Gale-Shapley and the Labor Market

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Abstract

We develop a proposal for a broader and more general microeconomic foundation for the job search theory and for the matching model. For this we apply the matching algorithm developed by Gale and Shapley to the reciprocal preference lists of jobs and workers which are derived from the degree of correlation between worker attribute profiles and job requirement profiles. The proposed procedure is decentralized and leads to results which are stable in the sense of Nash equilibrium. Approaches proposed up to now for establishing the microeconomic foundation of the job search theory and the matching model have proven to be unnecessarily restrictive.

Contents:

1. Introduction

With the contribution presented here the attempt is made to put the microfoundations of both the job search model[1] and the matching model[2] onto a broader and more general basis. A central and trivial aspect here is that not just a uniform good called “labor“ is being traded, but rather that the traded good is heterogeneous and composed of a combination of several traits with their respective values. For the strongly stylized case of two traits the economic idea underlying it can be outlined as follows: “Suppose there are two types of skills in the labor market, for example analytical and manual skills. Both skills are general in the sense that they are productive in many occupations. Occupations combine these two skills in different ways. For example, one occupation might rely heavily on analytical skills, a second more on manual skills, and a third combines the two in equal proportion. Human capital … while working in an occupation is then specific to that occupation to the extent that occupations place different values on combinations of skills” (Gathmann and Schönberg, 2006). Common to all three approaches discussed here is that they are strictly decentralized. No central clearinghouse is needed for the microeconomic-based meeting of jobs and workers; the processes occur through the autonomous behavior of the agents (seekers).

This work develops the combination of two elements, on one side the Gale-Shapley matching algorithm[3] and on the other side a special method for determining the distance between two

1 Cahuc and Zylberberg (2004), Chapter 3 “Job Search”.

2 Cahuc and Zylberberg (2004), Chapter 9.3 “The Matching Model”.

3 Gale and Shapley (1962).

objects with several traits. To the knowledge of the authors, the Gale-Shapley algorithm has only been applied to labor market problems up to now by Kelso and Crawford (1982). Hatfield and Milgrom (2005) characterize their approach as follows: "…consider the labor market model of Alexander Kelso and Vincent Crawford (1982), in which firms bid for workers in simultaneous ascending auctions. The Kelso-Crawford model assumes that workers have preferences over firm-wage pairs and that all wage offers are drawn from a prespecified finite set. If that set includes only one wage, then all that is left for the auction to determine is the match of workers to firms, so the auction is effectively transformed into a matching algorithm. The auction algorithm begins with each firm proposing employment to its most preferred set of workers at the one possible wage. When some workers turn it down, the firm makes offers to other workers to fill its remaining openings". In the approach proposed here, productivity – and derived from this the wage as well – of each combination of worker and job is dependent on their characteristics; there is not just one wage but potentially any number of them. In other words, complete heterogeneity of the labor market is initially assumed. Collective and group- or activity-oriented wage systems can be understood as aggregations of similar pairs.

Central to this paper is on one side the presentation of the model being proposed, and on the other its comparison with two standard models of the microeconomic foundation of labor market theories.

Gale and Shapley have developed a set of "game rules" (i.e., an algorithm) which foresees a sequence of steps through which pairs are matched based on reciprocal preference lists such that the result is Pareto optimal and in Nash equilibrium. This algorithm has found many applications and is easily transferable to labor market issues. These include the matching up of job-seekers and vacant positions, as well as to "fitting" already employed workers and tasks. The algorithm is also known as Stable Marriage Problem. "The Stable Marriage Problem has been widely studied as a model of economical systems, since it mimics agents maximizing their individual utility in a competing environment."4

In the following sections the algorithm's operation will first be explained. Following this, attention will be directed towards the formation of the reciprocal preference lists. A particular procedure will be proposed for properly determining the distance between objects with several traits (profiles). Next, the functionality of the model will be illustrated using a numerical example. In the following chapters it will be investigated how the approach developed here can set the assumptions of the job search model and the matching model onto a broader basis. Finally, a number of possible practical applications of the approach developed here will be discussed.

2. **Function and characteristics of the Gale-Shapley algorithm**[5]

The matching process presupposes that the seekers know of each other and that both sides have established preference lists about each other[6]. Accordingly, all job-seekers each have their own ideas in which order of preference they rank the vacant jobs, and for each of the

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4 Caldarelli, Capocci and Laureti (2006).

