The Optimal Inflation Rate under Downward Nominal Wage Rigidity*

Preliminary - Please do not quote!

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May 20, 2011

Abstract

We study the implications for optimal average inflation when there is both a role for money as a medium of exchange and when nominal wages are downwardly rigid The model also features transaction costs, as in Schmitt-Grohe and Uribe, 2004, and a non-Walrasian labor market with search frictions as in Trigari, 2009. Overall, we find an optimal inflation rate of about 1.2 percent.

Keywords: Optimal Monetary Policy, Inflation, Downward Nominal Wage Rigidities.

JEL classification: E42, E52, J30.

*We are grateful for comments and suggestions from Michael Dotsey, Pablo Guerron, Keith Kuester and seminar participants at the Federal Reserve Bank of Philadelphia and the Econometric Society World Congress in Shanghai. We would also like to thank Mats Levander for excellent research assistance. The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Executive Board of Sveriges Riksbank.

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1 Introduction

A robust empirical finding is that money wages do not fall to any significant degree during an economic downturn. A large number of studies report substantial downward nominal wage rigidity in the U.S. as well as in Europe and Japan.[1] Overall, the evidence points towards a sharp asymmetry in the distribution of nominal wage changes around zero. That is, money wages rise but they seldom fall.

Recently, Schmitt-Grohe and Uribe, 2010 raised the puzzle that most central banks targets an annual inflation rate of two percent, whereas current monetary models implies a Ramsey optimal inflation rate that is usually negative.[2] This paper addresses this puzzle by studying how far asymmetric nominal wage rigidities in combination other rigidities/frictions can take us towards understanding the stated inflation targets of central banks.

To this end, we develop a DSGE model that can account for several important factors in determining the optimal inflation rate. To capture the Friedman argument for deflation, to avoid inefficient economizing in money balances, we introduce a transaction cost (as in Schmitt-Grohe and Uribe, 2004). To include the Tobin argument for a positive rate of inflation in order to grease the wheels of wage formation in the presence of downward nominal wage rigidity (see Tobin, 1972), we introduce price- and wage-setting frictions.[3] Since our ultimate aim is to study the optimal inflation rate, it is important to allow optimal price- and wage-setting decisions to depend on the inflation rate. In order to do so we model price- and wage-setting decisions as state dependent. Specifically, price setting follows Dotsey, King, and Wolman, 1999, while wage setting is based on a modified version of the bargaining model in Holden, 1994. Beside costs stemming from the potential break up of the firm/worker match when initiating bargaining under disagreement, firms and workers also face a fixed costs of disagreement, such as disruptions in business relationships and deteriorating management-employee relationships.[4] Another key feature of the model is that, consistently with empirical evidence, work proceed at the old contract, if no party credibly can threaten with disagreement. Moreover, since the fixed disagreement costs need not be identical for workers and firms, this opens up for downward nominal wage rigidities as a rational outcome. Finally, to provide a scope for a surplus to be bargained over, the model

[1] The empirical evidence ranges from studies using data from personnel files presented in Altonji and Devereux, 2000, Baker, Gibbs, and Holmstrom, 1994, Fehr and Goette, 2005, and Wilson, 1999, survey/register data in Altonji and Devereux, 2000, Akerlof, Dickens, and Perry, 1996, Dickens et. al., 2007, Fehr and Goette, 2005, Holden and Wulfsberg, 2008, Kuroda and Yamamoto (2003a, 2003b) to interviews or surveys with wage setters like Agell and Lundborg, 2003, and Bewley, 1999, just to mention a few.

[2] An exception is Kim and Ruge-Murcia, 2010 who finds an optimal inflation rate of about 0.4 percent in a model with downward nominal wage rigidity. However, including productivity growth would, almost certainly push this figure substantially below zero. See Amano et. al., 2009, for the effect of productivity growth on optimal inflation. For a detailed overview of the literature, see Schmitt-Grohe and Uribe, 2010.

[3] Another reason for a positive steady state inflation rate is to avoid the non-negativity constraint in nominal interest rates to bind too frequently, see e.g. Billi and Kahn, 2008.

[4] We thus modify the bargaining set up in Holden, 1994 by assuming that disagreement can lead to a break up of the firm/worker match rather than a conflict period. This generates a bargaining formulation that is in line with standard search-matching models used in the macro literature (see e.g. Trigari, 2009 and others)

features a search-matching labor market akin to the model of Trigari, 2009 and Christoffel, Kuester, and Linzert, 2009.

To parametrize the distribution of disagreement costs in the model, we use a minimum distance estimation approach to match the nominal wage change distribution implied by the model to the empirical nominal wage change distribution observed in U.S. micro data. The estimated model yields a distribution of wage changes that captures the main features of the empirical wage change distribution. A key feature of our model that allows the model to fit the micro data with any precision is the introduction of firm-level heterogeneity in terms of productivity, as well as, aggregate productivity growth. The first feature is needed to capture the large variance of the distribution of nominal wage changes in the data and the second feature is needed to capture the fact that nominal wages increase more than the inflation rate on average. The introduction of these two features also has implications for the optimal inflation rate via effects through the steady state wage distribution.[5]

Two related papers are Kim and Ruge-Murcia, 2010, and Fagan and Messina, 2009. They analyze the effects on the optimal inflation target from downward wage rigidity. Both these papers rely on asymmetric adjustment costs in wages as in Rotemberg, 1982 to generate downward wage rigidities. It thus becomes key for the planner to avoid these costs in the design of optimal policy. We take a different stand on the underlying reason for downward wage rigidities. We think of this friction as stemming from disagreement costs and the implied effects on the threat points in the wage bargaining. Since disagreement will not occur in equilibrium, these costs are of no direct consequence for the planner when designing optimal policy. Though indirectly, via the effect on nominal wage formation through private sector behavior, these costs will affect the design of optimal policy. Importantly, we model wage dispersion explicitly and thus capture the associated inefficiencies that are due to suboptimal levels of hours and output across firms and workers. This strategy also implies that we can match the model to micro data, which allows us to put additional empirical discipline on the analysis. The paper by Fagan and Messina, 2009 also uses micro data when estimating their model, but in contrast to them, we allow for inflation to affect price- and wage-setting frequencies, a role for money as a medium of exchange and solve for the Ramsey policy. The study by Kim and Ruge-Murcia, 2010 also contains the latter two of these features, but relies only on macro data for estimation and evaluation.

We find that the optimal annual inflation rate under downward nominal wage rigidity is around 1.2%. The optimal annual inflation rate found here is significantly larger than in the baseline monetary models discussed in Schmitt-Grohe and Uribe, 2010 where the optimal inflation rate is generally at most around zero, and more in line with the targeted inflation central bank for most inflation-targeting

[5] The effect of aggregate productivity growth has been studied previously by Amano et. al., 2009, finding a negative impact on the optimal steady state inflation rate.

central banks. Thus, the features added in this model to the canonical monetary model can take us quite some distance in understanding the Schmitt-Grohe and Uribe, 2010 puzzle. However, we also show that varying the degree of flexibility in wage formation has large effects on this conclusion. Specifically, letting new hires wages to be perfectly flexible leads to an optimal annual inflation rate of around zero.

The increase in inflation relative to a model with flexible wages and price adjustment frictions is a little more than two percentage units. The introduction of downward nominal wage rigidities into a model with flexible wages can be decomposed into two effects. First, wage adjustment frictions are introduced and second, the frictions become asymmetric. The effects of introducing wage adjustment frictions is a little larger than when introducing asymmetries; wage adjustment frictions increases the yearly inflation rate by approximately 1.3 percentage units, while also introducing asymmetries increases the inflation rate by around 0.8 percentage units.

This paper is organized as follows, in section 2, we outline the model. In section 3, the optimal policy is described, in section 4 the calibration of the model is presented and in section 5 the results are presented. Finally, section 6 concludes.

