

Institute for Employment  
Research

The Research Institute of the  
Federal Employment Agency



# IAB-Discussion Paper

22/2015

Articles on labour market issues

## Factor structural time series models for official statistics with an application to hours worked in Germany

Roland Weigand  
Susanne Wanger  
Ines Zapf

ISSN 2195-2663

# Factor structural time series models for official statistics with an application to hours worked in Germany

Roland Weigand (IAB)  
Susanne Wanger (IAB)  
Ines Zapf (IAB)

Mit der Reihe „IAB-Discussion Paper“ will das Forschungsinstitut der Bundesagentur für Arbeit den Dialog mit der externen Wissenschaft intensivieren. Durch die rasche Verbreitung von Forschungsergebnissen über das Internet soll noch vor Drucklegung Kritik angeregt und Qualität gesichert werden.

The “IAB Discussion Paper” is published by the research institute of the German Federal Employment Agency in order to intensify the dialogue with the scientific community. The prompt publication of the latest research results via the internet intends to stimulate criticism and to ensure research quality at an early stage before printing.

**Contents**

Abstract . . . . . 4

Zusammenfassung . . . . . 4

1 Introduction . . . . . 5

2 A high-dimensional structural time series model . . . . . 6

    2.1 The factor model . . . . . 6

    2.2 Estimation by collapsing the factor space . . . . . 9

3 Observation schemes . . . . . 10

4 A Monte Carlo study . . . . . 12

5 Application to German hours worked statistics . . . . . 15

    5.1 Paid and unpaid overtime hours . . . . . 16

    5.2 Net flows on working time accounts . . . . . 19

6 Conclusion . . . . . 21

A Properties of the factor model in differences . . . . . 22

B The state space form . . . . . 23

References . . . . . 33

## Abstract

We introduce a high-dimensional structural time series model, where co-movement between the components is due to common factors. A two-step estimation strategy is presented, which is based on principal components in differences in a first step and state space methods in a second step. The methods add to the toolbox of official statisticians, constructing timely regular statistics from different data sources. In this context, we discuss typical measurement features such as survey errors, statistical breaks, different sampling frequencies and irregular observation patterns, and describe their statistical treatment. The methods are applied to the estimation of paid and unpaid overtime work as well as flows on working-time accounts in Germany, which enter the statistics on hours worked in the national accounts.

## Zusammenfassung

Wir stellen ein strukturelles Zeitreihenmodell für hochdimensionale Anwendungen zur Diskussion, bei dem Zusammenhänge zwischen den einzelnen Komponenten mithilfe gemeinsamer Faktoren modelliert werden. Es wird ein zweistufiges Schätzverfahren beschrieben, das in der ersten Stufe auf eine Hauptkomponentenanalyse in Differenzen und in der zweiten Stufe auf State-Space Methoden zurückgreift. Diese vorgeschlagene Methodik erweist sich insbesondere in der amtlichen Statistik als nützlich, wo regelmäßige Zeitreihen in Echtzeit aus den verschiedenen bis dahin vorliegenden Datenquellen konstruiert werden. In diesem Zusammenhang beschreiben wir übliche Eigenschaften der Messung, wie Surveyfehler, statistische Brüche, verschiedene Messfrequenzen sowie unregelmäßige Beobachtungsschemata, und erläutern deren Handhabung. Die Methoden werden auf die Schätzung bezahlter und unbezahlter Überstunden sowie von Saldenveränderungen auf Arbeitszeitkonten in Deutschland angewandt, die Eingang in die Volkswirtschaftlichen Gesamtrechnungen findet.

**JEL classification:** C14, C32, C51, C53, C58

**Keywords:** Factor model, structural time series, unobserved components, state space model, official statistics, national accounts, missing data, mixed frequencies

**Acknowledgements:** The authors are grateful for helpful comments by Enzo Weber, by participants of the IAB-Bundesbank seminar 2013 in Frankfurt am Main and of the German Statistical Week 2014 in Hannover.

# 1 Introduction

Factor models for high-dimensional time series have become an indispensable tool for macroeconomic fore- and nowcasting as well as structural modeling; see Bai/Ng (2008) as well as Stock/Watson (2011) for recent surveys. Typically, seasonally adjusted variables enter the model in first or second differences, while the factors are modeled as a stationary VAR process. Methods for handling nonstationary variables are also available (Bai/Ng, 2004), and unit-root versions of the factor-augmented VAR as well as error correction models are an area of active research; see, among others, Banerjee/Marcellino/Masten (2014).

We propose a concurrent parametrization for large factor models of nonstationary variables which we formulate in the structural time series framework of Harvey (1991). Factor structures on the trend, seasonal, cyclical and irregular components allow to model the co-movements of a large number of time series in a parsimonious, componentwise manner. The popular common trends or common cycle models emerge as special cases, but a common features assumption, restricting the idiosyncratic part to be stationary or even serially uncorrelated, is not necessarily imposed in our framework. Rather, the idiosyncratic part may be characterized by trend, cycle and seasonal components as well.

For a straightforward and computationally feasible implementation of the approach, a principal component analysis is combined with state space methods in the spirit of Brüning/Koopman (2014). We extract the principal components of suitably differenced data to account for nonstationarity of the idiosyncratic part. Re-cumulated factors are modeled jointly with the series of primary interest using likelihood-based techniques within a state space framework. In Monte Carlo simulations, we find that this method performs well, irrespectively of whether a common features assumption holds.

Our motivation and application of the model is from the viewpoint of official statistics, where several surveys and additional data sources are used to construct time series of a given concept on a regular basis in real time. The importance of timely and precise measures of the economy is emphasized by a large literature on real-time data analysis, which shows that data revisions pose a challenge to forecasters and policymakers; see, e.g., Croushore (2011). Hence, on the side of statistical agencies, most prominently for quarterly national accounts, effort is made to produce accurate statistics by bringing together a large amount of primary data sources, typically surveys; see Bureau of Economic Analysis (2014), Wood/Elliott (2007) and Federal Statistical Office (2008) for GDP calculation in the US, in the UK and in Germany. The current paper is a methodological contribution to these efforts.

From the perspective of data construction, we discuss several advantages and possible modifications of our model in state space form. Primary sources in official statistics are typically subject to survey errors and statistical breaks. They may be collected at different sampling frequencies, while changing survey designs lead to irregular measurement patterns. Since the key part of our model is formulated in state space form, it is well-suited to handle these patterns. It produces efficient estimates of the target series when different surveys measure the same underlying series. Information from the past of the series is

processed, and additional strength is borrowed from a large number of related series with correlated components. Seasonally adjusted time series, using all available data for the adjustment, are obtained as a by-product of the procedure.

The potential of the state space approach for official statistics has already been pointed out by other researchers. Uses in several areas of official statistics have been highlighted by Durbin (2000). Most prominently, state space methods are applied for seasonal adjustment, while Pfeiffermann (1991) and Tiller (1992) discuss signal extraction from repeated survey data. In small area statistics, state space models help obtain disaggregate figures from surveys by borrowing strength both over time and space, see Pfeiffermann/Tiller (2006) and Krieg/van den Brakel (2012). In that context, Bollinani-Balabay/van den Brakel/Palm (2015) pursue the estimation of aggregates along with the small-area domains in the presence of survey redesigns and variance breaks. Durbin/Quenneville (1997) and Quenneville/Gagné (2013) introduce benchmark constraints drawn from precise but low-frequency census data to correct the preliminary survey estimates, while Harvey/Chung (2000) discuss modeling data from different sources, and Moauro/Savio (2005) is concerned with temporal disaggregation as required by national statistical agencies.

We apply our methodology to the statistics of hours worked in Germany. High-quality data on hours worked are a key for understanding aggregate labor market dynamics, e.g., to track business cycles, to assess reactions to shocks such as the 2008/09 financial and economic crisis (Burda/Hunt, 2011), and to confront macroeconomic theory with time series evidence (Ohanian/Raffo, 2012). Timely figures on hourly labour productivity are considered as being important, e.g., for well-guided wage negotiations and monetary policy.

In Germany, working time statistics are constructed within the working time measurement concept of the Institute for Employment Research (IAB). The componentwise accounts provide a comprehensive figure of hours worked and contributes results to the German national accounts; see Wanger/Weigand/Zapf (2015). In the measurement of overtime hours and flows on working-time accounts (WTA), we use household and business surveys, while additionally drawing on several labor market and business cycle indicators. Lacking continuously available survey data on working-time account net flows, the latter is based on the unobserved trend and cycle components for transitory overtime hours as well as regular and actual hours worked.

The paper is structured as follows: Section 2 describes the model and its statistical treatment, Section 3 illustrates alternative measurement schemes faced in official statistics and Section 4 presents finite sample properties of the procedure. Section 5 applies the methods to the German statistics of hours worked, while the last section concludes.

## **2 A high-dimensional structural time series model**

### **2.1 The factor model**

This paper presents a model and its implementation for official statistics which extends the scope of multivariate structural time series models (STSM) discussed by Harvey/Koopman

(1997) to high-dimensional applications. As the point of departure, an  $N$ -dimensional vector time series  $y_t$  is decomposed into trend  $\mu_t$ , seasonal  $\gamma_t$ , cycle  $c_t$ , and irregular components  $u_t$ , according to

$$y_t = \mu_t + \gamma_t + c_t + u_t, \quad (1)$$

where the terms on the right are unobserved stochastic processes. After describing the dynamic specification of the components we introduce a factor structure, describing cross-series linkages within the groups of components, and the statistical treatment of the model.

We use a standard specification for the dynamics of each component and characterize the slow movements by local linear trends

$$\mu_{t+1} = \mu_t + \nu_t + \xi_t, \quad \nu_{t+1} = \nu_t + \zeta_t,$$

where  $\xi_t \sim \text{iid } N(0, \Sigma_\xi)$  and  $\zeta_t \sim \text{iid } N(0, \Sigma_\zeta)$  are independent Gaussian white noise sequences. For a model frequency of  $s$  observations per year, the seasonal components are

$$\gamma_{t+1} = - \sum_{j=0}^{s-2} \gamma_{t-j} + \omega_t, \quad \omega_t \sim \text{iid } N(0, \Sigma_\omega).$$

An individual cycle component  $\tilde{c}_{it}$ ,  $i = 1, \dots, N$  evolves jointly with the auxiliary process  $\tilde{c}_{it}^*$  as

$$\begin{pmatrix} \tilde{c}_{i,t+1} \\ \tilde{c}_{i,t+1}^* \end{pmatrix} = \rho_i \begin{pmatrix} \cos \lambda_i & \sin \lambda_i \\ -\sin \lambda_i & \cos \lambda_i \end{pmatrix} \begin{pmatrix} \tilde{c}_{it} \\ \tilde{c}_{it}^* \end{pmatrix} + \begin{pmatrix} \kappa_{it} \\ \kappa_{it}^* \end{pmatrix}, \quad \begin{pmatrix} \kappa_{it} \\ \kappa_{it}^* \end{pmatrix} \sim \text{iid } N(0, \Sigma_{\kappa,ii} I),$$

where  $\lambda_i$  is the dominant frequency and  $0 < \rho_i < 1$  denotes the dampening factor. As for the trends and seasonal components, linkages between the individual cycles are introduced through covariances between the disturbances, collected in  $\Sigma_\kappa$ . To gain flexibility on the temporal timing of the co-movement, we introduce phase shifts  $\delta_2, \dots, \delta_N$  between the cycles by setting  $c_{it} = \tilde{c}_{it} \cos \lambda_i \delta_i + \tilde{c}_{it}^* \sin \lambda_i \delta_i$ ,  $i = 1, \dots, N$ , where  $\delta_1 = 0$  as a normalization and  $\delta_i$  measures the lead time of cycle  $j$  against the cycle of the first variable; see Rünstler (2004) and Valle e Azevedo/Koopman/Rua (2006). Finally, the irregular noise term is given by  $u_t \sim \text{iid } N(0, \Sigma_u)$ . For simplicity we assume that all groups of shocks  $\xi_t$ ,  $\zeta_t$ ,  $\omega_t$ ,  $\kappa_t$  and  $u_t$  are mutually independent. Correlated components in the spirit of Morley/Nelson/Zivot (2003) could be straightforwardly adapted as long as suitable identification conditions are met.

Our focus is on cases where  $N$ , the number of series in  $y_t$  is large, and hence a curse of dimensionality occurs in the unrestricted model (1). For full covariance matrices  $\Sigma_\xi$ ,  $\Sigma_\zeta$ ,  $\Sigma_\omega$ ,  $\Sigma_\kappa$  and  $\Sigma_u$ , there are  $O(N(N+1))$  variance parameters to be estimated, which makes the application practically infeasible even for moderate values of  $N$ . In such situations, factor models have been found useful for different purposes in economics and finance. They allow a parsimonious representation of the cross-section dependencies between panel units or time series variables. Within our STSM setup, we consider common factors for each group of components. Denoting the common components by a  $C$  superscript and the

idiosyncratic terms by  $I$ , our model is given by

$$y_t = \Lambda_\mu \mu_t^C + \Lambda_\gamma \gamma_t^C + \Lambda_c c_t^C + \Lambda_u u_t^C + \mu_t^I + \gamma_t^I + c_t^I + u_t^I. \quad (2)$$

The common components are of dimensions  $r_\mu, r_\gamma, r_c$  and  $r_u$ , respectively, which are typically substantially smaller than  $N$ , while  $\Lambda_k, k \in \{\mu, \gamma, c, u\}$  are  $N \times r_k$  loading matrices of full column rank. All components follow the same dynamics as those described below (1), and are driven by shocks with covariance matrices  $\Sigma_l^C$  and  $\Sigma_l^I$  for  $l \in \{\xi, \zeta, \omega, \kappa, u\}$ . The idiosyncratic components are assumed mutually uncorrelated and hence  $\Sigma_l^I$  are diagonal, so that the number of parameters is reduced to an order  $O(N)$  for fixed factor dimensions. Clearly, the factor loadings  $\Lambda_k$  and covariance matrices of the common components  $\Sigma_l^C$  are not identified without further restrictions, which will be introduced in Section 2.2.