5 A clear and straightforward presentation of the algorithm with references to further literature is provided by Caldarelli and Capocci (2001).

6 The approach is concerned with how existing jobs and workers can be brought together in the best possible manner. It is not concerned with the creation or quality of jobs (technology, demand conditions, etc.) nor with the scope and quality of the labor supply (education, labor market participation, etc.).

vacancies there exists an individual order of preference regarding the job-seekers. The job-seekers' own profiles are known only to themselves and to the open positions to be filled, but not to the other job-seekers. Conversely, the requirement profiles of the open positions are known only to the job-seekers and the respective open positions themselves, but not to the other open positions which are competing for the same workers. This severely limits the possibilities for strategic behavior. The basis for the information and the evaluation processes for developing the preference lists are initially immaterial here. That is the subject of the following chapter.

The matching process begins with a random partner on one side (e.g., a vacant job, or "proposer") making an offer to form a pair to the top-ranked partner on its preference list of those on the other side (a job-seeker, or "responder"). The job-seeker accepts this first offer which results in a temporary match[7]. Next, another proposer (another open position) offers the responder top-ranked on its preference list (a job-seeker) to join it as a pair. If the job-seeker here is not already "engaged" in a temporary match, then a second temporary pair is formed. If, however, the job-seeker proposed to had already formed a temporary pair with the first open position, then the respective job-seeker determines which open position is preferred based on his/her rankings. If the first open position is ranked higher then this first temporary pair remains intact. The second open position is hence "rejected" and takes its proposal to the next job-seeker on its preference list. If, however, the second open position is preferred by the job-seeker, then the first temporary pair is dissolved. The job-seeker accepts the second job offer and establishes a new (but still temporary) pair. The first position is "single" again and so approaches the next-ranked job-seeker on its preference list. The matching process cycles through all of the preference lists of all participants and ends as soon as all seekers have been combined into pairs. This is provided that there are an equal number of seekers on both sides and that the preference lists are complete. Only when the matching process is finished do the temporary pairs become permanent ones.

The procedure will be illustrated in Chapter 4 with a concrete numerical example.

If the number of seekers on one side is less than the number on the other side, then the "shorter" side determines the number of possible matches. This results in some positions remaining unfilled or a number of job-seekers without work. A similar result occurs when the preference lists are incomplete. An example of this would be cases where certain minimum requirements of one or both sides about the quality of the pair to be formed are not met.

The Gale-Shapley algorithm exhibits the property that the solution it generates is stable in the sense of Nash equilibrium. This means that a condition has been achieved in which every attempt by an individual participant in the matching process to improve its situation through a change will only lead to a less favorable outcome for itself.

The algorithm itself generates a stable outcome. In general, however, several stable outcomes arise (Gusfield, 1989; Knuth, 1997). The total number of all stable outcomes here can be ranked from the respective views of the proposers and the responders: A stable outcome M1 is preferred to a stable outcome M2 from the view of one side if each member of this side has a partner in M1 which is at least just as preferred as its partner in M2. The preference ranking of the proposers here regarding stable outcomes runs directly counter to the preference ranking of the responders: If M1 is preferred to M2 from the jobs' perspective, then M2 is preferred from the view of the job-seekers. Furthermore, there is a most preferred

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7 The temporary nature of the matches entered into differentiates this approach from others, such as that of Gode and Sunder (1993) which foresees binding contracts being entered into.

stable outcome from each side's point of view which is furthermore explicitly determined. The Gale-Shapley algorithm has the further interesting property of each time producing the best of all stable outcomes for the proposing side (and hence the worst of all stable outcomes for the other side). The outcome of the Gale-Shapley algorithm is hence dependent on which side begins with proposing, with the proposing side better able to assert its priorities through this process than the responding side. Corresponding results will arise in a labor market situation characterized by a surplus of workers where it will more likely be the vacant jobs which act as proposers; in times of labor scarcity, on the other hand, the proposers will likely be the job-seekers.

The mechanism is fundamentally decentralized. With low numbers of cases and in a tractable time-frame it can arrive at a stable result without need for a centralized entity to collect and process the preference lists and announce the results. A "Walrasian auctioneer" is hence not required. The participants do not have to be more intelligent than being capable of setting up the described preference rankings and adhering to the rules laid out (Gode and Sunder, 1993). In practice up to now, however, it has mainly been applied in situations with larger numbers of cases and a clearinghouse which sends out notifications (Roth, 2007).