2 The Economic Environment

The basic framework shares many elements of standard DSGE models. There is a monopolistically competitive intermediate goods sector where producers set prices facing a known random (periodically) fixed cost of price adjustment as in Dotsey, King, and Wolman, 1999. Thus, for tractability, we assume that prices have a finite duration of at most J periods. There is also a wholesale sector that uses capital and labor to produce an input for the intermediate-goods sector.[6] The input is sold on a perfectly competitive market. The wholesale sector rents capital on a competitive capital market and post vacancies on a search and matching labor market. Wages are bargained between workers and the firm following a slightly modified version of Holden, 1994. In the model, the parties bargain every period. Each bargaining round starts with one of the parties making a bid, then the other party responds yes or no. If the response is no, there is a choice whether to continue bargaining in good faith and post a counter offer or enter into disagreement. If the latter choice is made, there is a probability that the match breaks down and the wage is determined in a standard Rubinstein-Ståhl fashion. Moreover, in case a party initiate bargaining under disagreement, both parties face their own known fixed disagreement cost (randomly drawn at the beginning of each period). This cost may be due to

[6] For simplicity, we abstract from capital accumulation, though.

deteriorating firm/worker and customer relationships.[7] In case none of the parties chooses to bargain under disagreement, but being unable to settle on a new wage, work continues according to the old contract. If the disagreement cost is sufficiently high, it is not credible for a party to threaten with disagreement in order to achieve a new wage contract. Instead, the outcome will be to continue to work according to the old contract already in place, thus endogenizing nominal wage rigidity. To capture the downward nominal wage rigidity observed in micro data it is required that firms, on average, face higher disagreement cost. As with prices, we assume that wage contracts last for at most J^w periods.

In order to introduce complete consumption insurance, we assume that there is a representative family as in Merz, 1995. Finally, notation is simplified by assuming a flexible price retail sector that repacks the intermediate goods in accordance with consumer preferences and sells them to consumers on a competitive market.

2.1 Retail firms

We follow Erceg, Henderson, and Levin, 2000 and Khan, King, and Wolman, 2003 and assume a competitive retail sector that buy intermediate goods and sell a composite final good. The composite good is combined from intermediate goods in the same proportions as households would choose. Given intermediate goods output levels Y_t^j produced by intermediate goods firms j, the amount of the composite good Y_t is

$$Y_t = \left[\sum_{j=0}^{J-1} \omega_t^j \left(Y_t^j \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where $\sigma > 1$ and ω^j is the share of retail firms producing Y_t^j at price P_t^j. The price P_t of one unit of the composite good is

$$P_t = \left[\sum_{j=0}^{J-1} \omega_t^j \left(P_t^j \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \tag{2}$$

2.2 Intermediate goods firms

The intermediate goods firms optimally choose whether to change prices, given a menu cost c_p of changing prices. Adjustment costs are drawn from a cumulative distribution function G_P. Let the probability of adjusting prices in a given period be denoted by α_t^j, given that the firm last adjusted it's price j periods ago. We assume that there is some $J > 1$ such that $\alpha^J = 1$.

[7] Note that there is no disagreement in equilibrium, and hence the equilibrium disagreement cost is zero. Thus, in contrast to state-dependent pricing, these cost neither enter resource constraints nor firm/worker value functions.

2.2.1 Prices

Given that an intermediate goods firm last reset prices in period $t-j$, the maximum remaining duration of a price contract is $J-j$, where J is the maximum duration of a price contract and α_t^j is the adjustment probability j periods after the price was last reset. The intermediate goods firms buys a homogeneous input from the wholesale firms at the real price p_t^w. Finally, the average or expected (real) adjustment cost, in terms of aggregate output, is given by

$$\Xi_{j,t} = \frac{1}{\alpha_t^j} \int_0^{G_P^{-1}(\alpha_t^j)} x dG_P(x). \tag{3}$$

Note that the upper bound is given by the maximum menu cost c_p that induces price adjustment, i.e., the c_p that solves $\alpha_t^j = G_P(c_p)$. As in Khan, King, and Wolman, 2003, but extended as in Lie, 2010 to allow for state-dependent pricing, an intermediate producer chooses the optimal price P_t^0 so that

$$\begin{aligned} v_t^0 &= \max_{P_t^0} \left[\frac{P_t^0}{P_t} - p_t^w\right] Y_t^0 + E_t \Lambda_{t,t+1} \beta \left(\alpha_{t+1}^1 v_{t+1}^0 + \left(1-\alpha_{t+1}^1\right) v_{t+1}^1 \left(\frac{P_t^0}{P_{t+1}}\right)\right) \\ &\quad - E_t \Lambda_{t,t+1} \beta p_{t+1}^w \alpha_{t+1}^1 \Xi_{1,t+1}, \end{aligned} \tag{4}$$

where

$$Y_t^j = \left(\frac{P_t^j}{P_t}\right)^{-\sigma} Y_t, \tag{5}$$

where P_t is the aggregate price level, β the discount factor and $\Lambda_{t,t+1}$ the ratio of Lagrange multipliers in the problem of the consumer tomorrow and today (i.e., relative value of consumption today versus tomorrow). The values v_t^j evolve according to

$$\begin{aligned} v_t^j \left(\frac{P_t^j}{P_t}\right) &= \left[\frac{P_t^j}{P_t} - p_t^w\right] Y_t^j + E_t \Lambda_{t,t+1} \beta \left(\alpha_{t+1}^{j+1} v_{t+1}^0 + \left(1-\alpha_{t+1}^{j+1}\right) v_{t+1}^{j+1} \left(\frac{P_t^j}{P_{t+1}}\right)\right) \\ &\quad - E_t \Lambda_{t,t+1} \beta p_{t+1}^w \alpha_{t+1}^{j+1} \Xi_{j+1,t+1}, \\ v_t^{J-1} \left(\frac{P_t^{J-1}}{P_t}\right) &= \left[\frac{P_t^{J-1}}{P_t} - p_t^w\right] Y_t^{J-1} + E_t \Lambda_{t,t+1} \beta v_{t+1}^0 - E_t \Lambda_{t,t+1} \beta p_{t+1}^w \Xi_{J,t+1}. \end{aligned} \tag{6}$$

Note that the term within the square brackets is just the firm's per unit profit in period $t+k$, given that prices were last reset in period t.

The first-order condition to the problem (4) is

$$\left[(1-\sigma)\frac{P_t^0}{P_t} + \sigma p_t^w\right] Y_t^0 \frac{1}{P_t} + E_t \Lambda_{t,t+1} \beta \left(\left(1-\alpha_{t+1}^1\right) D_1 v_t^1 \left(\frac{P_t^0}{P_{t+1}}\right) \frac{1}{P_{t+1}}\right) = 0, \tag{7}$$

where, noting that $P_{t+j}^{j}=P_{t}^{0}$, the derivative $D_1 v_t^1$ can be computed by using

$$D_1 v_t^j\left(\frac{P_t^j}{P_t}\right) = \left[(1-\sigma)\frac{P_t^j}{P_t}+\sigma p_t^w\right]\frac{Y_t^j}{P_t}+E_t\Lambda_{t,t+1}\beta\left(\left(1-\alpha_{t+1}^{j+1}\right)D_1 v_{t+1}^{j+1}\left(\frac{P_t^j}{P_{t+1}}\right)\frac{1}{P_{t+1}}\right),$$

$$D_1 v_t^{J-1}\left(\frac{P_t^{J-1}}{P_t}\right) = \left[(1-\sigma)\frac{P_t^{J-1}}{P_t}+\sigma p_t^w\right]\frac{Y_t^{J-1}}{P_t}. \tag{8}$$

Thus, optimal pricing behavior is fully characterized by expressions (7) and (8).

We model price adjustment probabilities as in Dotsey, King, and Wolman, 1999 and others. Thus, adjustment probabilities are chosen endogenously by the firm and is one if $c_p < \frac{v_t^0-v_t^j}{p_t^w}$ and zero if $c_p > \frac{v_t^0-v_t^j}{p_t^w}$. Adjustment costs are drawn from a cumulative distribution function G_P and the share of firms among those that last adjusted the price j periods ago that adjusts the price today is given by

$$\alpha_t^j = G_P\left(\frac{v_t^0-v_t^j}{p_t^w}\right). \tag{9}$$

Moreover the shares of firms with duration j since the last price change is denoted by ω_t^j. For $j\geq 1$ we have

$$\omega_t^j=\left(1-\alpha_t^j\right)\omega_{t-1}^{j-1}, \tag{10}$$

and, for $j=0$,

$$\omega_t^0=\sum_{j=1}^{J-1}\alpha_t^j\omega_{t-1}^{j-1}. \tag{11}$$

Assume that G_P follows a beta distribution, i.e., the probability density is $g_P=\frac{1}{\beta(l_p+1,r_p+1)}x^{l_p}\left(1-x\right)^{r_p}$.