The factor STSM can be represented in the notation of a standard multivariate STSM (1) if a similar cycle assumption holds, i.e., if all  $\rho_i$  and  $\lambda_i$  are identical for both the common and idiosyncratic components.<sup>1</sup> However, the factor structure imposes restrictions on the disturbance covariance matrices, which are given by

$$\begin{aligned} \Sigma_\xi &= \Lambda_\mu \Sigma_\xi^C \Lambda_\mu' + \Sigma_\xi^I, & \Sigma_\zeta &= \Lambda_\mu \Sigma_\zeta^C \Lambda_\mu' + \Sigma_\zeta^I, & \Sigma_\omega &= \Lambda_\gamma \Sigma_\omega^C \Lambda_\gamma' + \Sigma_\omega^I, \\ \Sigma_\kappa &= \Lambda_c \Sigma_\kappa^C \Lambda_c' + \Sigma_\kappa^I, & \Sigma_u &= \Lambda_u \Sigma_u^C \Lambda_u' + \Sigma_u^I. \end{aligned}$$

If one or more of the columns of  $\Lambda_i$  are linearly dependent with those of  $\Lambda_j, i \neq j$ , the stacked loadings  $(\Lambda_\mu, \Lambda_\gamma, \Lambda_c, \Lambda_u)$  have a reduced column rank denoted by  $r < r_\mu + r_\gamma + r_c + r_u$ . Then, the cross-section correlations between variables in  $y_t$  can be traced back to a smaller number of common sources than there are common structural time series components. This possibly smaller dimensional latent process is given by the  $r$ -dimensional compound factors denoted by  $f_t$  with a corresponding full-rank  $N \times r$  loading matrix  $\Lambda$ , such that  $y_t = \Lambda f_t + \mu_t^I + \gamma_t^I + c_t^I + u_t^I$ . The compound factors are related to the common components by

$$f_t = \Gamma_\mu \mu_t^C + \Gamma_\gamma \gamma_t^C + \Gamma_c c_t^C + \Gamma_u u_t^C,$$

where  $\Gamma_k = (\Lambda' \Lambda)^{-1} \Lambda' \Lambda_k$  are  $r \times r_k$  matrices of full column rank. Again, factors  $f_t$ , loadings  $\Lambda$  and  $\Gamma_k$  are only identified up to linear combinations, but for a chosen rotation the common components  $\mu_t^C, \gamma_t^C, c_t^C$  and  $u_t^C$  are identified (up to rotation) through their different dynamics, and hence can be estimated from  $f_t$  by the state space approach described by Harvey (1991). As an example with the richest dynamic structure possible for a given  $r$ , consider the case with  $r = r_\mu = r_\gamma = r_c = r_u$  and  $\Lambda = \Lambda_\mu = \Lambda_\gamma = \Lambda_c = \Lambda_u$ . The factors  $f_t = \mu_t^C + \gamma_t^C + c_t^C + u_t^C$  then follow a structural time series process and consist of trend, irregular, seasonal and cyclical components themselves.

Factor structures in the multivariate STSM framework have been studied before in the econometrics literature, albeit with a different scope. Models for a moderate number of

<sup>1</sup> Linear combinations of ARIMA processes with identical parameters have the same ARIMA structure. Identity of the ARIMA parameters is trivially the case for trend, season and irregular components, while under a similar cycle assumption, it holds also for the cyclical components.

series have been used to investigate common trends (and thus cointegration) or common cycles in their dynamics; see e.g., Harvey (1991: sec. 8.5) and Valle e Azevedo/Koopman/Rua (2006). Similar restrictions are not imposed in model (2), since the idiosyncratic part of any series may have the same types of components as the common part. In this way, we may obtain a more parsimonious structure with less factors when a larger panel of data is considered. Eickmeier (2009), among others, finds unit roots in the idiosyncratic part of many macroeconomic time series, so that a common trends assumption fails for a reasonable factor dimension. Our model is rather general in that it allows for co-movements in each of the components, while a common features restriction is possible by setting the respective idiosyncratic components, say trends or cycles, to zero.

Model (2) has several benefits relative to recent VAR-based factor approaches. Firstly, the structural approach offers insights into the nature of co-movements between the series, which can be assigned to specific components: Is it because of business cycles or rather correlated trends that macroeconomic time series co-move? Are there joint sources of changing seasonal patterns in several branches of the economy? Can common irregular components like weather effects be identified that transitorily hit several output measures? Secondly, in the context of filtering a signal from sparsely available data, the structural time series setup imposes a parsimonious parametrization which stabilizes the estimates. In the application to official statistics, all components help estimate the different features of the target series while taking into account all relevant information from related series. Thirdly, using information from many series may also lead to important improvements of seasonal adjustment procedures over univariate approaches.

## 2.2 Estimation by collapsing the factor space

We suggest an estimation procedure of the model given by a combination of principal component and state space techniques along the lines of Bräuning/Koopman (2014). Assume that we are primarily interested in a low-dimensional sub-process  $z_t$  holding  $N_z$  series of the available data, while the complete set of time series is separated according to  $y_t = (x_t', z_t')'$ . In forecasting applications,  $z_t$  will hold at least the series to be predicted, while the estimation of official statistical figures typically requires the series  $z_t$  to consist of the major surveys measuring the target series. Unlike Bräuning/Koopman (2014), we assume that all variables in  $y_t$  are generated by the same model, (2) in our case, and hence variables in  $x_t$  and  $z_t$  are treated symmetrically in terms of the model but not in terms of the estimation procedure.

To estimate the space of compound factors  $f_t$  in a first step, we apply a suitable principal components analysis to  $x_t$ . By using the data  $x_t$  in differences, we avoid possible inconsistencies due to nonstationary idiosyncratic components, and thus adapt ideas of Bai/Ng (2004) to our setting. More concretely, denoting by  $L$  the lag operator, by  $\Delta := (1 - L)$  the standard difference and by  $\Delta_s := (1 - L_s)$  the seasonal difference operator, we obtain factor loadings  $\bar{A}$  as  $\sqrt{N_x}$  times the orthonormal eigenvectors corresponding to the  $r$  largest eigenvalues of  $\sum_{t=1}^T (\Delta \Delta_s x_t)(\Delta \Delta_s x_t)'$ . Estimated factors are obtained by re-cumulating the principal components in differences, or from the level data as  $\bar{f}_t = \bar{A}' x_t$ , which differs from the re-cumulation approach through the effects of initial values. Under an additional

assumption on the factor loadings, the results of Bai/Ng (2002) are applicable to the variables in differences; see Appendix A. Among other things, this assures consistency (up to rotation and net of the effects of starting values) of  $\bar{f}_t$  for  $f_t$  at a fixed  $t$  as  $N$  and  $T$  tend to infinity.

To gain information on the common components and their relation to the variables in  $z_t$ , we consider the joint model of  $f_t$  and  $z_t$  within the state space setup. Replacing the compound factors by their estimates, the model is given by

$$\begin{pmatrix} \bar{f}_t \\ z_t \end{pmatrix} = \begin{pmatrix} \Gamma_\mu \\ \Lambda_\mu \end{pmatrix} \mu_t^C + \begin{pmatrix} \Gamma_\gamma \\ \Lambda_\gamma \end{pmatrix} \gamma_t^C + \begin{pmatrix} \Gamma_c \\ \Lambda_c \end{pmatrix} c_t^C + \begin{pmatrix} \Gamma_u \\ \Lambda_u \end{pmatrix} u_t^C + \begin{pmatrix} e_t \\ \mu_t^I + \gamma_t^I + c_t^I + u_t^I \end{pmatrix}, \quad (3)$$

where  $e_t$  is the error of  $\bar{f}_t$  estimating  $f_t$ . As a slight abuse of notation, the idiosyncratic components and loadings are those corresponding to the elements in  $z_t$  only. While the compound factors are hence identified by a normalization which is standard in principal components analysis, the common structural time series components are made unique by setting

$$\Gamma_\mu = \begin{pmatrix} I_{r_\mu} \\ \Gamma_\mu^{(2)} \end{pmatrix}, \quad \Gamma_\gamma = \begin{pmatrix} I_{r_\gamma} \\ \Gamma_\gamma^{(2)} \end{pmatrix}, \quad \Gamma_c = \begin{pmatrix} I_{r_c} \\ \Gamma_c^{(2)} \end{pmatrix}, \quad \Gamma_u = \begin{pmatrix} I_{r_u} \\ \Gamma_u^{(2)} \end{pmatrix},$$

while the common components may have unrestricted disturbance covariance matrices.

Under these restrictions, the model can be operationalized by ignoring the error from principal components estimation, and hence setting  $e_t = 0$ , which is justified as an approximation especially for large  $N$ . The unknown parameters of (3) are estimated by maximum likelihood within the state space approach. Alternatively, an unrestricted multivariate STSM can be fitted to the principal components and variables of interest. This second strategy allows for correlation between the idiosyncratic terms of  $z_t$ , while the model nests the factor STSM specification (3). Empirically, the compound factor dimension can be inferred from the data  $y_t$  in suitable differences, e.g., by the methods proposed in Bai/Ng (2002). Alternatively, different (small) values of  $r$  can be considered and robustness with respect to this choice can be assessed in practice. Subsequently, for a given  $r$ , beginning from  $r_\mu = r_\gamma = r_c = r_u = r$ , the dimension of each common component may be determined in a general-to-specific sequential testing procedure based on (3).

### 3 Observation schemes

The factor STSM introduced in this paper has advantages in filtering latent series from incomplete measures which is a key issue in official statistics. For this purpose, we assume that a latent  $N_\theta$  dimensional process  $\theta_t$  of target series instead of observed  $z_t$  is modelled to follow the factor STSM (2), and that the observations collected in  $z_t$  are related to  $\theta_t$  through a dynamic measurement relationship

$$z_t = d_t + M_t(L)\theta_t + \varepsilon_t, \quad \varepsilon_t \sim (0, H_t), \quad (4)$$

where  $d_t$  holds possible survey bias terms and statistical breaks,  $M_t(L) = M_{t0} + M_{t1}L + \dots + M_{tt}L^t$  are  $N_z \times N_\theta$  matrix lag polynomials holding the dynamic measurement coefficients, while  $\varepsilon_t$  is a vector of survey errors with possibly time-varying covariance matrices  $H_t$ . The latter need not necessarily follow a white noise process, but can, e.g., contain autocorrelation due to survey overlap, which may be treated by the methods of Pfeffermann/Tiller (2006). We review some of the cases that the general mechanism (4) captures, and propose its implementation in the state space form which is given in Appendix B.

The measurement scheme (4) is sufficiently flexible to allow for several surveys estimating the same underlying concept, for missing data and for time-varying observation patterns. Consider an example where  $\theta_{1t}$ , e.g., paid overtime hours per week and employee, is measured by two surveys  $z_{1t}$  and  $z_{2t}$ , e.g., the German Socio-Economic Panel (GSOEP), and the German Microcensus, as it is the case in the application to German hours worked data below. The measurement mechanism is then given by

$$z_{1t} = \theta_{1t} + \varepsilon_{1t}, \quad z_{2t} = d_2 + \theta_{1t} + \varepsilon_{2t}. \quad (5)$$

In this simple example, with  $M_t(L) = (1, 1)'$ , the scheme brings together contradicting surveys, where differences are explained by the survey errors  $\varepsilon_{1t}$  and  $\varepsilon_{2t}$ . The variances of these errors depend on the design and size of the survey and are likely to change over time. By including an unknown constant  $d_2$  in the second measurement equations, it is possible to correct for a bias in one of the sources. Similarly, if statistical breaks, like changes in the survey questionnaire, occur in one or more of the data sources, these may be explicitly accounted for by level shifts in  $d_t$ , and hence leave the measured  $\theta_t$  unaffected.

Different sampling frequencies of regular surveys, or data missing for other reasons, are also covered by the measurement scheme (4). Returning to the bivariate example, if in period  $t$  no survey  $z_{1t}$  is conducted, we obtain a trivial equation

$$0 = 0 \cdot \theta_{1t} + 0, \quad z_{2t} = d_2 + \theta_{1t} + \varepsilon_{2t} \quad (6)$$

by specifying  $M_t(L) = (0, 1)'$  and  $H_{11,t} = 0$ . Hence, no information is gained by the first survey in that period. Therefore, information about  $\theta_{1t}$  stem firstly from other surveys  $z_{2t}$ , secondly from past and future values of  $z_{1t}$  through the dynamics of the system, or thirdly from additional indicators correlated with  $\theta_{1t}$  through the common components.