3. Profiles as the basis for developing preference lists

The prevailing literature applies the Gale-Shapley algorithm to already existing preference lists. For the application intended here in the labor market, however, it is important to investigate precisely how the preference lists come about. We propose deriving preference lists from the distance between requirement and attribute profiles using a special distance measure, and in so doing put them on a microeconomic foundation.

A job is characterized by a requirement profile, and a worker by an attribute profile. Both profiles consist of a limited number of characteristics (requirements and attributes) which apply to all vacant positions (jobs) and job-seekers (workers). An individual job and individual worker is distinguished by the values indicating the manifestations of his/her attributes.

Examples of the components of the respective profiles include:

1. The labor market segment defined, for example, in terms of the learned vocation and occupation (Tiemann et al., 2008). Involved in this case is affiliation with an expression of a categorical trait rather than a profile.
2. Intelligence and integrity (profile with two traits: Schmidt and Hunter, 2000)
3. Activity and qualification (profile with two traits: Schlicht, 2007)
4. Skill characteristics of the job (profile with four traits: Ingram and Neumann, 2006):
    - Intelligence
    - Fine motor skills
    - Coordination
    - (Physical) strength
5. Activity fields (profile with six traits: Spitz, 2003):
    - Analytical tasks, such as research, planning, or evaluation activities
    - Interactive tasks, such as coordination and delegation of work
    - Repetitive cognitive tasks, such as accounting and costing

- Repetitive manual tasks, such as feeding parts into a machine
- Non-repetitive manual tasks, such as household management or repair work
- Activities on the computer

6. Interdisciplinary qualification demands (profile with 10 traits: Dickerson and Green, 2004):
   - Literacy
   - Physical skills
   - Numeracy
   - Technical know-how
   - High-level communication skills
   - Planning skills
   - Client communication skills
   - Horizontal communication skills
   - Problem-solving
   - Checking skills
7. Work conditions[8] (Noll and Weick, 2003):
   - A job which allows for ample leisure time
   - A secure job
   - A high-paying job
   - A job which allows for autonomy
   - Colleagues with whom one can work well
   - A job which offers good potential for advancement
   - A job where one can apply ones own initiative
   - A job with flexible working hours
   - A job where one can apply ones skills
   - Convenient working hours
   - Good opportunities for further education and training
   - A job which allows for partial or full-time work at home
   - A job with great task variation
   - A job with little stress.

The profiles can not be “unbundled”[9] for a short observation time-frame (Heckman and Scheinkman, 1987). A worker is only to be regarded as a whole person with all of his/her

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8 Response options in the “Eurobarometer” to the following question: For each of the following points please tell me how important they are to you personally in choosing a job (very important, important, neither/nor, unimportant, completely unimportant, don’t know).

characteristics, and a vacant position is first defined in terms of the inner-company division of labor. In the medium term dynamic aspects are brought to bear here such as job orientation, learning on the job, the learning curve, continuous training, initial uncertainty about job performance, reallocation of the inner-company workload, and other factors.

It is useful when taking these considerations further to imagine that the job requirements and the attributes of the workers are counterparts to each other, i.e., a requirement corresponds to an attribute and vice versa such that the traits can be measured in like manner (e.g., on an ordinal scale from 1 to 5, or on a cardinal scale from 1 to 100). Involved here is a simplified assumption in the face of the problems which arise from a concrete determination and quantification process[10]. Under these conditions the aggregated distance across the profile traits, or the similarity of job and worker, is the simple sum of the (ordinal or cardinal) distances between the traits.

If the conditions referred to are fulfilled then both profiles (that of the job and that of the job-seeker) are directly comparable with each other. Resulting from the comparison is the statement per trait whether

(1) the requirement of the job and the attribute of the worker as currently expressed are in alignment,

(2) the requirement exceeds the attribute (current ability), or

(3) the attribute (ability) goes beyond the requirement.

Matching can occur without there having been access to information about the profiles, for example through a random connection. Such matching mechanisms are frugal in their need for information and in the costs of data collection and preparation. Matching processes which use profile information, on the other hand, are complex and “expensive”.

If profile information is used then it stands to reason that similarity measures should be used through which the “proximity” of two objects to each other is calculated. In the case here, jobs and job-seekers are coming together which are as similar in nature as possible. Many similarity measures orient themselves around the difference between the values of a trait among two objects and sum up the differences[11]. For contextual reasons another similarity measure will be used here.