2.3 Households

Households have preferences

$$E_t\sum_{r=t}^{\infty}\beta^{r-t}\left[u\left(c_r\right)-\int_i \kappa^L\frac{\left(h_{ir}\right)^{1+\xi}}{1+\xi}di\right], \tag{12}$$

where h_{it} denotes the households hours worked at firm i. In contrast to Christoffel, Kuester, and Linzert, 2009 and Khan, King, and Wolman, 2003, consumption purchases are subject to a proportional transaction cost $s=Av+\frac{B}{v}-2\sqrt{AB}$ as in Schmitt-Grohe and Uribe, 2004.[8] The budget constraint

[8] Note that the transaction cost function have a satiation point $\underline{v}$ ensuring that transactions demand is bounded and that, since $v_t=\frac{c_t}{m_t}$, the transaction cost increases in c_t and decreases in m_t as long as $\frac{c_t}{m_t}$ is above the satiation point.

of the family is given by

$$P_t c_t \left(1 + s\left(\frac{c_t}{m_t}\right)\right) + P_t m_t + \frac{b_t}{R_t} + \theta_{t+1} P_t \left(F_t - Z_t\right) \geq P_{t-1} m_{t-1} + \varpi_t + \mathcal{W}_t, \tag{13}$$

where P_t is the price level, M_t is money holdings and $m_t = \frac{M_t}{P_t}$ is real money balances, b_t bonds, θ_{t+1} is the share of intermediate product firms, F_t the value of firms (measured on a pre-dividend basis $F_t - Z_t$)[9] and Z_t nominal dividends, ϖ_t is wealth at the start of time t, R_t is the one period nominal interest rate, b_t denotes one period nominal bonds and where

$$\mathcal{W}_t = \int_0^1 E_t W_{it} di + \left(1 - n_t\right) b_r, \tag{14}$$

with b_r representing the value of home production. Moreover, W_{it} denotes the households nominal wage. Each family own an equal share of all firms and of the aggregate capital stock. Finally, note that $1 - n_t$ is equal to the unemployment rate. Moreover, we have

$$\varpi_t = \theta_t P_t F_t + b_{t-1}.$$

The household first-order conditions are

$$\begin{aligned} c_t &: u_c\left(c_t\right) = \lambda_t \left(1 + s\left(\frac{c_t}{m_t}\right) + c_t s'\left(\frac{c_t}{m_t}\right)\frac{1}{m_t}\right), \\ b_t &: E_t \beta V_b\left(b_t, m_t, n_{t+1}\right) = \frac{\lambda_t}{R_t}, \\ m_t &: -\lambda_t \left(-\frac{c_t^2}{m_t^2} s'\left(\frac{c_t}{m_t}\right) + 1\right) + E_t \beta V_m^c\left(b_t, m_t, n_{t+1}\right) = 0, \end{aligned} \tag{15}$$

where $V_b^c\left(b_t, m_t, n_{t+1}\right)$ and $V_m^c\left(b_t, m_t, n_{t+1}\right)$ are the derivatives of the household value function.

Using the envelope theorem and the first-order condition with respect to b_t we can write the household Euler equation as

$$\frac{\lambda_t}{R_t} = \beta E_t \frac{\lambda_{t+1}}{1 + \pi_{t+1}}. \tag{16}$$

2.4 Search and matching

In each period wholesale firm i post v_{it} vacancies and employs n_{it} workers. The aggregate number of vacancies is

$$v_t = \int_0^1 E_t v_{it} di, \tag{17}$$

[9]Note that the net cost of buying a unit of claims is $F_t - Z_t$.

and aggregate employment is

$$n_t = \int_0^1 E_t n_{it} di. \tag{18}$$

As in Christoffel, Kuester, and Linzert, 2009, the number of unemployed workers is

$$u_t = 1 - n_t. \tag{19}$$

We assume that the number of matches m_t is given by the following constant-returns matching function

$$m_t^a = \sigma_m u_t^{\sigma_a} v_t^{1-\sigma_a}, \tag{20}$$

where u_t is unemployment and v_t the number of vacancies. The probability that a worker is matched to a firm

$$s_t^a = \frac{m_t^a}{u_t}, \tag{21}$$

and the probability that a vacancy is filled is

$$q_t = \frac{m_t^a}{v_t}. \tag{22}$$

Finally, a match is broken with probability $1 - \rho$.

2.5 Wage determination

Wage determination also closely follows the model in Christoffel, Kuester, and Linzert, 2009. Thus wholesale firms bargains with workers with some positive probability $\alpha_t^{j_w}$ in the j_w'th period following the last renegotiation. Though, in the model at hand, these probabilities are endogenous.

In the wholesale sector, wage adjustment probabilities are given by $\alpha_t^{j_w}$ with $\alpha_t^{J_w} = 1$ for some $J_w > 1$. In the model, these adjustment probabilities may depend on whether wages increase or decrease.

2.6 Value functions

The wholesale firm i use capital k_{it} and labor h_{it} as inputs to produce output y_{it} using a constant returns technology

$$y_{it} = (a_{it} h_{it})^{1-\gamma}. \tag{23}$$

where $a_{it} = e^{\gamma_r t} \varepsilon_{it}^a$ with γ_r being the growth of technology and ε_{it}^a an idiosyncratic shock. Also, let A denote the set of productivity levels. For simplicity, however, we will suppress the idiosyncratic productivity dimension in the notation in what follows.

The value for the family of a worker at wholesale firm i is in period t is, letting $\vartheta\left(a_{t+1}, a_t\right)$ denote the transition probability from productivity state a_t to a_{t+1},

$$
\begin{aligned}
V_t^{j_w}\left(w_t^{j_w}, a_t\right) &= w_t^{j_w} h_t\left(w_t^{j_w}, a_t\right) - \kappa^L \frac{\left(h_t\left(w_t^{j_w}, a_t\right)\right)^{1+\xi}}{1+\xi} + \beta \sum_{a_{t+1} \in A} E_t \Lambda_{t,t+1} \vartheta\left(a_{t+1}, a_t\right) \\
&\times \Big[\left(\rho \alpha_{t+1}^{j_w+1}\left(w_{t+1}^{j_w+1}, a_{t+1}\right) V_{t+1}^0\left(w_{t+1}^0, a_{t+1}\right) + (1-\rho) U_{t+1}\right) \\
&+ \left(\rho\left(1-\alpha_{t+1}^{j_w+1}\left(w_{t+1}^{j_w+1}, a_{t+1}\right)\right) V_{t+1}^{j_w+1}\left(w_{t+1}^{j_w+1}, a_{t+1}\right) + (1-\rho) U_{t+1}\right)\Big],
\end{aligned} \tag{24}
$$

where $w_t^{j_w} = \frac{W_t^{j_w}}{P_t}$ is the real wage and, since the firm has the right to manage, hours $h_t\left(w_t^{j_w}, a_t\right)$ are determined by the firm by maximizing the per-period payoff in

$$
p_t^w y_{it} - w_t^{j_w} h_{it},
$$

with respect to h_{it}, taking technology (23), wages and p_t^w as given. The value when being unemployed is

$$
U_t = b_r + \beta E_t \Lambda_{t,t+1}\left(s_{t+1}^a V_{x,t+1} + \left(1 - s_{t+1}^a\right) U_{t+1}\right). \tag{25}
$$

where, letting $\omega_t^{j_w}\left(w_{t+1}^{j_w+1}, a_t\right)$ denote the share of workers with wage $w_t^{j_w}$ and productivity a_t,

$$
V_{x,t} = \sum_{j_w=0}^{J_w-1} \sum_{a_t \in A} \omega_t^{j_w}\left(w_{t+1}^{j_w+1}, a_t\right) V_t^{j_w}\left(w_t^{j_w}, a_t\right), \tag{26}
$$

is the average value of employment, where the expectation is taken over all firms. Then the bargaining surplus (defined by $j_w = 0$) for the worker is, as usual in bargaining models with a probability of match breakdown, given by the difference between the value of employment and unemployment

$$
H_t^{j_w}\left(w_t^{j_w}, a_t\right) = V_t^{j_w}\left(w_t^{j_w}, a_t\right) - U_t, \tag{27}
$$

and hence, the value of an additional employee for the household can then be written as

$$
\begin{aligned}
H_t^{j_w}\left(w_t^{j_w}, a_t\right) &= w_t^{j_w} h_t\left(w_t^{j_w}, a_t\right) - \kappa^L \frac{\left(h_t\left(w_t^{j_w}, a_t\right)\right)^{1+\xi}}{1+\xi} - b_r + \beta \sum_{a_{t+1} \in A} E_t \Lambda_{t,t+1} \vartheta\left(a_{t+1}, a_t\right) \\
&\times \Big[\left(\rho \alpha_{t+1}^{j_w+1}\left(w_{t+1}^{j_w+1}, a_{t+1}\right) H_{t+1}^0\left(w_{t+1}^0, a_{t+1}\right) - s_{t+1}^a H_{x,t+1}\right) \\
&+ \left(\rho\left(1-\alpha_{t+1}^{j_w+1}\left(w_{t+1}^{j_w+1}, a_{t+1}\right)\right) H_{t+1}^{j_w+1}\left(w_{t+1}^{j_w+1}, a_{t+1}\right) - s_{t+1}^a H_{x,t+1}\right)\Big].
\end{aligned} \tag{28}
$$

