions of the subsidy limit *H*, the reservation costs T_P and T_S, the tightness Θ, and the design $\tau \in \{0,1\}$ of the hiring subsidy (s. App. III).

7 The general results are not affected if we set $\phi = \Phi = \beta$. In the standard matching model this condition internalises the effects of the search externalities on the equilibrium outcome (Hosios 1990, Pissarides 1990).

8 This wage penalty increases to 14 % in the fourth year after the unemployment spell and then decreases again (Arulampalam 2001).

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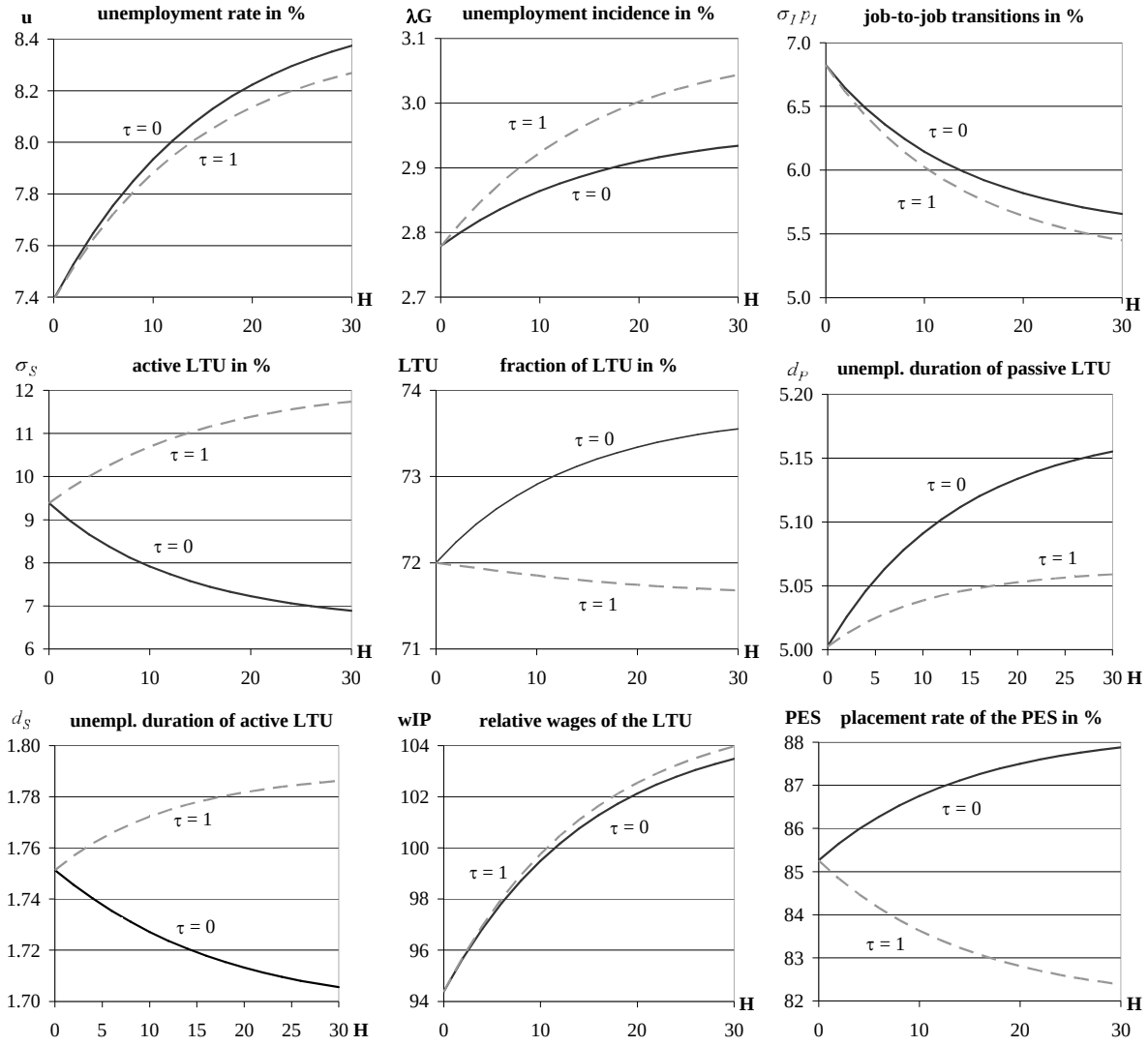
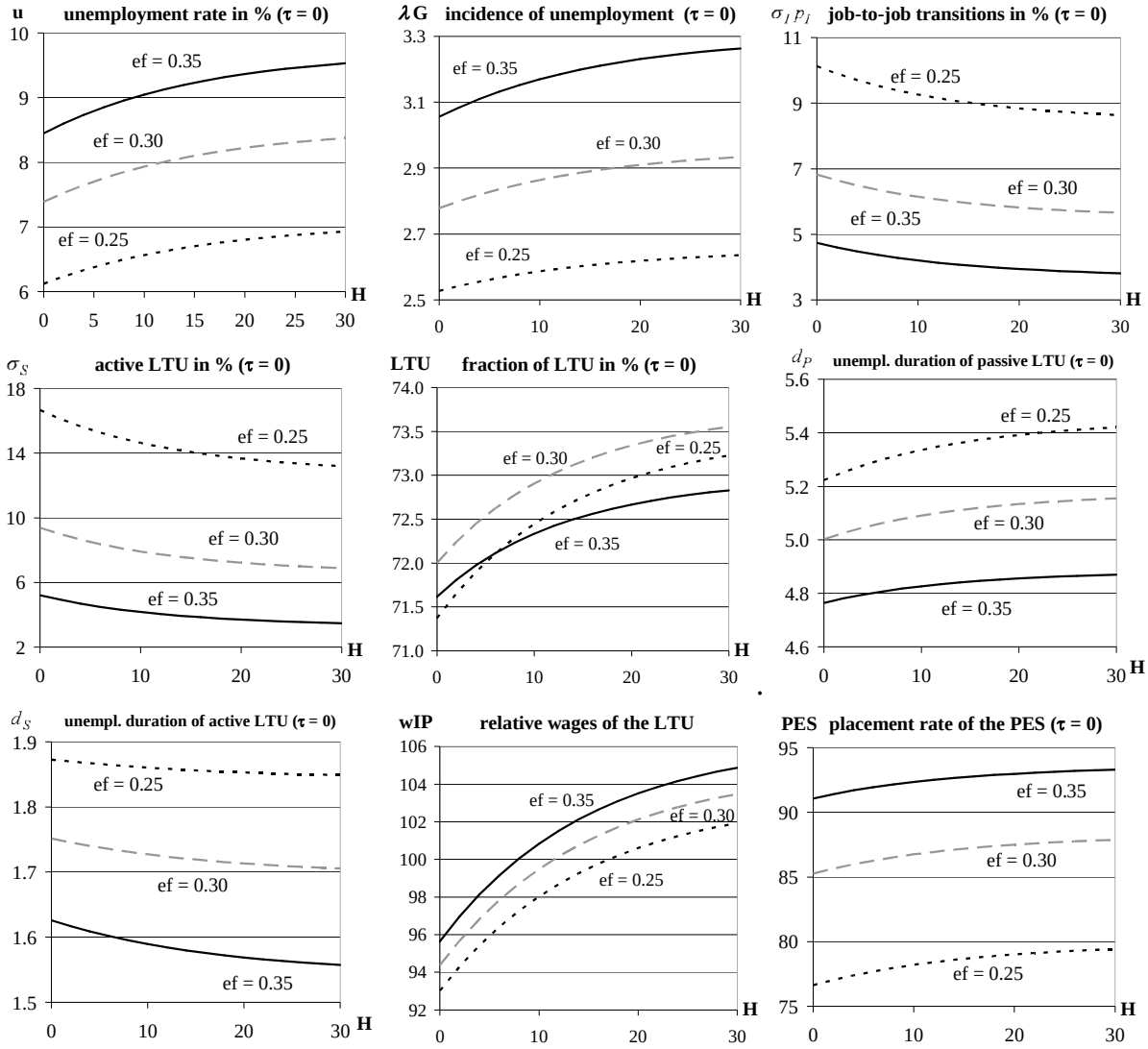
APPENDIX I: BASIC SCENARIO ($\tau = 0$, $\tau = 1$)



APPENDIX II: EFFECIVENESS ($\tau = 0$)