The productivity of a job-worker pair is determined by the respective lower value per element. This can be seen under case (2) above where the requirements are met only to the extent that the worker fulfills them, and under case (3) where the worker’s abilities which go beyond the requirements of the job are initially not being exploited (i.e., the person is overqualified). The “value” of a job-worker pair is calculated by adding up the minima of the element pairs. From the viewpoint of the individual job the most similar job-seeker is the one with whom the highest sum of minima is yielded (and vice versa). In the given configuration this job-seeker will take top place on that job’s preference list. Whether this also holds true for the job on the

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9 Schlicht (2007) approaches the problem by postulating two types of competition (quantity and quality competition) in accordance with the two attributes, which alternatively are brought to bear dependent on general labor market conditions.

10 The personnel and organizational consulting firm Hay Group uses this approach for the evaluation of remuneration structures. See O’Shaughnessy et al. (2001).

11 See Chapter 3.3 “Auswahl eines Unähnlichkeits- oder Ähnlichkeitsmaßes” in Bacher (2002).

respective job-seeker's list, or whether from his/her view one or more jobs are available which are more attractive (with higher minima sums), is at first uncertain. Whether this specific job is able to form a stable pair with this particular job-seeker as its partner of choice is established only at the end of the Gale-Shapley matching process.

In addition to the sum of the minima, the definition used here includes further indicators for the quality of the matched pair: Starting from the requirement profile, "undercoverage" (i.e., "underqualification") indicates the differences among the traits by which the job requirement exceeds the attributes of the job-seeker; and "overqualification" based on the attribute profile of the worker. The sum of undercoverage and overqualification corresponds to the sum of the differences between the trait values of both objects as applied in standard distance measures.

The greater undercoverage and overqualification turn out to be, the further worker abilities and job requirements are from each other, and in turn the greater is the "dissatisfaction" on both sides with the situation and the lower the pair's "bond strength". Although the overall situation for all matched pairs (i.e., with currently available potential partners) is stable, such weakly bonded pairs can arise. They will return more quickly to the labor market to seek out more appropriate partners than will strongly bonded partners with low values for undercoverage and overqualification.

The model provides starting points for modeling search intensities using these indicators about existing connections. On both sides the amount of reasonably expected search effort depends on the expected "difference" between the current and a potentially better search result. If this difference is small then the effort for further searching generally is not worthwhile as soon as a "satisfactory" solution is found. This aspect is partially addressed with the reservation wage of the job search model (see Chapter 5 below) and could be complemented with "minimal job requirements" on the parts of businesses.

The development potential of a job-worker pair is determined by the differences which exist between requirements and attributes. If an improvement of the "weaker" part of the element pair can be achieved through appropriate measures, then the total "value" of the pair as a whole increases. In any event it can be observed in the short-term perspective that only an improvement of the respectively weaker side of the pair results in an improvement in productivity. The further increase in a requirement which is already not being met leads nowhere, as does the further improvement of an ability which is not being fully utilized.

Determining the distance of objects with several traits gives rise to a variety of problems: Which traits are relevant in the current context, what dimensions are used for measuring them, how is the distance per trait measured, and how are the trait-related distances aggregated to a total distance? Distance determination here serves the formulation of preference lists which in turn make up the starting point for the matching process. A closer examination of the interaction between the type of distance measure and the matching process leads to a number of interesting results which are presented in the following.

With a given number of participants on each side (in the following there are seven workers and jobs per side, and in Chapter 4 there will be five in each case), there is – independent of the preferences – a multitude of situations each with differing combinations of possible pairs (n! possible situations). Each of the possible situations possesses a certain "macroeconomic" value which under the set assumptions (cardinal measurement of the trait values, and all traits being of the same dimension) equals the sum of the values (productivities) of the individual pairs[12]. In the following it will be investigated

- whether only one stable outcome exists, or several;
- how the stable outcome is ranked within the distribution of all possible situations; and
- how the choice of the distance measure influences the form of the distribution and the condition of the stable outcome.

If the preference lists for the Gale-Shapley process are derived from cardinally or ordinally measured distances between requirement and attribute profiles then only one unique stable outcome arises. This means that in these cases the outcome arrived at from the algorithm is the optimal one from among the stable outcomes in the views of both the proposing and responding sides. In these cases it makes no difference which side begins with proposing. This result traces back to the fact that the derivation of the preferences from a distance measure leads to single-peaked preference lists which allow only a single stable outcome in the Gale-Shapley algorithm (Bartholdi and Trick, 1986). The derivation of the preferences from distance measures generally leads to there being only one stable outcome.