where

$$H_{x,t} = V_{x,t} - U_t.$$

For the firm wholesale firm, the value of an additional employee is

$$\begin{aligned} J_t^{j_w}\left(w_t^{j_w}, a_t\right) &= p_t^w\left(a_t h_t\left(w_t^{j_w}, a_t\right)\right)^{1-\gamma} - w_t^{j_w} h_t\left(w_t^{j_w}, a_t\right) - \Phi \\ &+\beta \sum_{a_{t+1}\in A} \Lambda_{t,t+1}\vartheta\left(a_{t+1}, a_t\right)\alpha_{t+1}^{j_w+1}\left(w_{t+1}^{j_w+1}, a_{t+1}\right)\left(\rho J_{t+1}^0\left(w_{t+1}^0, a_{t+1}\right)\right) \\ &+\beta \sum_{a_{t+1}\in A} \Lambda_{t,t+1}\vartheta\left(a_{t+1}, a_t\right)\left(1-\alpha_{t+1}^{j_w+1}\left(w_{t+1}^{j_w+1}, a_{t+1}\right)\right)\rho J_{t+1}^{j_w+1}\left(w_{t+1}^{j_w+1}, a_{t+1}\right), \end{aligned} \tag{29}$$

where Φ are fixed consisting of a fixed labor cost Φ_L and a fixed capital cost Φ_K as in Christoffel, Kuester, and Linzert, 2009. A firm that last renegotiated wages j periods ago can credibly disagree if the gain from adjusting the wage

$$dJ_t^{j_w}\left(w_t^{j_w}, a_t\right) = J_t^0\left(w_t^0, a_t\right) - J_t^{j_w}\left(w_t^{j_w}, a_t\right),$$

is larger that the disagreement cost. Similarly, the worker disagree if

$$dH^{j_w}\left(w_t^{j_w}, a_t\right) = H_t^0\left(w_t^0, a_t\right) - H_t^{j_w}\left(w_t^{j_w}, a_t\right),$$

is larger that the workers disagreement cost.

Wage determination Wages are determined in bargaining between firms and the household member employed by the firm. Akin to Holden, 1994, if it is not credible to threaten with disagreement the parties settle on the previous periods wage. If it is credible to threaten with disagreement the wage is determined in a standard Rubinstein-Ståhl barging game. Since there is equivalence between the standard non-cooperative approach in Rubinstein, 1982 and the Nash bargaining approach, we use the latter method. The nominal wage W_{it}^0 is then chosen such that is solves the following problem

$$\max_{W_{it}^0}\left(H_t^0\left(w_t^0, a_t\right)\right)^{\varphi}\left(J_t^0\left(w_t^0, a_t\right)\right)^{1-\varphi}, \tag{30}$$

and φ denotes the bargaining power of workers. The first-order condition with respect to the nominal wage W_{it}^0 corresponding to (30) is

$$\varphi J_t^0\left(w_t^0, a_t\right) D_W H_t^0\left(w_t^0, a_t\right) + (1-\varphi) H_t^0\left(w_t^0, a_t\right) D_W J_t^0\left(w_t^0, a_t\right) = 0, \tag{31}$$

where $D_W H_t^0\left(w_t^0, a_t\right)$ and $D_W J_t^0\left(w_t^0, a_t\right)$ are computed using expressions (28) and (29).

2.6.1 Adjustment probabilities

The disagreement costs for the firm follow the cumulative distribution function $G^J : [0, \mathcal{B}^J] \to [0, 1]$ and the disagreement cost of workers follow the cumulative distribution function $G^H : [0, \mathcal{B}^H] \to [0, 1]$ with upper bounds $\mathcal{B}^J$ and $\mathcal{B}^H$, respectively. The adjustment probabilities are given by

$$\alpha_t^{j_w}\left(w_t^{j_w}, a_t\right), \tag{32}$$

and depend on both $G^J\left(dJ_t^{j_w}\left(w_t^{j_w}, a_t\right)\right)$ and $G^H\left(dH_t^{j_w}\left(w_t^{j_w}, a_t\right)\right)$. A detailed description on how there are computed are given in the appendix.

2.6.2 The hiring decision and employment flows

Firms chooses it's hiring so that the hiring cost of an additional employee is equal to the value. Thus, hiring is determined by

$$\kappa_t v_t = m_t^a \beta \sum_{j_w=0}^{J_w-1} \sum_{a_{t+1}\in A} E_t \omega_t^{j_w}\left(w_{t+1}^{j_w+1}, a_{t+1}\right) \Lambda_{t,t+1} J_{t+1}^{j_w}\left(w_{t+1}^{j_w}, a_{t+1}\right), \tag{33}$$

where the expectation is taken across all firms. This assumption makes wage setting in our model akin to Gertler and Trigari, 2009 and Christoffel, Kuester, and Linzert, 2009, where newly hired workers is paid the going wage in the firm. Here, however, we need to adjust that assumption to fit into the single-employee firm assumption, which we use as a modeling short cut.[10] Below, we also analyze the case with fully flexible wages for new hires.

Since there has been a significant controversy in the literature whether the wages of newly hired workers are more flexible than for incumbent workers, we find it important to motivate this assumption. Micro-data studies, summarized in Pissarides, 2009, seem to indicate that newly hired workers wages are substantially more flexible than incumbents wages. However, answering the question wether newcomers wages are more cyclical than incumbents wages is associated with severe identification problems. Especially, the studies summarized in Pissarides, 2009 generally fail to control for effects stemming from variations in the composition of firms and match quality over the cycle. It might thus be that the empirical evidence just reflect that workers move from low wage firms (low quality matches) to high wage firms (high quality matches) in boom periods and vice versa in recessions. The approach taken by e.g. Gertler and Trigari, 2009 to address this issue is introduce a job-specific

[10] One rationale for the formulation of the hiring condition is to think of a two-level structure in wholesale firms. First, large wholesale firms, who write staggered contracts with intermediate firms, produce a homogenous labor input according to a constant returns technology and buys input from small wholesale firms with no mark down. Secondly, small (single employee) wholesale firms who post vacancies and who are randomly assigned to a large wholesale firm after a match is found, which in turn determines the wage in the match.

fixed effects in a regression of individual wages on the unemployment rate and the interaction of the unemployment rate and dummy variable indicating if the tenure of the worker is short. This should control for composition effects in workers, firms and match quality. The problem, however, is that the interaction effect is then only identified with the within-match variation. It answers the question wether wages for workers with short tenure responds more to cyclical factors than wages for workers with longer tenure *after* that the worker has already been hired. Albeit an interesting question in itself, it is not the question at hand. Thus, existing micro-data studies can only takes us so far. If we instead turn to survey evidence, like Bewley, 1999, Bewley, 2007 for the U.S. and the study performed within the Eurosystem Wage Dynamics Network (WDN) covering about $17,000$ firms in 17 European countries, we see strong evidence of that the wages of new hires are tightly linked to those of incumbents. As reported by Galuscak et al., 2010, about 80% percent of the firms in the WDN survey respond that internal factors (like the internal pay structure) are the more important factor driving wages of new hires rather than external or market conditions. Finally, turning to the macro evidence, De Walque, 2009, develops a DSGE model that allows for a separate analysis of the flexibility of new and incumbent workers wages via different probabilities of being able to negotiate the wage. Estimates of this model relying on the European AWM database (presented in the final report of the WDN; see Lamo and Smets, 2009), indicate that new hires negotiate their wage in the same proportion as incumbents, in line with the survey evidence. Thus, all in all, we view the assumption underlying (33) as the natural baseline. However, we also explore the implications of modeling new hires wages as perfectly flexible.