APPENDIX III

Option values V_P and V_S. 1. When firms decide whether to post a vacancy they know the c.d.f. of the training costs $F(t)$, the reservation costs T_i and the subsidy design $\tau \in \{1,0\}$. Before the training costs are revealed the asset value of a vacancy expecting a contact with a LTU worker placed by the PES is

$$V_P = \int_0^H \Pi_P \, dF(t) + \int_H^{T_P} \left[\Pi_P(t) + H - t\right] dF(t). \qquad \text{(A1)}$$

If the training costs of the LTU are fully refunded, the job has the value Π_P, see equation (30). The second term in (A1) denotes the expected value of the job if the training costs are higher than the subsidy limit H but below the reservation costs T_P. Finally, if the training

costs exceed T_P, the match partners separate immediately. Analogously, before training costs are known the conditional option value of a vacancy that meets an active jobseeker among the LTU is

$$V_S = \int_0^{\tau H} \Pi_S \, dF(t) + \int_{\tau H}^{T_S} \left[\Pi_S(t) + \tau H - t\right] dF(t) + \int_{T_S}^{\infty} QV_P \, dF(t), \tag{A2}$$

where $\tau = 1$ if the PES also subsidises the matches formed by random search, otherwise $\tau = 0$. If the match specific training costs exceed T_S, the match dissolves and, in view of the third stage of the matching process, the vacancy takes on the value of the outside option QV_P.

2. With respect to the asset equations (28) – (33) and the reservation costs, Lemma 4, the option values (A1) and (A2) of a vacancy are determined by

$$V_P(T_P, H) = (1-\beta)\left[F(H)(T_P - H) + \int_H^{T_P} (T_P - t) dF(t)\right] \qquad \text{and} \tag{A3}$$

$$V_S(\Theta, T_P, T_S, H, \tau) = (1-\beta)\left[F(\tau H)(T_S - \tau H) + \int_{\tau H}^{T_S} (T_S - t) dF(t)\right] + Q(\Theta) V_P(T_P, H) \tag{A4}$$

APPENDIX IV

The model equations in implicit form are:

$$J^1(\Pi(1), \Pi_P, \Theta, \theta_I, \theta_S, R, T_P, T_S; H) \equiv \Pi(1) - (1-\beta) y \frac{1-R}{\lambda + r} = 0 \tag{A5}$$

$$J^2(\cdot) \equiv \Pi(1) - \rho(1-\beta)(1-z)y - \Pi_P - \frac{\beta P(1-q_I)(1-q_S)}{1 - P(1-q_I)(1-q_S)F(T_P)} V_P(T_P, H) = 0 \tag{A6}$$

$$J^3(\cdot) \equiv \Pi_P - (1-\beta)(T_P - H) = 0 \tag{A7}$$

$$J^4(\cdot) \equiv \frac{k - V_I(\Theta, T_P, T_S, \theta_S, H, \tau)}{q_I(\theta_I)} - \left[\Pi(1) - (1-\beta) V_I(\Theta, T_P, T_S, \theta_S, H, \tau)\right] = 0 \tag{A8}$$

$$J^5(\cdot) \equiv R - \frac{b}{y} - \frac{\Pi(1) - \rho(1-\beta)(1-z)y - \Pi_P}{(1-\beta)y} + \frac{\lambda}{\lambda + r} \int_R^1 (h - R) dG(h) = 0 \tag{A9}$$

$$J^6(\cdot) \equiv T_P - T_S - (1-\tau)H - Q(\Theta) V_P(T_P, H) = 0 \tag{A10}$$

$$J^7(\cdot) \equiv \frac{c_I}{p(\theta_I)} - \frac{\beta}{1-\beta}\left[\Pi(1) - (1-\beta) V_I(\Theta, T_P, T_S, \theta_S, H, \tau)\right] = 0 \tag{A11}$$

$$J^8(\cdot) \equiv \frac{c_S}{(1-q_I)p(\theta_S)} - \frac{\beta}{1-\beta}\left[V_S(\Theta, T_P, T_S, H, \tau) - Q(\Theta)V_P(T_P, H)\right] = 0\,. \qquad \text{(A12)}$$