Although there is always only one single stable outcome with the proposed procedure, the question of *which* pair assignment will prove to be stable is dependent on the distance measure applied with the given attribute and requirement profiles (cf. Figures 1 and 2). If the sum of the arithmetic means of the trait values is used instead of the sum of the minima then all possible results of the matching process summed for the pairs will indicate the same value. The same holds for the simple sum of the trait values. The distribution of the possible situations collapses to a single value. In other words, all random combinations of jobs and workers are of the same value in overall economic terms. This is an implausible result in this context.

If one observes the sum of the maxima instead of the sum of the minima, then the stable outcome of the distribution of all possible pair assignments tends towards being closer to the arithmetic mean than as with the minimum measure. It also makes a difference for the stable outcome whether the simple deviations or the quadratic deviations are used for measuring distance.

The measure used must hence be contextually supported for the corresponding applications. The minimum measure appears to be a sensible distance function for applications in the labor market for the above-mentioned reasons.

Figure 1: Identification of the stable outcome using different distance measures

12 With ordinally measured traits (i.e., all traits are scaled identically so that a simple aggregation in terms of the traits per pair is possible), the overall situation can be measured as overall satisfaction in that the “bond strength” of the individual pairs is summed. The bond strength of a pair in this case is the sum of the rankings which the partners of the formed pairs gave each other before matching was completed: If the pair consists of two partners who in their respective preference lists gave each other high rankings, then the bond is strong (the sum of the rankings is small). If, on the other hand, the result of the matching process turns out such that, for example, a job pairs up with a worker who was ranked far down on its preference list, then the bond is weak (the sum of the rankings is high), and the satisfaction of the pair elements with each other is low.