Finally, the employment flow between categories $n_t^{j_w}$ is given by

$$n_t^0(a_t) = \sum_{j_w=1}^{J_w} \sum_{a_{t-1}\in A} \vartheta(a_t, a_{t-1})\, \rho \alpha_{t-1}^{j_w-1}\left(w_{t-1}^{j_w-1}, a_{t-1}\right) n_{t-1}^{j_w-1}(a_{t-1}) + \frac{n_t^0(a_t)}{n} m_t^a, \tag{34}$$

and, for $j_w > 0$,

$$n_t^{j_w}(a_t) = \sum_{a_{t-1}\in A} \vartheta(a_t, a_{t-1})\, \rho \left(1 - \alpha_{t-1}^{j_w-1}\left(w_{t-1}^{j_w-1}, a_{t-1}\right)\right) n_{t-1}^{j_w-1}(a_{t-1}) + \frac{n_t^{j_w}(a_t)}{n} m_t^a. \tag{35}$$

When the wages of newly hired workers are completely flexible, hiring is determined by

$$\kappa_t v_t = m_t^a \beta \sum_{a_{t+1}\in A} E_t \frac{\omega_t^0\left(w_{t+1}^0, a_{t+1}\right)}{\sum_{a_{t+1}\in A} \omega_t^0\left(w_{t+1}^0, a_{t+1}\right)} \Lambda_{t,t+1} J_{t+1}^0\left(w_{t+1}^0, a_{t+1}\right), \tag{36}$$

instead of (33) and the value of getting a job is

$$V_{x,t} = \sum_{a_t \in A} \frac{\omega_t^0 \left(w_t^0, a_t\right)}{\sum_{a_t \in A} \omega_t^0 \left(w_t^0, a_t\right)} V_t^0 \left(w_t^0, a_t\right), \tag{37}$$

instead of (26). Moreover, the flow equations for employment will change to

$$n_t^0 \left(a_t\right) = \sum_{j_w=1}^{J_w} \vartheta \left(a_t, a_{t-1}\right) \rho \alpha_{t-1}^{j_w-1} \left(w_{t-1}^{j_w-1}, a_{t-1}\right) n_{t-1}^{j_w-1} \left(a_{t-1}\right) + \frac{n_t^0 \left(a_t\right)}{\sum_a n_t^0 \left(a_t\right)} m_t^a, \tag{38}$$

and, for $j_w > 0$,

$$n_t^{j_w} \left(a_t\right) = \sum_{a_{t-1} \in A} \vartheta \left(a_t, a_{t-1}\right) \rho \left(1 - \alpha_{t-1}^{j_{we-1}} \left(w_{t-1}^{j_w-1}, a_{t-1}\right)\right) n_{t-1}^{j_w-1} \left(a_{t-1}\right). \tag{39}$$

3 Optimal Policy

In the model we have several distortions. First, there is imperfect competition in the product market. There is also a distortion due to transactions costs in the final goods market. Furthermore, there are relative price and relative wage distortions. Finally, there are distortions in the hiring decision on the labor market.

The policymaker maximizes (12) subject to the constraints (7), (8), (16) the resource constraint, equating supply with demand[11]

$$\begin{aligned} \sum_{j=0}^{J-1} \omega_t^j y_t^j - \sum_{j=0}^{J-1} \omega_t^j \alpha_t^j \Xi_{j,t} \quad = \quad & \sum_{j=0}^{J-1} \omega_t^j \left(p_t^j\right)^{-\sigma} \left[c_t \left(1 + s \left(\frac{c_t}{m_t}\right)\right)\right. \\ & \left. + m_t^a v_t + \Phi_L n_t - \left(1 - n_t\right) b_r\right], \end{aligned} \tag{40}$$

the flow equation of prices

$$p_t^j = \frac{p_{t-1}^{j-1}}{1 + \pi_t}, \tag{41}$$

expressions (2), (4), (6), (9), (15), (19), (21), (28), (29), (31), (32), (33), (34), (35) and the flow equation of wages

$$w_t^j = \frac{w_{t-1}^{j-1}}{1 + \pi_t}. \tag{42}$$

[11]Note that, since adjustment costs is in terms of aggregate output, the left-hand side is total output, net of these costs. The right-hand side consists of the weighted sum across firm demand $\left(p_t^j\right)^{-\sigma} y_t^d$ with

$$y_t^d = c_t \left(1 + s \left(\frac{c_t}{m_t}\right)\right) + m_t^a v_t + \Phi_L n_t - \left(1 - n_t\right) b_r.$$

See Christoffel, Kuester, and Linzert, 2009.

The problem of the policy maker is given by

$$U^*\left(\phi_{t-1},\varsigma_t\right) = \min_{\phi_t}\max_{d_t}\left[u\left(c_t\right) - \int_i \kappa^L \frac{\left(h_{it}\right)^{1+\xi}}{1+\xi} di + U^*\left(\phi_t,\varsigma_{t+1}\right) + A_t^c\left(\phi_t,\phi_{t-1},d_t\right) + B_t^c\left(\phi_t,\phi_{t-1},d_t\right)\right], \tag{43}$$

where ϕ_t a vector of lagrange multipliers, ς_t is a vector of state variables in period t, d_t a vector of control variables, B_t^c are the constraints from wage setting and A_t^c the remaining constraints. The problem is stated fully in the appendix, see Marcet and Marimon, 1998 for details. Numerically, we solve the problem using the method proposed in Schmitt-Grohe and Uribe, 2009.

4 Calibration

For our numerical exercises, we assume that $u\left(c_t\right) = \log c_t$. The calibration of the deep parameters are presented in Table 2. To find the steady state of the model, we set the capital share γ to 1/3 and average productivity growth is 1.004 on a quarterly basis. To model the idiosyncratic productivity process, we use a four-state Markov chain with a quarterly persistence of 0.6 (bounded from above due to numerical reasons) and with a ratio between the max and the min state of $\frac{3.8750}{5.1250} \approx 0.76$. The

Table 1: Baseline Calibration of the Model

Parameters			
β	0.9928	b_r	0.48
σ	10	ρ	0.9
ξ	2	σ_a	0.6
γ	1/3	σ_μ	0.83
$\bar{\varepsilon}^a$	1.004	A	0.0111
ρ_a	0.096	B	0.07524
κ	0.085	φ	0.5
Φ_K	1/3	κ^L	24.3
Φ_L	0.0069		

value of b_r implies a replacement rate (the ratio of home production value to the average wage) of around 0.6. The parameter κ implies that vacancy costs are around 0.14% of steady-state output. We set the bargaining power $\varphi = 0.5$ implying symmetrical bargaining. For the job separation rate $1-\rho$, we follow Gertler, Sala, and Trigari, 2008 and set $\rho = 0.895$. For the parameters Φ_K and Φ_L we follow Christoffel, Kuester, and Linzert, 2009. The values for A and B in the transaction cost function are collected from Schmitt-Grohe and Uribe, 2004. We assume that the distribution of menu costs in price setting follow the beta distribution with parameters as in Lie, 2010, i.e., letting $g_P = \frac{1}{\beta(l,r)} x^{l-1}\left(1-x\right)^{r-1}$ denote the probability density function we have $l = 2.1$, $r = 1.0$ and upper bound 0.015.

5 Results

To find the optimal rate of inflation, we first need to pin down the parameters of the disagreement cost distributions. These costs also follow the beta distribution $g^H = \frac{1}{\beta(l_H, r_H)} x^{l_H - 1} (1-x)^{r_H - 1}$ for workers and $g^J = \frac{1}{\beta(l_J, r_J)} x^{l_J - 1} (1-x)^{r_J - 1}$ for firms. As for price setting, we set $l_H = 2.1$, $r_H = 1$ and $l_J = 2.1$, $r_J = 1$. To find parameters,i.e., the bounds of the distribution, we fit the dispersion of yearly wage changes in the model (given a yearly inflation rate of 2%) to the (average) empirical dispersion of yearly wage changes in the US during the period $1993 - 1997$ using a minimum distance estimator.[12] The time period is chosen since it represents a period with stable inflation close to two percent.

In this version we estimated the upper bounds for the two distributions. This procedure yields parameters $\mathcal{B}^H = 0.0168$ for workers and for firms $\mathcal{B}^J = 0.2213$. When imposing a symmetry restriction, we find the upper bounds to equal $\mathcal{B}^H = \mathcal{B}^J = 0.0519$. Moreover, the maximum length of a wage (price) contract is set to 7 (9) quarters.[13]

Given the resulting disagreement cost distributions, we then compute the optimal steady state inflation rate. To analyze the effects of downward nominal wage rigidity, we compare the optimal inflation rate to the optimal rate in a model where these rigidities are not present. Moreover, it is interesting to try to distinguish between the effects of just adding (symmetric) wage setting frictions from the effect of adding asymmetries, i.e., downward nominal wage rigidity. We do this by also looking at third model; a model with sticky wages but symmetric adjustment probabilities (averaging parameters of the two disagreement cost distributions). Finally, we analyze the case with flexible wages for new entrants.