In contrast to the previous example where survey interviews reflect observations on one period  $t$ , in reality reference intervals may span more than one period in terms of the model frequency. For example, while the German Microcensus refers to one base week in each year before 2005, since then it has a continuous interview policy and allows a evaluation of quarterly averages. If the model is formulated at monthly frequency, an observation  $z_{1t}$  of a flow variable, corresponding to June 2006, refers to the mean of the underlying  $\theta_{1t}$ ,  $\theta_{1,t-1}$  and  $\theta_{1,t-2}$  of April, May and June. The measurement equation reflects this by selecting

$$z_{1t} = \frac{1}{3}\theta_{1t} + \frac{1}{3}\theta_{1,t-1} + \frac{1}{3}\theta_{1,t-2}, \quad (7)$$

where  $z_{1t}$  contain values only in the end of the quarter of each year, and where  $M_t(L) =$

$\frac{1}{3} + \frac{1}{3}L + \frac{1}{3}L^2$ . The change from a fixed reference week to continuous interviews is reflected by a change in the time-varying observation polynomial  $M_t(L)$ .

For other surveys, the observation scheme is still more general. E.g., for household panel studies such as the GSOEP or the U.S. panel study of income dynamics (PSID), the field period spans several months and changes from year to year. Assigning the resulting yearly figure to the December of each survey year, an observation equation

$$z_{1t} = M_{t,dec}\theta_{1t} + \dots + M_{t,jan}\theta_{1,t-11} \quad (8)$$

reflects the time-varying shares  $M_{tj}$  of observations in each month  $j$ , relative to all observations in that year. Figure 1 shows the distribution of the GSOEP interviews for selected years, namely for 1991 (solid line), for 2000 (dashed), for 2004 (dotted) and for 2012 (dash-dotted).

If one survey  $z_{1t}$  is used as a benchmark and hence the resulting estimate of  $\theta_{1t}$  should exactly match that survey, this is reached by setting  $\varepsilon_{1t} = 0$ . Further relevant methods for benchmarking are discussed in Durbin/Quenneville (1997) and Quenneville/Gagné (2013). The model (3) with the measurement scheme (4) can be stated in state space form (Appendix B). After estimating the model parameters by maximum likelihood, estimated  $\theta_t$ ,  $t = 1, \dots, T$ , using all available data, are obtained by a state smoothing algorithm (Durbin/Koopman, 2012: sec. 4.3).

## 4 A Monte Carlo study

A Monte Carlo study is conducted to shed light on the practical performance of the proposed methods in finite samples. Different aspects of the procedure are analysed. Firstly, the difference-based principal components approach is studied in the case of factor STSM processes for different data generating mechanisms and sample sizes, and compared to principal components in levels. Secondly, the filtering of latent processes by the proposed techniques is evaluated and compared to simple benchmarks.

Four data generating processes are chosen to mimic different situations of practical relevance. We consider (1) cases with linearly independent loadings for the distinct common components and (2) cases with identical loadings, where principal components estimate a compound factor process. Furthermore, while typically (A) the idiosyncratic components have a structural time series structure with trend, seasonal and possibly cycle components, we additionally consider a common features assumption with (B) serially uncorrelated idiosyncratic components. We introduce the data generating processes as combinations of these characteristics in turn.

**(1A)** To define the first data generation mechanism as the case with linearly independent loadings and without common features, we consider the process (2) with  $s = 4$  and where the cyclical components have frequency  $\lambda = 0.2$  and dampening factor  $\rho = 0.97$ . The parameters in the loading matrices are randomly chosen for each draw. Denoting the

uniform distribution between  $a$  and  $b$  by  $U(a, b)$ , they are given for  $i = 1, \dots, N$  and  $j = 1, \dots, r$  by

$$\Lambda_{\mu,ij} \sim U(0, \chi), \quad \Lambda_{\gamma,ij} \sim U(0, \chi), \quad \Lambda_{c,ij} \sim U(0, \chi\varsigma), \quad \Lambda_{u,ij} \sim U(0, \chi\varsigma),$$

where the parameter  $\chi$  captures the overall importance of the common components relative to the idiosyncratic ones, while  $\varsigma$  determines the relative size of stationary versus nonstationary components. The components are generated using innovation covariance matrices with

$$\Sigma_{\xi,ii}^I \sim U(0, 1)^2, \quad \Sigma_{\zeta,ii}^I \sim U(0, 1/10)^2, \quad \Sigma_{\omega,ii}^I \sim U(0, 1)^2, \quad \Sigma_{\kappa,ii}^I, \Sigma_{u,ii}^I \sim U(0, \varsigma)^2$$

for idiosyncratic components and  $\Sigma_{\xi}^C = 10\Sigma_{\zeta}^C = \Sigma_{\omega}^C = \Sigma_{\kappa}^C = \Sigma_u^C = I$  for common components, respectively.

**(1B)** The second data generating process is characterized by the same parameters for the common components as in (1A), but a common features assumption is imposed and hence the idiosyncratic components are subject to

$$\Sigma_{\xi}^I = \Sigma_{\zeta}^I = \Sigma_{\omega}^I = \Sigma_{\kappa}^I = 0, \quad \Sigma_{u,ii}^I \sim U(0, 5)^2.$$

**(2A)** To introduce cases with linearly dependent common components loadings, the third data generating process sets the compound loading matrix  $\Lambda$  according to

$$\Lambda_{\mu,ij} = \Lambda_{\gamma,ij} = \frac{1}{\varsigma} \Lambda_{u,ij} \sim U(0, \chi),$$

and drops the cyclical components from the processes. The remaining variances  $\Sigma_{\xi}^I, \Sigma_{\zeta}^I, \Sigma_{\omega}^I, \Sigma_u^I, \Sigma_{\xi}^C, \Sigma_{\zeta}^C, \Sigma_{\omega}^C$ , and  $\Sigma_u^C$  correspond to those in (1A).

**(2B)** The last data generating process drops the trend and season from the idiosyncratic components of the previous one, so that the only difference to (2A) is in the covariance matrices

$$\Sigma_{\xi}^I = \Sigma_{\zeta}^I = \Sigma_{\omega}^I = 0, \quad \Sigma_{u,ii}^I \sim U(0, 5)^2.$$

We first assess the performance of the principal components procedure based on differenced data  $\Delta\Delta_4 y_t$  which we have proposed as a first step in estimating the factor STSM. For all data generating processes and several values for the time and cross-section dimensions  $T$  and  $N$ , we simulate 1000 trajectories and repeatedly estimate the compound factor process  $f_t$  by  $\bar{f}_t$  as explained in Section 2.2. We compare the results to the principal component method using the data in levels. The estimation errors are assessed by the adjusted  $R^2$  from regressing the true compound factors on the estimated factors  $\bar{f}_t$ . To enforce stationarity for these evaluation equations, we apply the regressions in differences ( $\Delta_4 \Delta$ ) of the true factors and their estimates. These  $R^2$  are averaged over the iterations.

Table 1 gives results for the data generating process (1A) with linearly independent component loadings and without a common components structure. There, the true factor process consists of the common structural times series components,  $f_t = (\mu_t^C, \gamma_t^C, c_t^C, u_t^C)'$ , which allows for an evaluation of each component separately. Overall, the principal components in differences outperform the estimates based on levels data. The difference between the methods is most pronounced for larger  $N$  and  $T$ . The estimates in differences clearly improve with  $N$ , but also slightly with  $T$ , with  $R^2$  becoming close to one for each component in large samples. The level estimate, however, especially the stationary components in the baseline case with  $\chi = 1$  and  $\varsigma = 1$ , does not show a clear improvement with larger  $N$ . The precision typically even worsens with larger  $T$ , which is the result of inconsistency when the idiosyncratic components are nonstationary; see Bai/Ng (2004) for the I(1) framework. The results are robust to changing the scale of the stationary components to  $\varsigma = 2$  and of the common factors to  $\chi = 2$ . These changes lead to the expected results that the stationary common components are estimated more precisely in the former case, while the overall precision increases in the latter case.

In Table 2, we show results for the process (1B) which entails the common features assumption that the idiosyncratic components are white noise. Compared to (1A), the overall picture changes. Now, the estimates in levels are better than their difference-based counterparts, most strikingly for larger  $N$ . The difference-based estimates still improve both with  $N$  and with  $T$ . The precisions of the two estimators for the stationary components are closer to each other for  $\varsigma = 2$  and for  $\chi = 2$ , but still the level-based estimates dominate the difference-based ones almost uniformly.

Results for the data generating processes with identical loadings for all common components are depicted in Table 3. We evaluate the precision of  $r$  principal components estimating the  $r$  compound factors  $f_t = \mu_t^C + \gamma_t^C + u_t^C$  for  $r \in \{1, 2\}$  by means of the adjusted  $R^2$  as before. The mean of the adjusted  $R^2$  over both evaluation regressions is computed in the case  $r = 2$ .

For  $r = 1$ , the adjusted  $R^2$  are very close to one for all chosen specifications. Thus, when compared to tables 1 and 2, the performance is seemingly enhanced if the components can be estimated in aggregated form, which reduces the compound factor dimension relative to the first two data generating processes. However, the higher uncertainty of the first two cases likely recurs in the second step when distinct structural time series components are estimated from the compound factors in a state space framework. The outcomes for  $r = 2$  reveal a loss of precision and visible differences between the specifications and estimators. The patterns described for the first two data generating processes are confirmed here. Most notably, without the common feature assumption the difference estimator outperforms the level estimator again, while the latter is slightly better in case of common features.

These outcomes suggest that the estimator choice should be based on whether the idiosyncratic components are white noise or not, and that unnecessary differencing should be avoided. The difference-based estimator appears as a robust choice since it is consistent in both settings while the level-based estimator does not necessarily improve with sample size in the general framework of this paper.

In a second part of this Monte Carlo study, we assess the two-step procedure with respect to its ability to estimate a latent process  $\theta_{1t}$  from incomplete data  $z_{1t}$  and additional information in high-dimensional  $x_t$ . We delete  $N/4$  of the observations in  $z_{1t}$  which is generated together with  $x_t$  as a factor STSM for  $y'_t = (z_{1t}, x'_t)$ . Different alternative approaches are considered to estimate  $\theta_{1t}$  for each case where  $z_{1t}$  is missing. As an infeasible benchmark, (1) the factor STSM with known factor process  $f_t$  is considered in state space form, where parameters are determined by maximum likelihood and missing values are estimated by the state smoother. As the feasible counterpart, (2) the two-step estimate proposed in this paper is used, where  $f_t$  is estimated by the principal components based on data in differences  $\Delta\Delta_4x_t$ . As one further straightforward benchmark we use (3) a univariate STSM which neglects information from  $x_t$ . As a simple competitor that also uses time series information on  $z_{1t}$  only, we interpolate the series using (4) a local mean of available  $\Delta_4z_{1t}$  in the range of  $\pm 20$  observations near the period to be estimated. Cross-section information, but not the dynamics of the system are utilized by static regression-based predictions of  $\theta_{1t}$  using the difference-based principal components of  $x_t$  as predictors. The regression is run (5) in levels, (6) applying a yearly difference operator  $\Delta_4$  to  $z_{1t}$  and the principal component, or (7) applying the difference operator  $\Delta\Delta_4$  which is sufficient to make the variables stationary.

Table 4 shows the corresponding root mean squared errors (RMSE) from estimating  $\theta_{1t}$  according to the data generating process (1B) with  $r = 1$ . Not surprisingly, the infeasible estimator (1) outperforms the others, while the feasible two-step strategy (2) of utilizing the factor STSM comes a close second. The loss from having to estimate  $f_t$  is rather small in this specification, and amounts to less than 5 % of the overall RMSE in most cases. Clearly, this result may strongly depend on the data generating process and the corresponding precision of the principal components method. The differences vanish with larger  $N$ .

The univariate STSM approach (3) comes in third place, but missing information on the factors leads to an efficiency loss which is more pronounced if either  $\varsigma = 2$  which increases the noise which is unpredictable by univariate methods, or if  $\chi = 2$  where the information content of  $x_t$  is higher. However, taking the dynamics into account appropriately pays off, which turns out from a comparison to the naïve local averaging method which performs clearly worse than all STSM approaches. The static regression estimation with principal components as predictors (5) leads to very spurious results in levels, while it still does not lead to a relevant improvement even over the local averaging method when it is applied in differences (6-7).

## 5 Application to German hours worked statistics

We apply the proposed techniques to the measurement of several components of hours worked in Germany. Official statistics on hours worked per person and the overall volume of work are determined by the IAB which contributes the corresponding time series to the German national accounts. The working time measurement concept is a componentwise system where collective, calendar, cyclical, personal and other components are determined

separately on a quarterly basis since 1991, and results are disaggregated according to industries, regions, and employment status; see Wanger (2013) and Wanger/Weigand/Zapf (2015) for recent overviews.

During a major revision in 2014 which also affected the methodology of the working time measurement, state space techniques were introduced to enhance the estimation precision for components with incomplete data sources, or where more than one source is used in the measurement. In this section, we describe the computation of the cyclical components paid and unpaid overtime work as well as flows on working-time accounts. These are of primary importance when assessing the business cycle fluctuations of hours worked in real time.

## 5.1 Paid and unpaid overtime hours

The computations of overtime hours in the working-time measurement concept are primarily based on two yearly surveys. In the GSOEP, employed persons are asked for the number of performed overtime hours in the recent month and the way overtime work is typically compensated. From the responses, yearly time series on paid and unpaid overtime hours since the 1980s can be constructed, but as has been mentioned in Section 3, a changing distribution of interviews over the year has to be taken into account when considering a target series of higher frequency. As a second primary data source, the Microcensus offers information on paid and unpaid overtime hours since 2010 on the basis of quarterly averages.