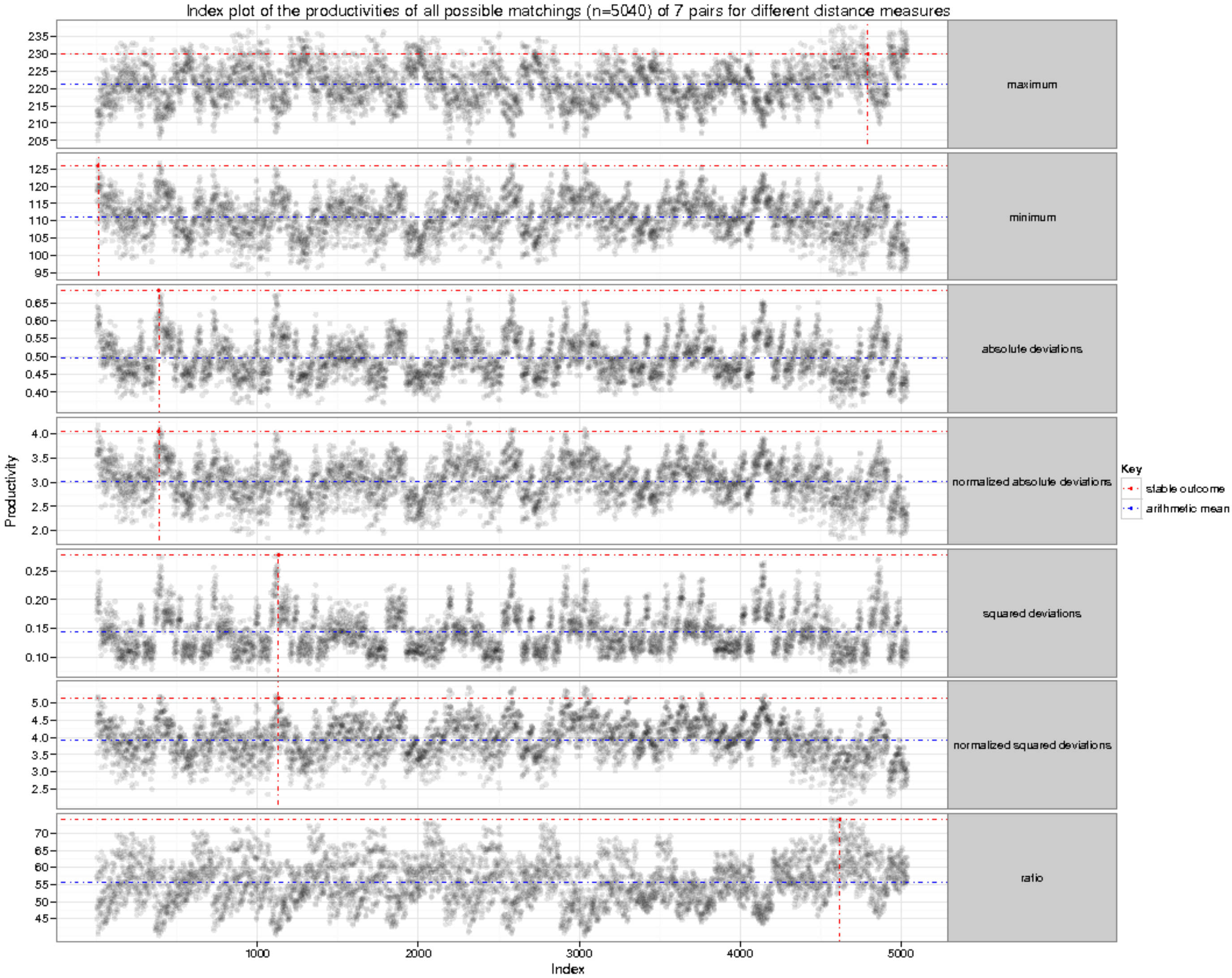
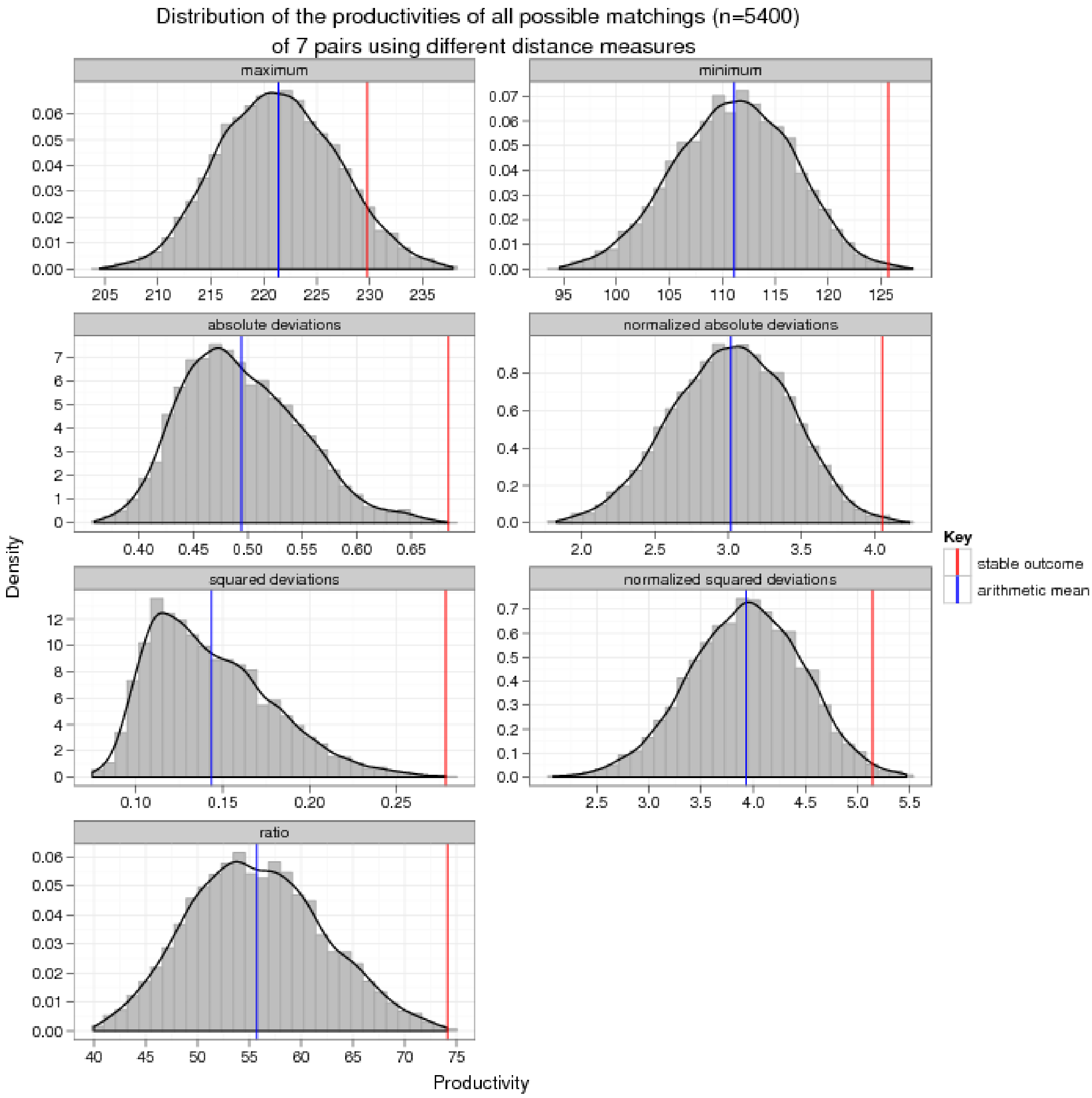