Figure 1 (2) illustrates the model (empirical) distribution of nominal wage changes for stayers. Comparing the two figures, we see that the model captures key features of the empirical wage distribution fairly well. For example, the spike at zero nominal wage change and the peak around 5 % as well as the absence of any substantial mass on nominal wage cuts.

Table 2: Yearly optimal inflation rate under the Ramsey policy

	Asymmetric wage frictions	Symmetric wage frictions	Flexible wages
Baseline	1.21	0.36	−0.96
Flex wages for new hires	0.00		−0.96

We find that the optimal annual inflation rate under downward nominal wage rigidities is about

[12] The micro data on wages is collected from the the Panel Study of Income Dynamics and is corrected from measurement errors as described by Dickens *et. al.*, 2007.

[13] The difference in contract length between prices and wages is due to the increase in the computational burden of increasing the maximum length of wage contracts.

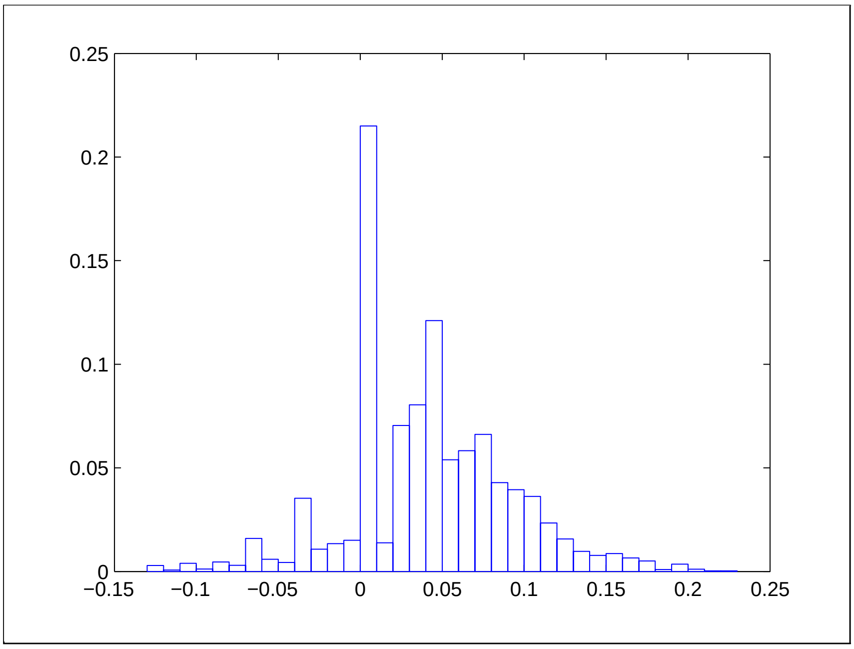

Figure 1: The nominal wage change distribution implied by the model.

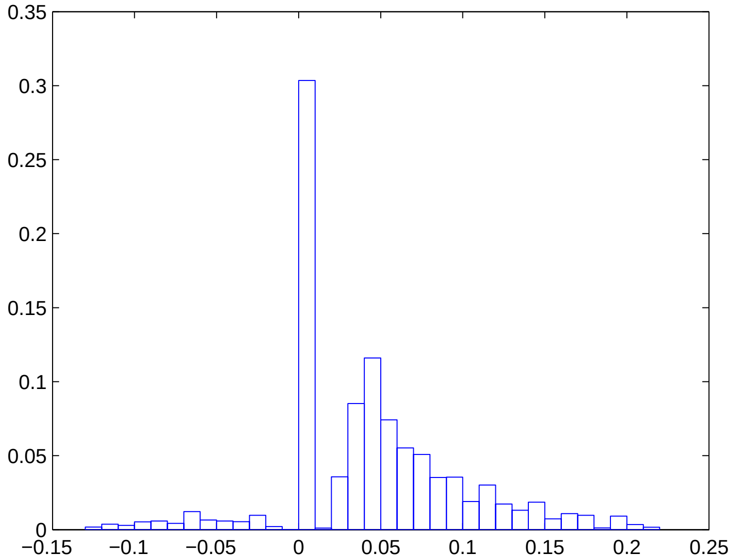

Figure 2: Empirical distribution of nominal wage changes in the US during the period 1993-1997

1.2% in the baseline calibration. The optimal annual inflation rate found here is significantly larger than in the baseline monetary models discussed in Schmitt-Grohe and Uribe, 2010 where the optimal inflation rate is at most around zero.

The increase in inflation relative to a model with flexible wages and price adjustment frictions is about 2.2 percentage units. The introduction of downward nominal wage rigidities into a model with flexible wages can be decomposed into two effects. First, wage adjustment frictions are introduced and second, the frictions become asymmetric. The effects of introducing wage adjustment frictions is larger than when introducing asymmetries; wage adjustment frictions increases the yearly inflation rate by approximately 1.3 percentage units, while also introducing asymmetries increases the inflation rate by around 0.8 percentage units. [14]

Next, we experiment by letting newly hired workers become flexible. We then find an optimal annual inflation rate of zero. Thus the treatment of the wage flexibility of newly hired workers has a large impact on the optimal policy prescription.

6 Concluding Discussion

We develop a DSGE model where there is a role for money as a medium of exchange, as well as, when declining nominal wages might not be a viable margin for adjustment. To capture the Friedman argument, we introduce a transaction cost (as in Schmitt-Grohe and Uribe, 2004). To include the Tobin argument, we introduce price- and wage-setting friction. Since our ultimate aim is to study the optimal inflation rate, it is important to allow optimal price- and wage-setting decision to depend on the inflation rate. To this end, both price and wage decisions are modeled as state dependent. Price-setting frictions are introduce as in Dotsey, King, and Wolman, 1999). Wage setting is based on the bargaining model in Holden, 1994, where downward nominal wage rigidities can arise as a rational outcome. Finally, the model feature a search-matching labor market akin to the model of Christoffel, Kuester, and Linzert, 2009. To parametrize the distribution of wage adjustment costs in the model, we use a minimum-distance estimation approach to match the nominal wage change distribution implied by the model to the empirical nominal wage change distribution observed in U.S. micro data. The estimated model yields a distribution of wage changes that captures the overall shape of the empirical wage distribution. An important feature that allows the model to fit the data with any precision, is the introduction of firm-level heterogeneity in terms of productivity, as well as, aggregate productivity growth.

[14] Note that the result for flexible wages is very much in line with Lie, 2010.

We find that the optimal annual inflation rate under downward nominal wage rigidities is 1.2%. The optimal annual inflation rate found here is significantly larger than in the baseline monetary models discussed in Schmitt-Grohe and Uribe, 2010 where the optimal inflation rate is at most around zero. However, we also show that the flexibility of the wage formation has large effects on this conclusion. Letting new hires wages to be perfectly flexible leads to an optimal annual inflation rate of around zero.

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Appendix

This appendix briefly describes wage adjustment probabilities and the optimal policy problem stated in section 3. For detailed derivations, see the accompanying technical appendix.

A Wage Adjustment Probabilities

The fraction of firms that disagree is

$$\begin{array}{cc} 1 & \text{if } \mathcal{B}^J < dJ_t^{j_w}\left(w_t^{j_w}, a_t\right), \\ G^J\left(dJ_t^{j_w}\left(w_t^{j_w}, a_t\right)\right) & 0 \leq dJ_t^{j_w}\left(w_t^{j_w}, a_t\right) \leq \mathcal{B}^J, \\ 0 & dJ_t^{j_w}\left(w_t^{j_w}, a_t\right) < 0. \end{array}$$

Similarly, the fraction of workers that has an incentive to disagree to force a renegotiation of the wage contract is

$$\begin{array}{cc} 1 & \text{if } \mathcal{B}^H < dH_t^{j_w}\left(w_t^{j_w}, a_t\right), \\ G^H\left(H_t^{j_w}\left(w_t^{j_w}, a_t\right)\right) & 0 \leq dH_t^{j_w}\left(w_t^{j_w}, a_t\right) \leq \mathcal{B}^H, \\ 0 & dH_t^{j_w}\left(w_t^{j_w}, a_t\right) < 0. \end{array}$$