The main problem of constructing a quarterly time series in real time is the substantial publication lag of each of the sources, since results from the GSOEP are available approximately 12 months after the end of a reference year, while the Microcensus results typically come in July of the following year. Hence, information regarding the first quarter of each year is available only after about 21 months (GSOEP) and 16 months (Microcensus), respectively. Additionally, the determination of intra-year fluctuations before 2010 is challenging, since until then only yearly GSOEP data are available. In response, we gather additional indicators to tackle these problems and to achieve the highest possible precision for the given available data.

As an additional data source, we consider the Ifo Business Survey. There, in the last month of each quarter, establishments are asked whether their employees currently perform overtime work. Along with the log of the GSOEP and the Microcensus measures of overtime hours per week ( $z_{1t}$  and  $z_{2t}$ , respectively), the logarithmic fraction of establishments with overtime work enter the model as a third series of interest,  $z_{3t}$ .

Further economic and labor market indicators ( $x_t$ ) are used to compute principal components which enter the factor STSM. Here, we use real gross domestic product, the production index, new orders for all manufacturing industries, the number of employed persons, real compensation per employee (all from the Federal Statistical Office), registered unemployment (from the Federal Employment Agency), business expectations, business assessment and the employment barometer (from the Ifo Institute) as well as the willingness

to buy index (from GfK Nuremberg). These variables are considered informative when assessing the current business cycle and labor market development, and hence for the amount of overtime work. We refrain from using a dataset of higher dimension, since the additional data are likely to introduce irrelevant information and require a higher number of factors. At the same time, we keep the updating process simple by this choice.

Principal component estimates are computed after applying the natural logarithm to all variables except business expectations, business assessment, the employment barometer and the willingness to buy index. Seasonally adjusted data are used in  $x_t$ , so that yearly differences are not needed to remove seasonal nonstationarity. Additionally, there is no evidence for a changing slope in the processes: The p-values for tests of  $\Sigma_\zeta = 0$  in univariate STSM is 0.97 and 0.07 for the first and second principal component (based on second differences), respectively. Hence, we base the subsequent analysis on re-cumulated principal components from first differences of the raw data. Data gaps and mixed frequency issues in  $x_t$  are resolved by the algorithm described by Stock/Watson (2002).

The resulting models for paid and unpaid overtime hours, respectively, are formulated in terms of a monthly model frequency to precisely capture the timing of the measurement process. Along with the  $r$  estimated factors  $\bar{f}_t$ , which capture the compound common components  $\theta_t^C$  on a monthly basis, the measurement model is given by

$$\begin{pmatrix} \bar{f}_t \\ \log(\text{ot\_gsoep}_t) \\ \log(\text{ot\_mc}_t) \\ \log(\text{ot\_ifo}_t) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ d_2 \\ 0 \end{pmatrix} + \begin{pmatrix} I & 0 & 0 \\ 0 & M_{11,t}(L) & 0 \\ 0 & M_{21,t}(L) & 0 \\ 0 & 0 & M_{33,t}(L) \end{pmatrix} \begin{pmatrix} \theta_t^C \\ \theta_{1t} \\ \theta_{2t} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \varepsilon_{2t} \\ 0 \end{pmatrix}.$$

The GSOEP measurement scheme  $M_{11,t}(L)$  is determined by the changing proportion of interviews in each month as in (8), the Microcensus measures the same underlying  $\theta_{1t}$ , but by quarterly averages according to (7), while the Ifo measure of overtime refers to a single month, and hence  $M_{33,t}(L) = 1$  if data are available in month  $t$  and  $M_{33,t}(L) = 0$ , otherwise. Since for a long sample of data prior to 2010, the GSOEP is the only available statistic directly measuring  $\theta_{1t}$ , we implement this source as a benchmark, and force a weighted average of  $\theta_{1t}$  to fit the yearly GSOEP figure exactly. Recent Microcensus figures, in contrast, enter the model with an adjustment term  $d_2$ , and the survey error  $\varepsilon_{2t}$  is modeled serially uncorrelated with a fixed variance which is estimated within the state space model.

We choose an unrestricted multivariate STSM formulation of  $\theta_t^C$  and  $\theta_t$  as the dynamic model. In contrast to the formulation (3), this approach allows for a correlation between  $\theta_{1t}$  and  $\theta_{2t}$  (fraction of establishments with overtime) beyond their dependence on the common components, while nesting the strict factor specification. The gain in flexibility is reasonable since the Ifo survey measures a concept relatively close to the target series, and may provide specific information beyond the overall business cycle.

We pre-select the dynamic components to be included based on each series individually. Augmented Dickey Fuller tests, with lag lengths determined by AIC, fail to reject unit roots for each of the series considered in the models (the exception being  $\bar{f}_{1t}$ , with a p-value of

0.0029). We hence include unit root components for all series and let  $\xi_{it} \neq 0$  in general. We test the presence of slope changes  $\zeta_t$ , white noise terms  $u_t$ , cyclical components  $c_t$ , and changes to the seasonal pattern  $\omega_t$  in univariate STSMs, and present the p-value of the corresponding hypotheses in Table 5. The time series of Microcensus data is not sufficiently long for univariate analyses so that we base the specification for paid and unpaid overtime on the yearly GSOEP series.

There is no evidence for a changing slope in neither of the series on a 5% significance level, which again supports the computation of principal components based on first differenced data. We hence set  $\Sigma_\zeta$  to zero in the models for both paid and unpaid overtime hours. Following the outcomes, we include a white noise term for the principal components, but not for the series in  $z_t$  in what follows. Relatively strong evidence is in favor of cyclical components which seems to be present in each of the observed series. Finally, the Ifo survey, which is the only series due to a seasonal component is reasonably modeled with a fixed seasonal pattern. The seasonal figure in overtime hours come in only trough the short Microcensus time series and has therefore to be set fixed.

Considering the joint dynamic process introduced above, a decomposition

$$\begin{pmatrix} \theta_t^C \\ \theta_{1t} \\ \theta_{2t} \end{pmatrix} = \begin{pmatrix} \mu_t^C \\ \mu_{1t} \\ \mu_{2t} \end{pmatrix} + \begin{pmatrix} 0 \\ \gamma_{1t} \\ \gamma_{2t} \end{pmatrix} + \begin{pmatrix} c_t^C \\ c_{1t} \\ c_{2t} \end{pmatrix} + \begin{pmatrix} u_t^C \\ 0 \\ 0 \end{pmatrix}$$

applies, with dynamic components driven by the processes introduced below equation (1). There,  $\Sigma_\kappa$  and  $\Sigma_\xi$  are full symmetric  $(r + 2) \times (r + 2)$  parameter matrices, while  $\Sigma_u$  is a scalar and  $\Sigma_\zeta = \Sigma_\omega = 0$ .

Both for paid and unpaid overtime hours, models with  $r = 1$  are estimated as the baseline specifications, which appears reasonable due to the relatively small number of indicators in  $x_t$  and avoids parameter abundance. Setting  $r = 2$  while using the same modeling strategy does not change the estimated time series in a relevant way.<sup>2</sup> We assess whether the data are consistent with a similar cycles assumption ( $\rho_i = \rho$ ,  $\lambda_i = \lambda$ ) and whether the overtime measures  $\theta_{1t}$  and  $\theta_{2t}$  have the same cycle shift with respect to the business cycle factor ( $\delta_2 = \delta_3$ ). These restrictions are rejected neither for paid, nor for unpaid overtime hours on a 5% significance level, so that they are maintained. The estimated cyclical parameters are shown in the left two columns of Table 6.

For both models, we find that the cycles are relatively persistent, with a dampening factor close to one, and that a typical cycle lasts about four and a half years. The cycles are shifted by approximately three months to the right relative to the business cycle of the principal component, so that a peak in overtime hours typically lags behind that of the factor. Paid overtime hours appear to be more pro-cyclical, since the standard deviation of the factor (log-scale  $\times 100$ ) is more than twice as large as that for unpaid overtime hours. At the same time, paid overtime hours exert a stronger correlation with the business cycle.

Figure 2 shows the smoothed estimate for paid overtime hours. The observations of the

<sup>2</sup> Results can be obtained from the authors upon request.

GSOEP (round points, placed in March of each year) and Microcensus (crosses, net of the constant  $d_2$ ) are shown along with the trend  $\mu_{1t}$  (dotted), the seasonally adjusted estimate  $\mu_{1t} + c_{1t}$  (dashed) and the overall smoothed series including the seasonal component (solid). The ordinate axis is depicted on a logarithmic scale to reflect the logarithmic model formulation. The nearly linear long-term downward trend is visibly superimposed by stochastic cycles which had a pronounced effect during the 2008/09 financial and economic crisis and reflects well-known patterns from cyclical output movements. The fixed seasonal component, which shows higher overtime usage in the second half of the year, stems mostly from the short sample of Microcensus observations, and should therefore be treated with care.

The unpaid overtime hours, shown in Figure 3, are driven by a rather volatile trend which closely follows the observations. There are several periods of longer upward or downward movements, and although unpaid hours rose in tendency over the whole sample, there is a decline since about 2006 until now. The cycle is rather small, which reflects the low business cycle sensitivity of this working-time component, while the seasonal component is positive in the first and fourth quarter.

## 5.2 Net flows on working time accounts

Not all additional hours worked by employees in a given period lead to a definitive increase in the amount of labor over a longer time span. Some of them, termed transitory overtime hours, are compensated by leisure time in a future period. The number of these additional hours worked hence raise the credits on WTA, which are formal arrangements to record such additional hours worked. When measuring hours actually worked per period, the statistician has to track such inflows on WTA which raise hours worked, but also the outflows from WTA which reduce the overall hours worked.

Only few data sources are available which allow to measure in- and outflows from WTA on a regular basis. Besides paid and unpaid overtime hours, the GSOEP questionnaire asks for overtime hours which are compensated with time-off, and which we hence treat as inflows on WTA. A question regarding the reduction of such hours has been included in the questionnaire only in 2014 and the results are not yet available. A similar objection is faced by a new question regarding balances on WTA in the IAB Job Vacancy Survey. It has been included in the establishment survey in 2013 and therefore still lacks a sufficient history to base long time series estimates thereupon.

The Microcensus holds additional information on WTA flows over a longer time span, which we exploit in our estimation strategy. Each employed household member is asked for the regular weekly hours worked and for hours worked last week. If both differ, the main reason for that difference is inquired, where possible answers include “compensation for more hours worked (e.g. flexible working hours)” if actual hours were lower and “hours for the accumulation of the time credit or for the reduction of time debt” if they were higher than usual. These or analogous questions are available for the whole estimation period.

Since only the *main* reason for a difference is asked for in the Microcensus, there are likely further WTA in- or outflows that are not revealed by the survey participants and hence the

results are biased. Our strategy thus combines information on the level of gross inflows from the GSOEP with cyclical variations of the Microcensus figures on in- and outflows around their trends to arrive at a final estimate of net flows. The maintained assumption is that even if both WTA in- and outflows follow (possibly stochastic) trends, the latter should be identical so that there is no long-run discrepancy between the both, and the net flows average to zero in the long run. This allows us to estimate the trend by use of the GSOEP series, while relative deviations from it are determined from the Microcensus. Stated jointly with the estimated factors, the measurement model is

$$\begin{pmatrix} \bar{f}_t \\ \log(in\_mc_t) \\ \log(out\_mc_t) \\ \log(in\_gsoep_t) \end{pmatrix} = \begin{pmatrix} 0 \\ d_{1t} \\ d_{2t} \\ 0 \end{pmatrix} + \begin{pmatrix} I & 0 & 0 & 0 \\ 0 & M_{11,t}(L) & 0 & 0 \\ 0 & 0 & M_{22,t}(L) & 0 \\ 0 & 0 & 0 & M_{33,t}(L) \end{pmatrix} \begin{pmatrix} \theta_t^C \\ \theta_{1t} \\ \theta_{2t} \\ \theta_{3t} \end{pmatrix}.$$

We do not model survey uncertainty explicitly and set  $\varepsilon_t = 0$ , but take the restructuring of the Microcensus in 2005 into account, when a fixed reference week each year (or less frequently before 1995) was replaced by a continuous interviewing policy. This leads to a structural break in the series which we model by  $d_{1t}$  and  $d_{2t}$  set to nonzero constants before 2005 and to zero afterwards.

We again include a unit root component to all series in order to reflect results from Augmented Dickey Fuller tests. A univariate analysis of the individual components similar to Table 5 reveals that the GSOEP inflow series has a significant slope change (p-value 0.04), while a noise term finds more support from the data than a cycle (for which the p-value is 0.16). For the Microcensus series, we set  $\zeta_t = 0$  and include a noise term along with the cycle and random walk trends, since this specification is supported in the multivariate model. Correlations between the GSOEP and other series are not considered and hence the former is used solely to extract its trend by univariate filtering and smoothing. The model is thus given by

$$\begin{pmatrix} \theta_t^C \\ \theta_{1t} \\ \theta_{2t} \\ \theta_{3t} \end{pmatrix} = \begin{pmatrix} \mu_t^C \\ \mu_{1t} \\ \mu_{2t} \\ \mu_{3t} \end{pmatrix} + \begin{pmatrix} 0 \\ \gamma_{1t} \\ \gamma_{2t} \\ 0 \end{pmatrix} + \begin{pmatrix} c_t^C \\ c_{1t} \\ c_{2t} \\ 0 \end{pmatrix} + \begin{pmatrix} u_t^C \\ u_{1t} \\ u_{2t} \\ u_{3t} \end{pmatrix},$$

where  $\Sigma_\zeta$  has a single nonzero element associated with  $\theta_{3t}$ ,  $\Sigma_u$  is diagonal, and  $\Sigma_\xi$  as well as  $\Sigma_\kappa$  are block diagonal with a full upper left  $3 \times 3$  submatrix.