APPENDIX V

Proof of Lemma 1. (i) At the time at which the job search of the public placement service for a given unemployed begins, one period has already passed since the first day of the current unemployment spell *of* the jobseeker. Since the PES arranges contacts with probability *P*, $1+1/P$ periods elapse between the onset of the unemployment spell and the first job contact. $(1-q_I)(1-q_S)$ is the probability that the vacancy found is neither filled with an active STU nor with an active jobseeker among the LTU. Therefore the average duration of data processing and search for an unfilled vacancy amounts to $1+1/P(1-q_I)(1-q_S)$ periods. If firm and applicant meet, the firm observes the training costs $t \geq 0$. If $t \leq T_P$, the applicant is accepted, an event, which has the probability $F(T_P)$. Therefore the average length of time required for data registration, processing, and for a successful job search by the PES as well as for a positive assessment of the worker's skills is equal to D_P.

(ii) The reasoning *is similar to that in the former case.*

*(iii) An STU wor*ker, who lost his job at the end of the previous period and opts for the active search strategy, meets a vacancy immediately with probability p_I. In this case, the duration of job search is equal to zero and the worker makes a job-to-job transition. With probability $1-p_I$ the search fails and the worker becomes an active LTU, with an average duration of unemployment equal to D_S. Therefore the average duration of unemployment of an active STU is $(1-p_I)D_S$.

Proof of Lemma 2. (i) Write the sharing rule used for the negotiations with an insider (20) as

(P1) $$(1-\beta)U_I = (1-\beta)W(x) - \beta\Pi(x)\,.$$

Substitute $\Pi(x)$ and $W(x)$ with the asset pricing equations (4) and (5) out of (P1) and the inside-wage (21) follows.

(ii) From the sharing rule used for the negotiations with the STU (17) follows

(P2) $$(1-\beta)U_I - \beta V_I = (1-\beta)W_I - \beta\Pi_I\,.$$

Now the outside-wage (22) for STU workers follows from (P2), the asset pricing equations (6) and (7) and (P1).

(iii) Write the sharing rule (19) as

(P3) $$(1-\beta)U_P - \beta(t-H) = (1-\beta)W_P(t) - \beta\Pi_P(t).$$

Substitute the values of the filled job and the employed worker with (8) and (9) out of (P3), and take into account (P1) and (21) to get the wage equation

(P4) $$w_P(t) = w(1) - (1-z)\beta y + (1-\beta)(U_P - U_I)\rho^{-1} - \beta(t-H)\rho^{-1}.$$

The wage equations (23) and (24) follow from (P4). Notice that the last term on the RHS of (P4) is equal to zero for $t \le H$.

(iv) As in (iii), the wage equations (25) and (26) follow from the asset pricing equations (8) and (9), (P1) and the sharing rule (18), which we can write as

(P5) $$(1-\beta)U_S - \beta(t-\iota H) - \beta QV_P = (1-\beta)W_S(t) - \beta\Pi_S(t).$$

Proof of equation (27). Rearrange the asset pricing equation (15), and take account of the equilibrium condition $U_I = \rho(b+U)$ to get

(P6) $$(U_P - U_I)[1 - P(1-q_I)(1-q_S)F(T_P)] = P(1-q_I)(1-q_S)\left[F(H)(W_P - U_P) + \int_H^{T_P}(W_P(t) - U_P)dF(t)\right]$$

Substitute the sharing rule (19) into the worker's share of the match rent on the RHS of (P6) and take account of the asset equation of the outside option (A1), to find the equation of the differential rent (27).

Proof of Lemma 3. (i) Equations (4) and (5) imply $0 = yR - w(R) + \lambda\int_R^1 \Pi(h)dG(h)$ and $(\lambda + r)\Pi(x) = yx - w(x) + \lambda\int_R^1 \Pi(h)dG(h)$. From these two equations together with the wage equation (21) the statement follows.

(ii) Insert the wage equation (22) into the asset equation (6) and take account of equation (4) to derive the asset pricing equation (29).

(iii) The asset pricing equations (30) and (31) follow from substituting the wage equation (P4) into (8) and rearranging terms with respect to the asset equation (4).

(iv) Similar to the above argument we can derive the asset pricing equations (32) and (33) from (8) by taking into account the wage equations (25) and (26).