The adjustment probabilities are then

$$\alpha_t^{j_w}\left(w_t^{j_w}, a_t\right) = 1.$$

if if $\mathcal{B}^J < dJ_t^{j_w}\left(w_t^{j_w}, a_t\right)$ or if $\mathcal{B}^H < dH_t^{j_w}\left(w_t^{j_w}, a_t\right)$,

$$\begin{aligned} \alpha_t^{j_w}\left(w_t^{j_w}, a_t\right) &= G^J\left(dJ_t^{j_w}\left(w_t^{j_w}, a_t\right)\right) + G^H\left(dH_t^{j_w}\left(w_t^{j_w}, a_t\right)\right) \\ &\quad - G^H\left(dH_t^{j_w}\left(w_t^{j_w}, a_t\right)\right) G^J\left(dJ_t^{j_w}\left(w_t^{j_w}, a_t\right)\right), \end{aligned}$$

if $0 \leq dJ_t^{j_w}\left(w_t^{j_w}, a_t\right) \leq \mathcal{B}^J$ and $0 \leq dH_t^{j_w}\left(w_t^{j_w}, a_t\right) \leq \mathcal{B}^H$,

$$\alpha_t^{j_w}\left(w_t^{j_w}, a_t\right) = G^J\left(dJ_t^{j_w}\left(w_t^{j_w}, a_t\right)\right),$$

if $0 \leq dJ_t^{j_w}\left(w_t^{j_w}, a_t\right) \leq \mathcal{B}^J$ and $dH_t^{j_w}\left(w_t^{j_w}, a_t\right) < 0$,

$$\alpha_t^{j_w}\left(w_t^{j_w}, a_t\right) = G^H\left(dH_t^{j_w}\left(w_t^{j_w}, a_t\right)\right),$$

if $dJ_t^{j_w}\left(w_t^{j_w}, a_t\right) < 0$ and $0 \leq dH_t^{j_w}\left(w_t^{j_w}, a_t\right) \leq \mathcal{B}^H$ and zero otherwise.

B Optimal Policy

Note first that, following the method in Khan, King, and Wolman, 2003, we can write the price setting first-order condition (7) and intermediate goods firms value derivatives (8) as

$$
\begin{aligned}
& \omega_0 x\left(p_t^0,.\right)+\beta E_t \chi_{1,t+1}, \qquad (44)\\
\chi_{j,t} &= \omega_j x\left(p_t^j,.\right)+\beta E_t \chi_{j+1,t+1},
\end{aligned}
$$

and

$$
\chi_{J-1,t}=\omega_{J-1} x\left(p_t^{J-1},.\right)
$$

where

$$
x\left(p_t^j,.\right)=\lambda_t p_t^j\left[(1-\sigma) p_t^j+\sigma p_t^w\right] Y_t^j .
$$

Similarly, for the bargaining problem, we define

$$
\begin{aligned}
y^J\left(w_t^{j^w}, a_t,.\right) &= \lambda_t\left(p_t^w\left(a_t h_t\left(w_t^{j_w}, a_t\right)\right)^{1-\gamma}-w_t^{j_w} h_t\left(w_t^{j_w}, a_t\right)-\Phi\right), \\
y^H\left(w_t^{j^w}, a_t,.\right) &= \lambda_t\left(w_t^{j_w} h_t\left(w_t^{j_w}, a_t\right)-\kappa^L \frac{\left(h_t\left(w_t^{j_w}, a_t\right)\right)^{1+\xi}}{1+\xi}-b_r\right), \\
x^J\left(w_t^{j^w}, a_t,.\right) &= \frac{\partial y^J\left(w_t^{j^w}, a_t,.\right)}{\partial w_t^{j^w}}, \\
x^H\left(w_t^{j^w}, a_t,.\right) &= \frac{\partial y^H\left(w_t^{j^w}, a_t,.\right)}{\partial w_t^{j^w}},
\end{aligned}
$$

and

$$
\begin{aligned}
\varkappa_t^{J,j^w}\left(a_t\right) &= \lambda_t J_t^{j_w}\left(w_{it}^{j^w}, a_t\right), \\
\varkappa_t^{H,j^w}\left(a_t\right) &= \lambda_t H_t^{j_w}\left(w_{it}^{j^w}, a_t\right), \\
\tau_t^{J,j^w}\left(a_t\right) &= \lambda_t D_w J_t^{j_w}\left(w_{it}^{j^w}, a_t\right), \\
\tau_t^{H,j^w}\left(a_t\right) &= \lambda_t D_w H_t^{j_w}\left(w_{it}^{j^w}, a_t\right).
\end{aligned}
$$

and

$$
\varkappa_{x,t}^H=\lambda_t H_{x,t}.
$$

Then we can write

$$
\begin{aligned}
0 &= \varphi \varkappa_t^{H,0}(a_t)\,\tau_t^{J,0}(a_t) + (1-\varphi)\,\varkappa_t^{J,0}(a_t)\,\tau_t^{H,0}(a_t),\\
\varkappa_{j^w,t}^{J}(a_t) &= y^J\left(w_t^{j^w}, a_t, .\right) + \beta\rho \sum_{a_{t+1}\in A} E_t\Lambda_{t,t+1}\vartheta(a_{t+1}, a_t)\\
&\quad \times \left[\alpha_{t+1}^{j^w+1}(a_{t+1})\,\varkappa_{0,t+1}^{J}(a_{t+1}) + \beta E_t\rho\left(1-\alpha_{t+1}^{j^w+1}(a_{t+1})\right)\varkappa_{j+1,t+1}^{J}(a_{t+1})\right],\\
\varkappa_{j^w,t}^{H}(a_t) &= y^H\left(w_t^{j^w}, a_t, .\right) + \beta\rho \sum_{a_{t+1}\in A} E_t\Lambda_{t,t+1}\vartheta(a_{t+1}, a_t)\\
&\quad \times \left[\alpha_{t+1}^{j^w+1}\varkappa_{0,t+1}^{H}(a_{t+1}) + \beta E_t\rho\left(1-\alpha_{t+1}^{j^w+1}(a_{t+1})\right)\varkappa_{j+1,t+1}^{H}(a_{t+1}) - s_{t+1}^{a}\varkappa_{x,t+1}^{H}\right],\\
\varkappa_{J^w-1,t}^{i}(a_t) &= y^i\left(w_t^{J^w-1}, a_t, .\right),\\
\tau_{j^w,t}^{J}(a_t) &= x^J\left(w_t^{j^w}, a_t, .\right) + \beta \sum_{a_{t+1}\in A} E_t\Lambda_{t,t+1}\vartheta(a_{t+1}, a_t)\,\rho\left(1-\alpha_{t+1}^{j^w+1}(a_{t+1})\right)\tau_{j+1,t+1}^{J}(a_{t+1}),\\
\tau_{j^w,t}^{H}(a_t) &= x^H\left(w_t^{j^w}, a_t, .\right) + \beta \sum_{a_{t+1}\in A} E_t\Lambda_{t,t+1}\vartheta(a_{t+1}, a_t)\,\rho\left(1-\alpha_{t+1}^{j^w+1}(a_{t+1})\right)\tau_{j+1,t+1}^{H}(a_{t+1}),\\
\tau_{J^w-1,t}^{i}(a_t) &= x^i\left(w_t^{J^w-1}, a_t, .\right).
\end{aligned}
$$

The optimal policy problem (43) is

$$
\begin{aligned}
U^*(\phi_{t-1}, \varsigma_t) &= \min_{\phi_t}\max_{d_t}\left[u(c_t) - \int_i \kappa^L \frac{(h_{it})^{1+\xi}}{1+\xi}di + U^*(\phi_t, \varsigma_{t+1})\right.\\
&\quad \left. + A_t^c(\phi_t, \phi_{t-1}, d_t) + B_t^c(\phi_t, \phi_{t-1}, d_t)\right],
\end{aligned} \tag{45}
$$

where, using the household Euler equation (16), the constraints (44), (45) from price setting, the resource constraint (40), the price aggregator (2), the flow equation of prices (41) and the first-order condition of the household with respect to c_t and m_t in expression (15) gives