The properties of the cyclical components of WTA in- and outflows are summarized in the right two columns of Table 6. Again, we cannot reject the similar cycles restriction, and the common period and the dampening factor are similar to the case of overtime hours. Both components have a relatively strong cyclical pattern, and WTA inflows have the highest cycle standard deviation among the variables under consideration. Not surprisingly, shocks to inflows are positively, while outflow shocks are negatively related to business cycle shocks. The phase shifts mean that typically seven months after employees have built up most credit on the accounts, the outflows peak and reduce the savings on WTA.

The cyclical patterns of in- and outflows are shown in Figure 4, where  $c_{jt} + u_{jt}$ ,  $j = 1, 2$ , is depicted for inflows (solid line) and outflows (dashed line) in logarithmic scale  $\times 100$ , as annotated on the left axis. The mentioned phase shift between the cycles becomes evident here. At most of the visible peaks of WTA inflows, the outflows are rising and reach their highest value a few months later. Before the building up of credits beginning in 2005, the outflows dropped, while the credits were used up afterwards during the 2008/09 crisis, where outflows peaked again. The trending behavior of transitory overtime hours from the GSOEP, which is used as the trend in both, in- and outflows from WTA, is shown in hours per week as the thin dash-dotted line with annotation at the right axis. It shows a flattening growth from below 0.5 hours per week to over 1 hour until 2010, and has diminished slightly over the recent years.

The trend and cycles are combined to yield the net flow on WTA, which is the relevant statistic measuring the effect on hours worked per period. We compute this effect as

$$\Delta WTA_t \approx \exp(\mu_{3t}^z)(\gamma_{1t}^z + c_{1t}^z + u_{1t}^z - \gamma_{2t}^z - c_{2t}^z - u_{2t}^z).$$

This overall effect is plotted in Figure 5, where also the seasonal patterns are assessed. The overall increase in the scale of the fluctuations over time is partly due to the increased overall importance of WTA corresponding to the upward trend of gross flows described above, while the cyclical patterns from Figure 4 are closely reflected by the overall net flows.

As for the results of paid and unpaid overtime hours, further processing of the data is performed within the working-time measurement concept to yield quarterly results which are partly decomposed for several groups of employees. These are published by the IAB in the form of working time components tables, and also enter the publication of national accounts by the German Federal Statistical Office.

## 6 Conclusion

We have proposed a factor structural time series model and discussed its implementation for possibly high-dimensional problems. The main motivation of the approach was for smoothing latent series using surveys and several other indicators, as is of foremost importance for official statistical agencies. Its usefulness for the measurement of working time components was illustrated by the empirical application in the paper. However, the methods may reveal their strength also for other tasks such as exploration of the component-wise dynamic properties and co-movements of several macroeconomic time series, and forecasting. Additional research may also be concerned with correlated unobserved components models in high dimensions, which allow for a more flexible modelling of spillovers and structural identification.

## A Properties of the factor model in differences

In this appendix, we show that the variables generated by a factor STSM satisfy strong assumptions on time and cross section dependence when suitably differenced. These are sufficient to ensure key results of Bai/Ng (2002). Denoting  $X_{it} := \Delta\Delta_s y_{it}$ ,  $i = 1, \dots, N$ , and  $F_{jt} := \Delta\Delta_s f_{jt}$ ,  $j = 1, \dots, r$ , where  $y_t$  is the vector of observed variables and  $f_t$  is the compound factor process introduced in the main text, the factor model can be stated as

$$X_{it} = \Lambda_i \cdot F_t + e_{it}.$$

Here,  $e_{it}$  is cross-sectionally uncorrelated and independent from  $F_t$  by assumption, while both  $F_{jt}$  and  $e_{it}$  follow strictly stationary linear Gaussian processes with absolutely summable coefficients, as we discuss in the following.

To see the dynamic properties more clearly, a generic element from  $e_{it}$  is stated as

$$e_{it} = \Delta\Delta_s \mu_{it} + \Delta\Delta_s \gamma_{it} + \Delta\Delta_s c_{it} + \Delta\Delta_s u_{it},$$

where the  $I$  superscript is suppressed for notational simplicity. Since  $\mu_{it} = \mu_{i,t-1} + \nu_{i,t-1} + \xi_{i,t-1}$ , we have  $\Delta\mu_{it} = \nu_{i,t-1} + \xi_{i,t-1}$ , while from  $\nu_{it} = \nu_{i,t-1} + \zeta_{i,t-1}$ , it follows that  $\nu_{it} = \nu_{i,t-s} + \zeta_{i,t-1} + \dots + \zeta_{i,t-s}$ . Hence,

$$\Delta\Delta_s \mu_{it} = \Delta_s \nu_{i,t-1} + \Delta_s \xi_{i,t-1} = \zeta_{i,t-2} + \dots + \zeta_{i,t-s-1} + \xi_{i,t-1} - \xi_{i,t-s-1},$$

where  $\xi_{it}$  and  $\zeta_{it}$  are mutually independent Gaussian iid processes, and hence a finite-order moving average structure is obtained for the differenced trend component, with coefficients straightforwardly obtained from the  $s$  nonzero autocovariances.

A similar result is obtained for the seasonal component  $\gamma_{it} = -\gamma_{i,t-1} - \dots - \gamma_{i,t-s+1} + \omega_{i,t-1}$ . Applying first differences to both sides of this equation yields  $\gamma_{it} = \gamma_{i,t-s} + \omega_{i,t-1} - \omega_{i,t-2}$ , and hence

$$\Delta\Delta_s \gamma_{it} = \Delta^2 \omega_{i,t-1},$$

which is again a (over-differenced) finite-order moving average that trivially has absolutely summable coefficients.

Regarding the cycle, Harvey (1991: sec. 2.5.6) gives the stationary ARMA(2,1) representation for  $|\rho_i| < 1$ , which leads directly to

$$\Delta\Delta_s c_{it} = \Delta\Delta_s \frac{1 + \theta_i L}{1 - 2\rho_i \cos(\lambda_i) L - \rho_i^2 L^2} \tilde{\kappa}_{i,t-1},$$

where  $\theta_i$  is a moving average parameter and  $\tilde{\kappa}_{it}$  is composed of the two jointly Gaussian iid processes  $\kappa_{it}$  and  $\kappa_{it}^*$ . Since as a stationary ARMA process the fraction expands to a polynomial with absolutely summable coefficients, also the entire expression for  $\Delta\Delta_s c_{it}$  shares this property while inheriting stationarity and Gaussianity. The same is true for differenced noise term  $\Delta\Delta_s u_{it}$ . Hence, any linear combination of  $\Delta\Delta_s \mu_{it}$ ,  $\Delta\Delta_s \gamma_{it}$ ,  $\Delta\Delta_s c_{it}$  and  $\Delta\Delta_s u_{it}$  is strictly stationary, Gaussian and has absolutely summable coefficients. The statement is applicable both to the differenced idiosyncratic components  $e_{it}$  and to series

of the differenced factor process  $F_t$ .

The properties of  $F_t$  and  $e_{it}$  are clearly sufficient to assure Assumptions A (by a law of large numbers drawing on ergodicity of  $F_t$ ), C (since absolutely summable autocovariances follow from absolutely summable Wold coefficients), and D (due to the independence between  $e_{it}$  and  $F_t$ ) of Bai/Ng (2002), while their Assumption B on the factor loadings has to be imposed additionally to obtain the main results of that paper. Clearly, the squared autocorrelations of  $e_{it}$  are also summable in our setup, and hence Bai/Ng (2002: eq. (6)) yields mean-square convergence of estimated  $F_t$  to the true values for a given  $t$ . Naturally, the consistency holds also for a cumulation of finitely many estimated  $F_s$ ,  $s \leq t$ . Hence, also the factors  $\bar{f}_t$  in level are found consistent for a fixed  $t$ . The effects of the initial values are lost due to the differencing, however.

## B The state space form

The model given by (3) with measurement scheme (4) can be easily represented in linear state space form which allows to use the techniques described in Durbin/Koopman (2012). We adopt their notation as far as possible and state the system as

$$\begin{pmatrix} \bar{f}_t \\ z_t \end{pmatrix} = Z_t \alpha_t + \begin{pmatrix} 0 \\ \varepsilon_t \end{pmatrix}, \quad \varepsilon_t \sim N(0, H_t) \quad (9)$$

$$\alpha_{t+1} = T \alpha_t + R \eta_t, \quad \eta_t \sim N(0, Q), \quad t = 1, \dots, n. \quad (10)$$

For simplicity of exposition we assume that  $l \geq s - 1$ , so that  $l$  lags of all components have to be included in the state vector to make the measurement equation (4) representable in state space form. Hence, the state vector  $\alpha_t$  holds the components  $\mu_{it}^I, \mu_{jt}^C, \nu_{it}^I, \nu_{jt}^C, \gamma_{it}^I, \gamma_{jt}^C, (\tilde{c}_{it}^I, \tilde{c}_{it}^{I*}), (\tilde{c}_{jt}^C, \tilde{c}_{jt}^{C*}), u_{it}^I$  and  $u_{jt}^C$ , each for  $i = 1, \dots, N_z$  and  $j = 1, \dots, r$ , along with  $l$  lags of each component. More precisely,

$$\alpha_t^I = \begin{pmatrix} \mu_{1t}^I, \dots, \mu_{N_z,t}^I, \\ \nu_{1t}^I, \dots, \nu_{N_z,t}^I, \\ \gamma_{1t}^I, \dots, \gamma_{N_z,t}^I, \\ \tilde{c}_{1t}^I, \tilde{c}_{1t}^{I*}, \dots, \tilde{c}_{N_z,t}^I, \tilde{c}_{N_z,t}^{I*}, \\ u_{1t}^I, \dots, u_{N_z,t}^I, \\ \mu_{1,t-1}^I, \mu_{2,t-1}^I, \\ \dots \text{lagged components} \dots, u_{r,t-l}^C \end{pmatrix} \begin{pmatrix} \mu_{1t}^C, \dots, \mu_{r,t}^C, \\ \nu_{1t}^C, \dots, \nu_{r,t}^C, \\ \gamma_{1t}^C, \dots, \gamma_{r,t}^C, \\ \tilde{c}_{1t}^C, \tilde{c}_{1t}^{C*}, \dots, \tilde{c}_{r,t}^C, \tilde{c}_{r,t}^{C*}, \\ u_{1t}^C, \dots, u_{r,t}^C, \\ \dots \text{lagged components} \dots, u_{r,t-l}^C \end{pmatrix}'$$

is the  $m := 6(N_z + r)(l + 1)$ -dimensional state vector. Accordingly, the  $6(N_z + r)(l + 1) \times 6(N_z + r)(l + 1)$  transition matrix is given by

$$T = \begin{pmatrix} \tilde{T} & 0 & \dots & 0 \\ I & & & \vdots \\ & \ddots & & \vdots \\ 0 & & I & 0 \end{pmatrix}, \quad \text{where } \tilde{T} = \begin{pmatrix} T_\mu & T_{\mu\nu} & 0 & 0 & 0 \\ 0 & T_\nu & 0 & 0 & 0 \\ 0 & 0 & T_\gamma & 0 & 0 \\ 0 & 0 & 0 & T_c & 0 \\ 0 & 0 & 0 & 0 & T_u \end{pmatrix}$$

is a  $6(N_z + r) \times 6(N_z + r)$  matrix with  $T_\mu = T_\nu = T_{\mu\nu} = I_{N_z+r}$ . Moreover,  $T_u = 0_{N_z+r}$  and  $T_c$  is a  $2(N_z + r) \times 2(N_z + r)$  block diagonal matrix with  $i$ th block given by

$$T_c^{(i,i)} = \rho_i \begin{pmatrix} \cos \lambda_i & \sin \lambda_i \\ -\sin \lambda_i & \cos \lambda_i \end{pmatrix},$$

for  $i = 1, \dots, N_z + r$ . Here,  $\rho_i$  and  $\lambda_i$  correspond to the individual cycle parameters for  $i = 1, \dots, N_z$ , while they correspond to the parameters of the joint cycles,  $\rho_i = \rho_{i-N_z}^C$  and  $\lambda_i = \lambda_{i-N_z}^C$  for  $i = N_z + 1, \dots, N_z + r$ . The transition innovation covariance matrix  $Q$  is block diagonal with block element given by  $\Sigma_\xi^I, \Sigma_\xi^C, \Sigma_\zeta^I, \Sigma_\zeta^C, \Sigma_\omega^I, \Sigma_\omega^C, \Sigma_\kappa^I \otimes I_2, \Sigma_\kappa^C \otimes I_2, \Sigma_u^I$  and  $\Sigma_u^C$ , respectively, while  $R$  is a vertical stacking of an identity and  $l$  quadratic zero matrices that selects the contemporaneous states.