Proof of Lemma 4. (i) Write the asset equation (31) as $\Pi_P(T_P)+H-T_P = \Pi_P-(1-\beta)(T_P-H)$, and take account of the condition of the reservations costs, $\Pi_P(T_P)+H-T_P=0$.

Write (32) and (33) as $\Pi_S(T_S)+H-T_S=\Pi_P+\beta QV_P-(1-\beta)(T_S-\iota H)$, take account of $\Pi_S(T_S)+H-T_S=QV_P$ to derive $T_S=\Pi_P/(1-\beta)+\tau H-QV_P$. Substitute (34) into the last equation and the statement follows.

Proof of Lemma 5. (i) Substitute $w_S=w_P-\beta QV_P\rho^{-1}$ into the wage equation (26) to get $w_S(T_S)=w_P-\beta(T_S-\tau H+QV_P)\rho^{-1}$. By Lemma 3 $T_S-\tau H+QV_P=T_P-H$. Therefore we can conclude taking into account the wage equation (24): $w_S(T_S)=w_P-\beta(T_P-H)\rho^{-1}=w_P(T_P)$.

(ii) First we define the auxiliary functions $z(x)$ and $K(x,\tau)$, $x\in[0,T_P-H)$, as

(P7) $$z(x)=T_P-H-x$$

(P8) $$K(x,\tau)=x+\frac{\int_{\tau H}^{z(x)+\tau H}(t-\tau H)\,f(t)dt}{F(z(x)+\tau H)}$$

$K(x,\tau)$ is continuously differentiable on $[0,T_P-H)$, if the p.d.f. of the training costs $f(t)$, $t>0$, is differentiable.

Inserting the wage equations (24) and (26) into the expectations of the wage distributions, $\overline{w}_P$ and $\overline{w}_S$, and taking account of $w_S=w_P-\beta QV_P\rho^{-1}$ and (35), we can rewrite $\overline{w}_P$ and $\overline{w}_S$ with respect to (P8) as

(P9) $$\overline{w}_P=w_P-\beta K(0,1)\rho^{-1}$$

(P10) $$\overline{w}_S=w_P-\beta K(QV_P,\tau)\rho^{-1}.$$

Now, (P9) and (P10) imply: $\overline{w}_P>\overline{w}_S \quad\Leftrightarrow\quad K(0,1)<K(QV_P,\tau)$, where $\tau\in\{0,1\}$.

Assume that

(P11) $$K(x,\tau)>K(0,\tau),$$ and

(P12) $$K(0,0)\geq K(0,1),$$

For $\tau = 1$ the statement follows from assumption (P11). For $\tau = 0$ the assumptions (P11), (P12) and $QV_P > 0$ imply that $K(QV_P,0) > K(0,0) \geq K(0,1)$. Using the inequalities again then $\overline{w}_P > \overline{w}_S$ follows.

If the training costs are exponentially distributed, $x \in [0, T_P - H)$, and $H \geq 0$, then the inequalities (P11) and (P12) hold.

1. Let $f(t) = \delta e^{-\delta t}$, $\delta > 0$, then

(P13) $$K(x,\tau) = x + \frac{e^{-\delta\tau H}\left[1 - (1 + \delta z(x))e^{-\delta z(x)}\right]}{\delta\left(1 - e^{-\delta\left(1 - e^{-\delta(z(x) + \tau H)}\right)}\right)}$$

2. From (P13) and $H \geq 0$ we can conclude that (P12) holds.

3. The function $K(x,\tau)$ is continuously differentiable on $[0, T_P - H)$. To prove that $K(x,\tau)$ is strictly monotonically increasing with respect to x, we compute the partial derivative of $K(x,\tau)$:

(P14) $$\frac{\partial K(x,\tau)}{\partial x} = 1 - \frac{e^{-\delta(z + \tau H)}\left[\delta z - e^{-\delta\tau H}\left(1 - e^{-\delta z}\right)\right]}{\left(1 - e^{-\delta(z + \tau H)}\right)^2}$$

For $z > 0$ it is true that $\partial K(x,\tau)/\partial x > 0 \quad \Leftrightarrow \quad 1 > e^{-\delta(z + \tau H)}\left[(2 + \delta z) - e^{-\delta\tau H}\right]$. As the inequality on the RHS of the equivalence holds for $\tau H \geq 0$ the statement follows.