$$\begin{aligned} A_t^c(.) &= \varphi_t\left(\frac{\lambda_t}{R_t}-\beta E_t\frac{\lambda_{t+1}}{1+\pi_{t+1}}\right)+\phi_t^0\left(\omega_0 x\left(p_t^0,.\right)+\beta E_t\chi_{1,t+1}\right) \\ &+\sum_{j=1}^{J-2}\phi_t^j\left(\omega_j x\left(p_t^j,.\right)+\beta E_t\chi_{j+1,t+1}-\chi_{j,t}\right) \\ &+\phi_t^{J-1}\left(\omega_{J-1}x\left(p_t^{J-1},.\right)-\chi_{J-1,t}\right) \\ &+\eta_t\left[\sum_{j^w=0}^{J^w-1}n_{j^w t}\left(a_t h_{j^w t}\right)^{1-\alpha}-\sum_{j=0}^{J-1}\omega_t^j\alpha_t^j\Xi_{j,t}-\sum_{j=0}^{J-1}\omega_j\left(p_t^j\right)^{-\varepsilon}\right. \\ &\left.\times\left(c_t\left(1+s\left(\frac{c_t}{m_t}\right)\right)+m_t^a v_t+\Phi_L n_t-(1-n_t)\,b_r\right)\right] \\ &+\gamma_t\left(1-\left(\sum_{j=0}^{J-1}\omega_j\left(p_t^j\right)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}\right)+\sum_{j=1}^{J-2}\varsigma_t^j\left(\frac{p_{j-1,t-1}}{1+\pi_t}-p_{j,t}\right) \\ &+\vartheta_t\left(u_C\left(c_t\right)-\lambda_t\left(1+s\left(v_t\right)+v_t s'\left(v_t\right)\right)\right) \\ &+\varrho_t\left(v_t^2 s'\left(v_t\right)-\frac{R_t}{1+R_t}\right)+\sum_{j=0}^{J-1}\varrho_t^{\alpha^j}\left(\alpha_t^j-G_P\left(\frac{v_t^0-v_t^j}{p_t^w}\right)\right) \\ &+\sum_{j=1}^{J}\varrho_t^{\omega^j}\left(\omega_t^j-\left(1-\alpha_t^j\right)\omega_{t-1}^{j-1}\right)+\varrho_t^{\omega^0}\left(\omega_t^0-\sum_{j=1}^{J-1}\alpha_t^j\omega_{t-1}^{j-1}\right) \end{aligned}$$

and

$$B_t^c(.)=\sum_{a_t\in A}\left[B_{t1}+B_{t2}+B_{t3}\right], \tag{46}$$

where, using the constraints (31), (28), (29), (33) and rewriting following the lines in Khan, King, and Wolman, 2003,

$$
\begin{aligned}
B_{t1} &= \sum_{j^w=0}^{J^w-1} \psi_t^{S,j^w} \left(\lambda_t \kappa_t v_t - m_t^a \beta \sum_{a_{t+1} \in A} E_t \frac{\omega_t^{j^w}\left(w_{t+1}^{j^w}, a_{t+1}\right)}{\sum_{a_{t+1}\in A} \omega_t^{j_w}\left(w_{t+1}^{j^w}, a_{t+1}\right)} \varkappa_{t+1}^{J,j^w}\left(a_{t+1}\right) \right) \\
&+ \psi_t^0 \left[\varphi \varkappa_t^{H,0}\left(a_t\right) \tau_t^{J,0}\left(a_t\right) + \left(1-\varphi\right) \varkappa_t^{J,0}\left(a_t\right) \tau_t^{H,0}\left(a_t\right) \right] \\
&+ \sum_{j^w=0}^{J^w-1} \psi_t^{J,j^w} \left[x^J\left(w_t^{j^w}, a_t, .\right) - \tau_t^{J,j^w}\left(a_t\right) \right. \\
&\left. + \beta \sum_{a_{t+1}\in A} E_t \vartheta\left(a_{t+1}, a_t\right) \rho \left(1 - \alpha_{t+1}^{j^w+1}\left(w_{t+1}^{j^w}, a_{t+1}\right)\right) \tau_{t+1}^{J,j_w+1}\left(a_{t+1}\right) \right] \\
&+ \sum_{j^w=0}^{J^w-1} \psi_t^{H,j^w} \left[x^H\left(w_t^{j^w}, a_t, .\right) - \tau_t^{H,j^w}\left(a_t\right) \right. \\
&\left. + \beta \sum_{a_{t+1}\in A} E_t \vartheta\left(a_{t+1}, a_t\right) \rho \left(1 - \alpha_{t+1}^{j^w+1}\left(w_{t+1}^{j^w}, a_{t+1}\right)\right) \tau_{t+1}^{H,j_w+1}\left(a_{t+1}\right) - \tau_t^{H,j^w}\left(a_t\right) \right] \\
&+ \psi_t^{J,J^w} x^J\left(w_t^{J^w}, .\right) + \psi_t^{H,J^w} x^H\left(w_t^{J^w}, .\right)
\end{aligned}
$$

and

$$
\begin{aligned}
B_{t2} &= \sum_{j^w=0}^{J^w-1} \Im_t^{J,j^w} \left[y^J\left(w_t^{j^w}, a_t, .\right) + \beta \sum_{a_{t+1}\in A} E_t \vartheta\left(a_{t+1}, a_t\right) \rho \left[\alpha_{t+1}^{j^w+1}\left(w_{t+1}^{j^w}, a_{t+1}\right) \varkappa_{t+1}^{J,0}\left(a_{t+1}\right) \right. \right. \\
&\left. \left. + E_t \left(1 - \alpha_{t+1}^{j^w+1}\left(w_{t+1}^{j^w}, a_{t+1}\right)\right) \varkappa_{t+1}^{J,j_w+1}\left(a_{t+1}\right) \right] - \varkappa_t^{J,j^w}\left(a_t\right) \right] \\
&+ \Im_t^{J,J^w} \left(y^J\left(w_t^{J^w}, a_t, .\right) - \varkappa_t^{J,J^w-1}\left(a_t\right) \right) \\
&+ \sum_{j^w=0}^{J^w-1} \Im_t^{H,j^w} \left[y^H\left(w_t^{j^w}, a_t, .\right) + \beta \sum_{a_{t+1}\in A} E_t \vartheta\left(a_{t+1}, a_t\right) \rho \left[\alpha_{t+1}^{j^w+1}\left(w_{t+1}^{j^w}, a_{t+1}\right) \varkappa_{t+1}^{H,0}\left(a_{t+1}\right) \right. \right. \\
&\left. \left. + E_t \left(1 - \alpha_{t+1}^{j^w+1}\left(w_{t+1}^{j^w}, a_{t+1}\right)\right) \varkappa_{t+1}^{H,j_w+1}\left(a_{t+1}\right) \right] - s_{t+1}^a \varkappa_{x,t+1}^H - \varkappa_t^{H,j^w}\left(a_t\right) \right] \\
&+ \Im_t^{H,J^w} \left(y^W\left(w_t^{J^w}, a_t, .\right) - \varkappa_{J^w-1,t}^H\left(a_t\right) \right),
\end{aligned}
$$

and, using the constraints (34), (35), (19), (21) and the flow equation of wages (42)

$$
\begin{aligned}
B_{t3} \;=\; & \aleph_t^s \left(s_t^a - \frac{m_t^a}{u_t}\right) + \aleph_t^u \left(u_t - 1 + n_t\right) + \aleph_t^x \left(m_t^a - \sigma_m u_t^\sigma v_t^{1-\sigma}\right) \\
& + \aleph_{0,t}^{\Delta n} \left[n_{0,t}\left(a_t\right) - \sum_{j_w=1}^{J_w} \sum_{a_{t-1}\in A} \vartheta\left(a_t, a_{t-1}\right) \rho \alpha_{t-1}^{j_w-1} \left(w_{t-1}^{j^w-1}, a_{t-1}\right) n_{t-1}^{j_w-1}\left(a_{t-1}\right) \right. \\
& \left. - \frac{n_{0,t}\left(a_t\right)}{n_t} m_t^a \right] \\
& + \sum_{j^w=1}^{J^w-1} \aleph_{j^w,t}^{\Delta n} \left[n_{j^w,t}\left(a_t\right) - \sum_{a_{t-1}\in A} \vartheta\left(a_t, a_{t-1}\right) \rho \left(1 - \alpha_{t-1}^{j_w-1}\left(w_{t-1}^{j^w-1}, a_{t-1}\right)\right) n_{t-1}^{j_w-1}\left(a_{t-1}\right) \right. \\
& \left. - \frac{n_{j^w,t}\left(a_t\right)}{n_t} m_t^a \right] + \left(\sum_{j_w=0}^{J_w-3} \aleph_{t+1}^{a,j_w+1} \frac{w_t^{j_w}}{1+\pi_{t+1}} - \sum_{j_w=1}^{J_w-2} \aleph_t^{a,j_w} w_t^{j_w} \right),
\end{aligned}
$$

and in addition the constraints imposed by the adjustment probabilities as described in section A.