The observation matrices  $Z_t$  reflect both the observation patterns for the variables and the loading of common components on the individual series. We denote

$$\tilde{A} = \begin{pmatrix} 0 & \Gamma_\mu & 0 & 0 & 0 & \Gamma_\gamma & 0 & \check{\Gamma}_c & 0 & \Gamma_u \\ I & \Lambda_\mu & 0 & 0 & I & \Lambda_\mu & \check{I} & \check{\Lambda}_c & I & \Lambda_u \end{pmatrix},$$

where the checked matrices reflect the phase shifts of the variables, so that the  $i$ th row of  $\check{I}$  is  $(\cos(\lambda_i \delta_i), \sin(\lambda_i \delta_i)) \otimes I_{i\cdot}$ , the  $i$ th row of  $\check{\Gamma}_c$  is  $(\cos(\lambda_i \delta_i), \sin(\lambda_i \delta_i)) \otimes \Gamma_{c,i}$ , and the  $i$ th row of  $\check{\Lambda}_c$  is  $(\cos(\lambda_i \delta_i), \sin(\lambda_i \delta_i)) \otimes \Lambda_{c,i}$ , which have twice the number of columns as the unchecked quantities. Then, for

$$\tilde{M}_t(L) = \tilde{M}_{t0} + \tilde{M}_{t1}L + \dots + M_{tl}L^l = \begin{pmatrix} I & 0 \\ 0 & M_{0t} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & M_{1t} \end{pmatrix} L + \dots + \begin{pmatrix} 0 & 0 \\ 0 & M_{lt} \end{pmatrix} L^l,$$

the time-varying observation matrices are given by

$$Z_t = (\tilde{M}_{t0}\tilde{A}, \tilde{M}_{t1}\tilde{A}, \dots, \tilde{M}_{tl}\tilde{A}),$$

which completes the state space representation for the general case with  $d_t = 0$ .

If constant terms or statistical breaks occur, the transition matrix is enriched by additional diagonal elements of 1, while the observation matrix reflects this by additional columns with corresponding element either set to the constant values, or switching from zero to that constant at a specified period. The state innovation error covariance matrix is unchanged and the matrix  $R$  holds additional rows of zeros.

Table 1: Precision of common component estimation by principal components in levels and differences ( $\Delta_4\Delta$ ) for process (1A) without common features. The mean of the adjusted  $R^2$  from regressions of true common components on estimated factors is given.

$\chi$	$\varsigma$	$T$	$N$	pca in levels				pca in differences			
				$\mu_t$	$\gamma_t$	$c_t$	$u_t$	$\mu_t$	$\gamma_t$	$c_t$	$u_t$
1	1	250	10	0.260	0.506	0.270	0.338	0.229	0.551	0.310	0.430
1	1	250	50	0.427	0.633	0.272	0.338	0.507	0.836	0.637	0.756
1	1	250	100	0.499	0.688	0.270	0.333	0.736	0.917	0.812	0.873
1	1	250	500	0.634	0.780	0.278	0.338	0.941	0.983	0.959	0.973
1	1	500	10	0.260	0.496	0.253	0.333	0.232	0.548	0.306	0.418
1	1	500	50	0.369	0.588	0.260	0.336	0.564	0.846	0.676	0.773
1	1	500	100	0.427	0.638	0.260	0.346	0.770	0.922	0.830	0.884
1	1	500	500	0.550	0.720	0.257	0.338	0.951	0.984	0.964	0.976
1	1	1000	10	0.254	0.469	0.257	0.339	0.230	0.548	0.317	0.422
1	1	1000	50	0.309	0.504	0.262	0.348	0.588	0.853	0.698	0.785
1	1	1000	100	0.316	0.503	0.260	0.352	0.783	0.924	0.839	0.888
1	1	1000	500	0.347	0.520	0.265	0.355	0.953	0.985	0.966	0.977
1	2	250	10	0.136	0.287	0.445	0.499	0.096	0.291	0.483	0.603
1	2	250	50	0.254	0.366	0.550	0.516	0.147	0.627	0.799	0.861
1	2	250	100	0.336	0.414	0.584	0.522	0.260	0.783	0.889	0.924
1	2	250	500	0.517	0.516	0.655	0.535	0.806	0.951	0.976	0.984
1	2	500	10	0.127	0.277	0.420	0.499	0.094	0.288	0.490	0.603
1	2	500	50	0.209	0.343	0.439	0.523	0.157	0.655	0.808	0.867
1	2	500	100	0.259	0.371	0.440	0.525	0.369	0.807	0.898	0.928
1	2	500	500	0.425	0.459	0.448	0.532	0.864	0.959	0.979	0.985
1	2	1000	10	0.125	0.263	0.403	0.502	0.091	0.289	0.483	0.600
1	2	1000	50	0.151	0.275	0.407	0.528	0.167	0.668	0.813	0.869
1	2	1000	100	0.160	0.274	0.420	0.528	0.453	0.819	0.901	0.931
1	2	1000	500	0.180	0.294	0.418	0.535	0.883	0.962	0.980	0.986
2	1	250	10	0.492	0.716	0.487	0.473	0.497	0.783	0.605	0.695
2	1	250	50	0.717	0.841	0.514	0.453	0.878	0.959	0.911	0.939
2	1	250	100	0.767	0.869	0.537	0.447	0.941	0.980	0.956	0.970
2	1	250	500	0.825	0.901	0.628	0.422	0.988	0.996	0.991	0.994
2	1	500	10	0.482	0.717	0.417	0.484	0.500	0.788	0.603	0.695
2	1	500	50	0.668	0.820	0.387	0.465	0.884	0.959	0.914	0.940
2	1	500	100	0.714	0.845	0.373	0.463	0.942	0.980	0.957	0.970
2	1	500	500	0.764	0.872	0.378	0.460	0.988	0.996	0.992	0.994
2	1	1000	10	0.453	0.682	0.413	0.493	0.514	0.791	0.609	0.700
2	1	1000	50	0.546	0.726	0.387	0.480	0.885	0.960	0.916	0.940
2	1	1000	100	0.566	0.740	0.396	0.480	0.943	0.980	0.958	0.971
2	1	1000	500	0.620	0.773	0.378	0.466	0.989	0.996	0.992	0.994

Table 2: Precision of common component estimation by principal components in levels and differences ( $\Delta_4\Delta$ ) for process (1B) with common features. The mean of the adjusted  $R^2$  from regressions of true common components on estimated factors is given.

$\chi$	$\varsigma$	$T$	$N$	pca in levels				pca in differences			
				$\mu_t$	$\gamma_t$	$c_t$	$u_t$	$\mu_t$	$\gamma_t$	$c_t$	$u_t$
1	1	250	10	0.150	0.331	0.160	0.185	0.079	0.230	0.101	0.163
1	1	250	50	0.300	0.556	0.343	0.237	0.120	0.381	0.157	0.263
1	1	250	100	0.414	0.673	0.468	0.275	0.136	0.440	0.177	0.305
1	1	250	500	0.756	0.900	0.802	0.742	0.176	0.631	0.234	0.519
1	1	500	10	0.145	0.331	0.154	0.180	0.075	0.221	0.098	0.156
1	1	500	50	0.297	0.559	0.343	0.234	0.117	0.383	0.156	0.252
1	1	500	100	0.410	0.677	0.471	0.276	0.132	0.444	0.179	0.298
1	1	500	500	0.763	0.905	0.810	0.794	0.199	0.741	0.302	0.618
1	1	1000	10	0.143	0.338	0.158	0.175	0.073	0.220	0.100	0.148
1	1	1000	50	0.296	0.564	0.342	0.231	0.115	0.382	0.159	0.246
1	1	1000	100	0.414	0.678	0.475	0.287	0.133	0.448	0.183	0.300
1	1	1000	500	0.767	0.907	0.816	0.818	0.247	0.821	0.460	0.710
1	2	250	10	0.110	0.249	0.386	0.402	0.057	0.163	0.281	0.411
1	2	250	50	0.257	0.494	0.651	0.606	0.069	0.213	0.412	0.566
1	2	250	100	0.403	0.659	0.785	0.785	0.074	0.272	0.535	0.688
1	2	250	500	0.767	0.905	0.947	0.957	0.101	0.653	0.843	0.918
1	2	500	10	0.109	0.252	0.391	0.399	0.054	0.158	0.285	0.403
1	2	500	50	0.261	0.506	0.654	0.630	0.063	0.211	0.426	0.577
1	2	500	100	0.402	0.666	0.790	0.801	0.069	0.288	0.595	0.732
1	2	500	500	0.768	0.906	0.947	0.959	0.112	0.786	0.901	0.940
1	2	1000	10	0.109	0.250	0.388	0.394	0.051	0.148	0.279	0.396
1	2	1000	50	0.260	0.509	0.658	0.645	0.060	0.208	0.440	0.589
1	2	1000	100	0.404	0.668	0.791	0.808	0.068	0.330	0.639	0.755
1	2	1000	500	0.768	0.908	0.948	0.960	0.194	0.835	0.922	0.949
2	1	250	10	0.286	0.541	0.331	0.299	0.162	0.442	0.216	0.315
2	1	250	50	0.565	0.792	0.633	0.534	0.213	0.636	0.303	0.499
2	1	250	100	0.716	0.884	0.774	0.764	0.288	0.784	0.437	0.663
2	1	250	500	0.929	0.974	0.946	0.955	0.759	0.953	0.850	0.926
2	1	500	10	0.281	0.540	0.337	0.296	0.156	0.433	0.216	0.306
2	1	500	50	0.571	0.793	0.632	0.564	0.217	0.656	0.322	0.512
2	1	500	100	0.721	0.885	0.776	0.783	0.329	0.818	0.519	0.710
2	1	500	500	0.929	0.975	0.946	0.958	0.868	0.966	0.911	0.947
2	1	1000	10	0.282	0.543	0.332	0.291	0.151	0.426	0.213	0.304
2	1	1000	50	0.571	0.799	0.635	0.582	0.225	0.677	0.347	0.527
2	1	1000	100	0.725	0.887	0.779	0.791	0.407	0.838	0.593	0.743
2	1	1000	500	0.930	0.975	0.947	0.959	0.900	0.970	0.931	0.954

Table 3: Precision of compound factor estimation by principal components in levels and differences ( $\Delta_4\Delta$ ) for processes (2A) and (2B) (without and with common features). The mean of the adjusted  $R^2$  from regressions of true common components on estimated factors is given.

$\chi$	$\varsigma$	$T$	$N$	$r = 1$				$r = 2$			
				(2A)		(2B)		(2A)		(2B)	
				level	diff	level	diff	level	diff	level	diff
1	1	250	10	0.970	0.994	0.998	0.985	0.644	0.760	0.463	0.283
1	1	250	50	0.973	0.999	1.000	0.999	0.798	0.951	0.752	0.494
1	1	250	100	0.969	0.999	1.000	1.000	0.843	0.976	0.850	0.614
1	1	250	500	0.978	1.000	1.000	1.000	0.932	0.995	0.964	0.919
1	1	500	10	0.977	0.995	0.998	0.986	0.628	0.764	0.469	0.279
1	1	500	50	0.983	0.999	1.000	0.999	0.772	0.952	0.753	0.507
1	1	500	100	0.987	0.999	1.000	1.000	0.825	0.976	0.850	0.684
1	1	500	500	0.990	1.000	1.000	1.000	0.915	0.995	0.964	0.945
1	1	1000	10	0.976	0.994	0.998	0.985	0.616	0.767	0.465	0.280
1	1	1000	50	0.990	0.999	1.000	1.000	0.761	0.953	0.754	0.527
1	1	1000	100	0.992	0.999	1.000	1.000	0.809	0.977	0.850	0.733
1	1	1000	500	0.996	1.000	1.000	1.000	0.907	0.995	0.964	0.954
1	2	250	10	0.965	0.994	0.998	0.992	0.642	0.746	0.586	0.414
1	2	250	50	0.972	0.999	1.000	1.000	0.801	0.951	0.850	0.694
1	2	250	100	0.978	0.999	1.000	1.000	0.843	0.975	0.916	0.857
1	2	250	500	0.974	1.000	1.000	1.000	0.928	0.995	0.981	0.972
1	2	500	10	0.970	0.993	0.998	0.995	0.627	0.750	0.593	0.414
1	2	500	50	0.985	0.999	1.000	1.000	0.772	0.952	0.847	0.731
1	2	500	100	0.987	0.999	1.000	1.000	0.829	0.976	0.916	0.878
1	2	500	500	0.991	1.000	1.000	1.000	0.913	0.995	0.981	0.977
1	2	1000	10	0.976	0.993	0.998	0.995	0.615	0.756	0.593	0.413
1	2	1000	50	0.986	0.999	1.000	1.000	0.759	0.952	0.849	0.756
1	2	1000	100	0.991	0.999	1.000	1.000	0.807	0.976	0.916	0.888
1	2	1000	500	0.996	1.000	1.000	1.000	0.905	0.995	0.981	0.979
2	1	250	10	0.985	0.999	0.999	0.999	0.867	0.930	0.710	0.538
2	1	250	50	0.990	1.000	1.000	1.000	0.948	0.988	0.914	0.872
2	1	250	100	0.988	1.000	1.000	1.000	0.965	0.994	0.955	0.941
2	1	250	500	0.988	1.000	1.000	1.000	0.985	0.999	0.991	0.988
2	1	500	10	0.991	0.998	1.000	0.999	0.844	0.931	0.708	0.540
2	1	500	50	0.994	1.000	1.000	1.000	0.933	0.988	0.915	0.885
2	1	500	100	0.997	1.000	1.000	1.000	0.955	0.994	0.955	0.945
2	1	500	500	0.998	1.000	1.000	1.000	0.982	0.999	0.991	0.990
2	1	1000	10	0.993	0.998	1.000	0.999	0.837	0.929	0.711	0.544
2	1	1000	50	0.996	1.000	1.000	1.000	0.928	0.988	0.915	0.890
2	1	1000	100	0.998	1.000	1.000	1.000	0.949	0.994	0.955	0.948
2	1	1000	500	0.998	1.000	1.000	1.000	0.980	0.999	0.991	0.990

Table 4: RMSE of estimating  $N/4$  randomly chosen missing values for  $z_{1t}$  in process (2A) with  $r = 1$ . (1) Factor STSM with known factors  $f_t$ , (2) factor STSM with differenced pca factor estimates, (3) univariate STSM, (4) mean of yearly differences of  $z_{1t}$  within  $\pm 20$  observations, (5-7) static OLS of  $z_{1t}$  on pca in levels and differences.

$\chi$	$\varsigma$	$T$	$N$	STSM $f_t$	STSM pca	STSM univar.	Mean $\Delta_4$	OLS level	OLS $\Delta_4$	OLS $\Delta\Delta_4$
1	1	250	10	1.126	1.210	1.627	2.518	4.743	2.217	2.490
1	1	250	50	1.148	1.166	1.653	2.550	4.772	2.189	2.411
1	1	250	100	1.151	1.159	1.634	2.512	4.584	2.133	2.416
1	1	250	500	1.160	1.161	1.663	2.558	4.575	2.136	2.397
1	1	500	10	1.187	1.261	1.644	2.548	10.621	2.642	2.555
1	1	500	50	1.181	1.193	1.612	2.502	9.964	2.498	2.420
1	1	500	100	1.180	1.187	1.644	2.567	9.733	2.498	2.393
1	1	500	500	1.165	1.166	1.598	2.501	9.803	2.454	2.359
1	1	1000	10	1.275	1.328	1.636	2.567	26.310	3.195	2.562
1	1	1000	50	1.255	1.265	1.614	2.526	24.397	2.955	2.388
1	1	1000	100	1.268	1.274	1.630	2.554	24.386	2.986	2.386
1	1	1000	500	1.267	1.270	1.624	2.549	24.330	2.970	2.368
1	2	250	10	1.522	1.650	2.245	3.182	4.899	2.649	3.310
1	2	250	50	1.556	1.582	2.283	3.224	4.923	2.597	3.203
1	2	250	100	1.565	1.576	2.254	3.187	4.728	2.561	3.209
1	2	250	500	1.568	1.570	2.290	3.238	4.727	2.555	3.194
1	2	500	10	1.569	1.692	2.274	3.238	10.705	3.064	3.428
1	2	500	50	1.546	1.568	2.220	3.158	10.062	2.881	3.198
1	2	500	100	1.545	1.557	2.272	3.243	9.826	2.878	3.166
1	2	500	500	1.543	1.545	2.212	3.167	9.889	2.836	3.139
1	2	1000	10	1.595	1.700	2.259	3.246	26.281	3.561	3.400
1	2	1000	50	1.555	1.575	2.220	3.183	24.436	3.292	3.149
1	2	1000	100	1.573	1.583	2.241	3.218	24.424	3.328	3.151
1	2	1000	500	1.587	1.589	2.252	3.232	24.356	3.323	3.154
2	1	250	10	1.200	1.277	2.514	3.837	4.756	2.225	2.496
2	1	250	50	1.232	1.248	2.543	3.886	4.768	2.189	2.412
2	1	250	100	1.221	1.229	2.497	3.800	4.577	2.131	2.414
2	1	250	500	1.244	1.247	2.563	3.909	4.575	2.136	2.397
2	1	500	10	1.404	1.466	2.534	3.884	10.652	2.654	2.563
2	1	500	50	1.384	1.394	2.456	3.781	9.972	2.502	2.422
2	1	500	100	1.395	1.402	2.542	3.915	9.732	2.498	2.393
2	1	500	500	1.357	1.361	2.440	3.771	9.811	2.455	2.358
2	1	1000	10	1.683	1.719	2.512	3.893	26.520	3.209	2.571
2	1	1000	50	1.664	1.673	2.488	3.849	24.408	2.955	2.388
2	1	1000	100	1.677	1.683	2.518	3.901	24.416	2.985	2.385
2	1	1000	500	1.687	1.692	2.505	3.888	24.327	2.969	2.368

Table 5: P-values from testing different null hypotheses on the presence of several components in univariate structural time series models. The tests refer to the full model in the alternative. Models are formulated at the original data frequency (monthly for  $\hat{f}_t$ , yearly for GSOEP, quarterly for Ifo Business Survey).

	$\hat{f}_{1t}$	$\hat{f}_{2t}$	Paid Ot. (GSOEP)	Unpaid Ot. (GSOEP)	Overtime (Ifo)
$H_0 : \zeta_t = 0$	0.4213	0.0898	0.6815	0.9735	1.0000
$H_0 : u_t = 0$	0.0008	0.0433	1.0000	0.0676	1.0000
$H_0 : c_t = 0$	0.0000	0.0010	0.0053	0.0133	0.0000
$H_0 : \omega_t = 0$	—	—	—	—	0.4396

Table 6: Estimated parameters for cyclical components in models for paid and unpaid overtime hours (first two columns) and flows on working time accounts (last two columns). A similar cycles assumption is imposed in each of the models.

	Paid Ot.	Unpaid Ot.	Inflow WTA	Outflow WTA
Dampening factor $\rho$	0.9832	0.9880	0.9835	0.9835
Angle frequency $\lambda$	0.1155	0.1128	0.1198	0.1198
Period $\frac{2\pi}{\lambda}$	54.42	55.70	52.45	52.45
Cycle standard deviation	8.45	3.89	15.04	9.34
Cycle shock correlation with $\kappa_t^C$	0.69	0.45	0.60	-0.42
Phase shift $\delta$	-2.60	-3.15	0.89	-7.79

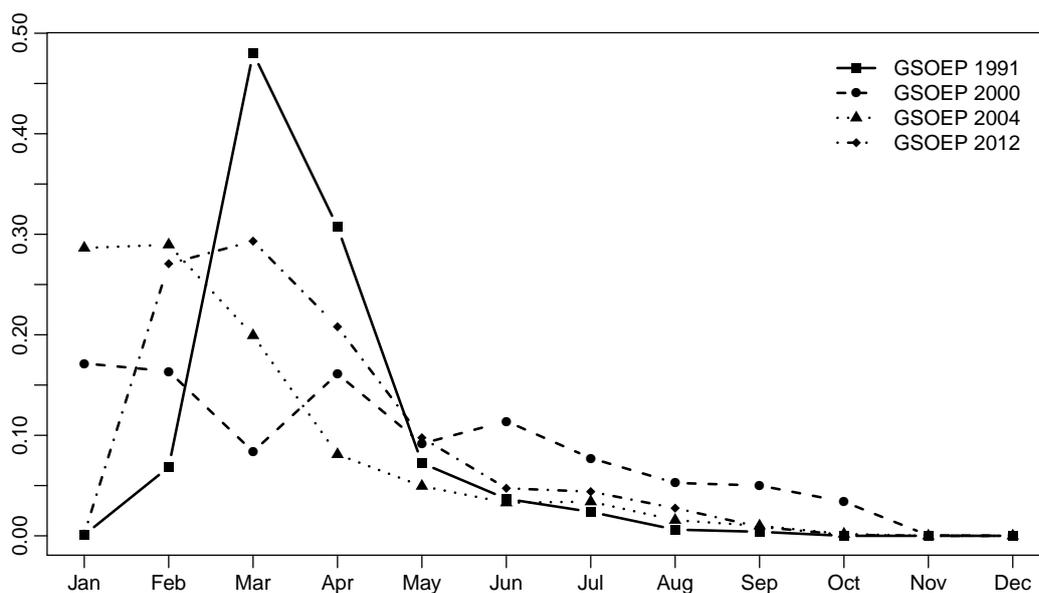


Figure 1: Distribution of GSOEP interviews over certain years. The fraction of interviews for each month is shown for 1991, 2000, 2004 and 2012.

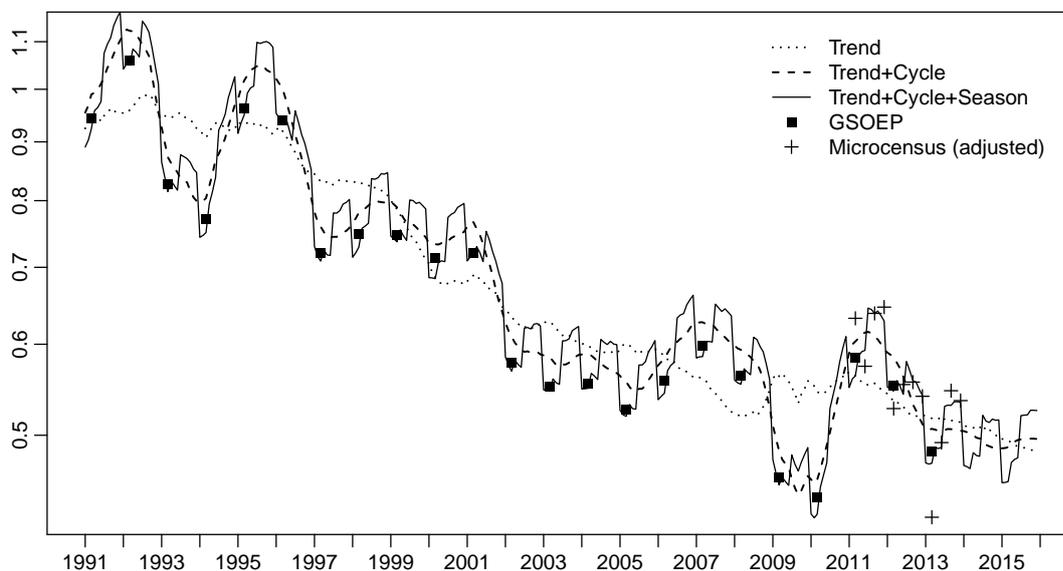


Figure 2: Paid overtime hours per week. The trend, cycle and seasonal figures are obtained by the state smoother and shown along with the GSOEP and Microcensus observations. The latter is adjusted for the constant  $d_2$ .

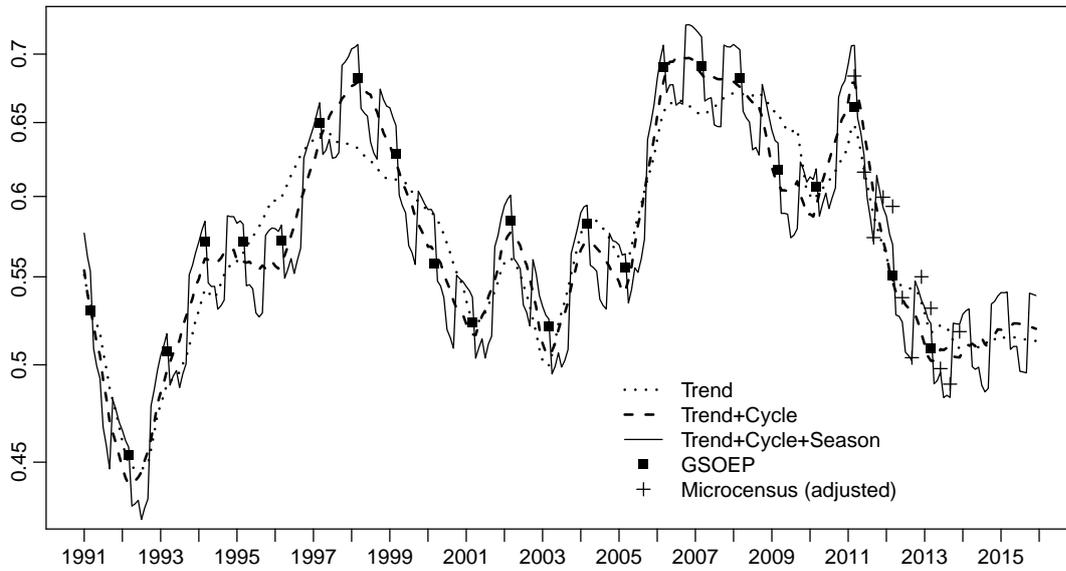


Figure 3: Unpaid overtime hours per week. The trend, cycle and seasonal figures are obtained by the state smoother and shown along with the GSOEP and Microcensus observations. The latter is adjusted for the constant  $d_2$ .

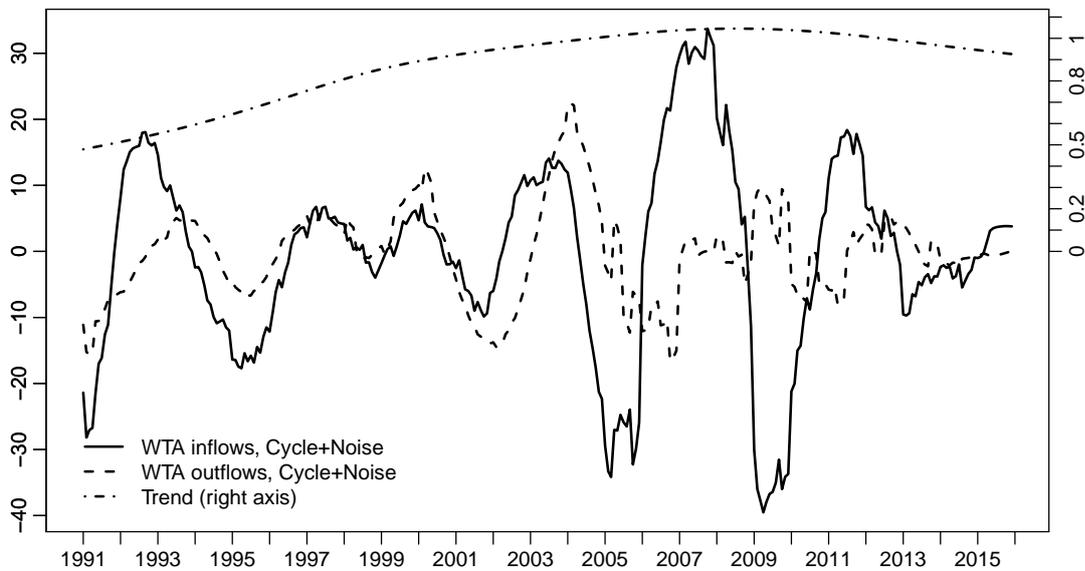


Figure 4: Cyclical and noise components of in- and outflows (left axis) and trend in flows on working time accounts (right axis). The cycle, noise and trend figures are obtained by the state smoother.

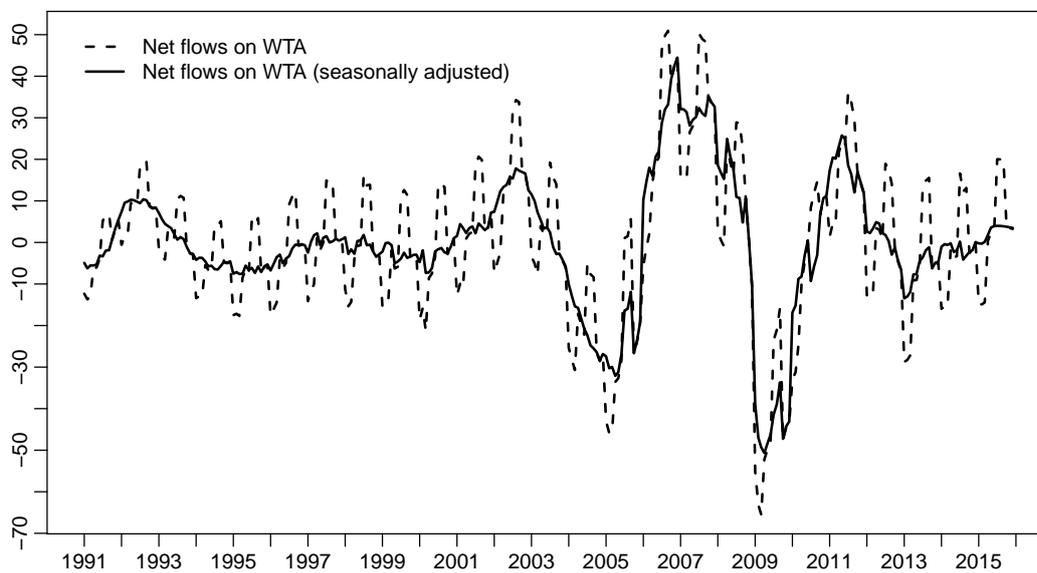


Figure 5: Working time account net flows in hours per week, computed from smoothed cycles and trends by  $\Delta WTA_t \approx \exp(\mu_{3t}^z)(\gamma_{1t}^z + c_{1t}^z + u_{1t}^z - \gamma_{2t}^z - c_{2t}^z - u_{2t}^z)$ .

## References

- Bai, J.; Ng, S. (2008): Large Dimensional Factor Analysis. In: Foundations and Trends in Econometrics, Vol. 3, No. 2, p. 89–163.
- Bai, J.; Ng, S. (2002): Determining the Number of factors in Approximate Factor Models. In: Econometrica, Vol. 1, No. 1, p. 191–221.
- Bai, Jushan; Ng, Serena (2004): A PANIC attack on unit roots and cointegration. In: Econometrica, Vol. 72, No. 4, p. 1127–1177.
- Banerjee, Anindya; Marcellino, Massimiliano; Masten, Igor (2014): Forecasting with factor-augmented error correction models. In: International Journal of Forecasting, Vol. 30, No. 3, p. 589–612.
- Bollineni-Balabay, Oksana; van den Brakel, Jan; Palm, Franz (2015): Multivariate state space approach to variance reduction in series with level and variance breaks due to survey redesigns. In: Journal of the Royal Statistical Society: Series A (Statistics in Society).
- Bräuning, Falk; Koopman, Siem Jan (2014): Forecasting macroeconomic variables using collapsed dynamic factor analysis. In: International Journal of Forecasting, Vol. 30, No. 3, p. 572–584.
- Burda, Michael C; Hunt, Jennifer (2011): What Explains the German Labor Market Miracle in the Great Recession? In: Brookings Papers on Economic Activity, Vol. Spring 2011, p. 273–335.
- Bureau of Economic Analysis (2014): Concepts and Methods of the U.S. National Income and Product Accounts. Tech. Rep., URL <https://www.bea.gov/national/pdf/all-chapters.pdf>.
- Croushore, Dean (2011): Frontiers of real-time data analysis. In: Journal of Economic Literature, Vol. 49, No. 1, p. 72–100.
- Durbin, James (2000): The State Space Approach to Time Series Analysis and its Potential for Official Statistics. In: The Australian & New Zealand Journal of Statistics, Vol. 42, No. 1, p. 1–23.
- Durbin, James; Koopman, Siem Jan (2012): Time Series Analysis by State Space Methods: Second Edition. Oxford: Oxford Statistical Science Series.
- Durbin, James; Quenneville, Benoit (1997): Benchmarking by State Space Models. In: International Statistical Review, Vol. 65, No. 1, p. 23–48.
- Eickmeier, Sandra (2009): Comovements and heterogeneity in the Euro area analyzed in a non-stationary dynamic factor model. In: Journal of Applied Econometrics, Vol. 24, No. 6, p. 933–959.
- Federal Statistical Office (2008): National Accounts: Quarterly Calculations of Gross Domestic Product in accordance with ESA 1995 - Methods and Data Sources. Subject-matter Series 18, S. 23, URL <https://www.destatis.de/EN/Publications/Specialized/Nationalaccounts/QuarterlyCalculationsGrossDomesticProductAccordance.pdf>.

Harvey, AC; Koopman, SJM (1997): Multivariate structural time series models. In: *Systematic Dynamics in Economic and Financial Models*, p. 269–298.

Harvey, Andrew; Chung, Chia-Hui (2000): Estimating the Underlying Change in Unemployment in the UK. In: *Journal of the Royal Statistical Society: Series A (Statistics in Society)*, Vol. 163, No. 3, p. 303–339.

Harvey, Andrew C. (1991): *Forecasting, Structural Time Series Models and the Kalman Filter*. New York: Cambridge University Press.

Krieg, Sabine; van den Brakel, Jan A (2012): Estimation of the monthly unemployment rate for six domains through structural time series modelling with cointegrated trends. In: *Computational Statistics & Data Analysis*, Vol. 56, No. 10, p. 2918–2933.

Moauero, Filippo; Savio, Giovanni (2005): Temporal disaggregation using multivariate structural time series models. In: *The Econometrics Journal*, Vol. 8, No. 2, p. 214–234.

Morley, James C; Nelson, Charles R; Zivot, Eric (2003): Why are the Beveridge-Nelson and unobserved-components decompositions of GDP so different? In: *Review of Economics and Statistics*, Vol. 85, No. 2, p. 235–243.

Ohanian, Lee E; Raffo, Andrea (2012): Aggregate hours worked in OECD countries: New measurement and implications for business cycles. In: *Journal of Monetary Economics*, Vol. 59, No. 1, p. 40–56.

Pfeffermann, Danny (1991): Estimation and Seasonal Adjustment of Population Means Using Data from Repeated Surveys. In: *Journal of Business & Economic Statistics*, Vol. 9, No. 2, p. 163–175.

Pfeffermann, Danny; Tiller, Richard (2006): Small-Area Estimation With State-Space Models Subject to Benchmark Constraints. In: *Journal of the American Statistical Association*, Vol. 101, No. 476, p. 1387–1397.

Quenneville, Benoit; Gagné, Christian (2013): Testing time series data compatibility for benchmarking. In: *International Journal of Forecasting*, Vol. 29, No. 4, p. 754 – 766.

Rünstler, Gerhard (2004): Modelling Phase Shifts among Stochastic Cycles. In: *Econometrics Journal*, Vol. 7, No. 1, p. 232–248.

Stock, J. H.; Watson, M. W. (2011): Dynamic Factor Models. In: Clements, M.; Hendry, D. (Eds.) *The Oxford Handbook of Economic Forecasting*, p. 35–60.

Stock, James H; Watson, Mark W (2002): Macroeconomic forecasting using diffusion indexes. In: *Journal of Business & Economic Statistics*, Vol. 20, No. 2, p. 147–162.

Tiller, Richard B (1992): Time series modeling of sample survey data from the US Current Population Survey. In: *Journal of Official Statistics*, Vol. 8, No. 2, p. 149–166.

Valle e Azevedo, João; Koopman, Siem Jan; Rua, António (2006): Tracking the business cycle of the Euro area: a multivariate model-based bandpass filter. In: *Journal of Business & Economic Statistics*, Vol. 24, No. 3, p. 278–290.

Wanger, Susanne (2013): Arbeitszeit und Arbeitsvolumen in Deutschland – Methodische Grundlagen und Ergebnisse der Arbeitszeitrechnung. In: Wirtschafts- und Sozialstatistisches Archiv, Vol. 7, No. 1-2, p. 31–69.

Wanger, Susanne; Weigand, Roland; Zapf, Ines (2015): Measuring hours worked in Germany: Contents, data and methodological essentials of the IAB working time measurement concept. IAB Discussion Paper 21/2015, Institute for Employment Research.

Wood, John; Elliott, Duncan (2007): Methods explained: Forecasting. In: Economic & Labour Market Review, Vol. 1, No. 12, p. 55–58, URL <http://www.ons.gov.uk/ons/rel/elmr/economic-and-labour-market-review/no--12--december-2007/methods-explained--forecasting.pdf>.

## Recently published

No.	Author(s)	Title	Date
<a href="#">7/2015</a>	Vallizadeh, E. Muysken, J. Ziesemer, T.	Offshoring of medium-skill jobs, polarization, and productivity effect: Implications for wages and low-skill unemployment	2/15
<a href="#">8/2015</a>	Busk, H. Jahn, E. J. Singer, C.	Do changes in regulation affect temporary agency workers' job satisfaction?	2/15
<a href="#">9/2015</a>	Eisermann, M. Moritz, M. Stockinger, B.	Multinational resilience or dispensable jobs? German FDI and employment in the Czech Republic around the Great Recession	3/15
<a href="#">10/2015</a>	Walwei, U.	From deregulation to re-regulation: Trend reversal in German labour market institutions and its possible implications	3/15
<a href="#">11/2015</a>	vom Berge, P. Schmillen, A.	Direct and indirect effects of mass layoffs: Evidence from geo-referenced data	3/15
<a href="#">12/2015</a>	Brenzel, H. Reichelt, M.	Job mobility as a new explanation for the immigrant-native wage gap: A longitudinal analysis for the German labor market	3/15
<a href="#">13/2015</a>	Schanne, N.	A Global Vector Autoregression (GVAR) model for regional labour markets and its forecasting performance with leading indicators in Germany	4/15
<a href="#">14/2015</a>	Reichelt, M. Abraham, M.	Occupational and regional mobility as substitutes: A new approach to understanding job changes and wage inequality	4/15
<a href="#">15/2015</a>	Zapf, I.	Individual and workplace-specific determinants of paid and unpaid overtime work in Germany	4/15
<a href="#">16/2015</a>	Horbach, J. Janser, M.	The role of innovation and agglomeration for employment growth in the environmental sector	6/15
<a href="#">17/2015</a>	Dorner, M. Fryges, H. Schopen, K.	Wages in high-tech start-ups – do academic spin-offs pay a wage premium?	6/15
<a href="#">18/2015</a>	Möller, J.	Verheißung oder Bedrohung? Die Arbeitsmarktwirkungen einer vierten industriellen Revolution	6/15
<a href="#">19/2015</a>	Hecht, V.	Location choice of German multinationals in the Czech Republic: The importance of agglomeration economies	7/15
<a href="#">20/2015</a>	Wiemers, J.	Endogenizing take-up of social assistance in a microsimulation model: A case study for Germany	7/15
<a href="#">21/2015</a>	Wanger, S. Weigand, R. Zapf, I.	Measuring hours worked in Germany: Contents, data and methodological essentials of the IAB working time measurement concept	8/15

As per: 2015-08-20

For a full list, consult the IAB website

<http://www.iab.de/de/publikationen/discussionpaper.aspx>



# Survey

on the Relaunch of the IAB website

## Online Survey of the IAB web presence

The IAB is conducting an Online Survey of its German- and English-language web presence until September 2015. The aim is to gather information on the quality and variety of what IAB offers, on comprehensibility, motivation for use, and new user requirements, with a view to improving IAB's web presence even further. For this purpose we would like to hear your opinion, wishes and suggestions. We kindly ask you to take about ten minutes to take part in this Online Survey.

Click here to get to the [Online Survey](#).  
[Information](#) for survey participants.



## Imprint

IAB–Discussion Paper 22/2015

### Editorial address

Institute for Employment Research  
of the Federal Employment Agency  
Regensburger Str. 104  
D-90478 Nuremberg

### Editorial staff

Regina Stoll, Jutta Palm-Nowak

### Technical completion

Gertrud Steele

### All rights reserved

Reproduction and distribution in any form, also in parts,  
requires the permission of IAB Nuremberg

### Website

<http://www.iab.de>

### Download of this Discussion Paper

<http://doku.iab.de/discussionpapers/2015/dp2215.pdf>

ISSN 2195-2663

### For further inquiries contact the author:

Roland Weigand  
Phone +49.911.179 3291  
E-mail [roland.weigand@iab.de](mailto:roland.weigand@iab.de)