Figure 2: Form of the distribution of all pair assignments using different distance measures and the condition of the stable outcome within it



The wage (work remuneration) can be an element of the requirement and attribute profile. It is smaller than productivity which is determined by the characteristics of the matched pair, and is the subject of either collective or individually bargained agreements. Due to institutional circumstances groups of pairs with differing productivities can be paid similar wages while wages can vary for pairs with similar productivities.

The extent of under- and overqualification defined above can play a role in wage determination. Job-worker pairs with equally high minima sums are differentiated during the determination of the similarity of the profiles, which is the basis of the matching process. Here it is established whether the attribute profile overall has proven to be (somewhat) stronger or (somewhat) weaker than the requirement profile turned out to be. A “stronger” attribute profile defined in this way leads to a wage premium, while a “weaker” one, in contrast, leads to a wage reduction (Daly, Büchel & Duncan, 2000; Büchel, 2002).

4. A numerical example for clarification of the proposed approach

In the following the entire procedure for priority setting and the matching process in the method proposed here will be described with a simple numerical example. The starting point of the numerical example is a set of data regarding five workers and five jobs. These each have four traits with differing values (cf. Table 1).

Table 1: Jobs and workers with their traits

	Trait				
Jobs	**1**	**2**	**3**	**4**	**Sum**
1	0.0	1.8	3.5	10.0	15.3
2	2.1	5.2	6.2	9.8	23.2
3	6.3	8.7	4.3	1.3	20.6
4	1.6	5.3	4.6	9.7	21.2
5	2.8	6.0	3.2	9.7	21.7
Workers	**1**	**2**	**3**	**4**	**Sum**
1	5.9	9.0	3.6	1.7	20.3
2	2.7	6.2	4.7	9.4	23.1
3	5.8	9.3	4.8	3.3	23.1
4	1.3	5.0	4.9	9.3	20.5
5	2.4	4.8	4.6	9.5	21.3

With this material the jobs' and workers' preference lists are formulated and the matching process run through. The "macroeconomic" result is established and the amount of information needed for the matching process indicated.

The first step is to establish the reciprocal preference lists. Every job evaluates each job-seeker by comparing its requirement profile with the workers' attribute profiles, and each job-seeker evaluates the jobs by comparing his/her attribute profile with the jobs' requirement profiles. The "values" of the individual pairs which arise from the evaluation process are calculated according to the rule described above: as the sum of the minima of the values of the respective trait. Table 2 shows the calculation operation in detail for the pair "Job 1" and "Worker 1". The economic value of this pair amounts to 7.0. In addition it is shown with which traits and to what extent the worker – measured in terms of the job's requirements – is under- or overqualified.

Table 2: Calculation of the value (of productivity) of the pair "Job 1" and "Worker 1" according to the minimum principle

	Trait No.				Sum
	1	2	3	4	
Job No. 1	0.0	1.8	3.5	10.0	15.3
Worker No. 1	5.9	9.0	3.6	1.7	20.3
Minimum	0.0	1.8	3.5	1.7	**7.0**
Underqualification				8.3	8.3
Overqualification	5.9	7.2	0.1		13.3

A value is arrived at for each possible combination of job and worker (see Appendix). These values are in turn the basis for the reciprocal preference lists which contain the information about the rankings the respective job has given the applicants, and conversely, that used by each applicant to evaluate the jobs. The preference lists for the numerical example are shown in Table 3.

Table 3: Preference lists of the jobs and workers

Ranking	Job 1	Job 2	Job 3	Job 4	Job 5
	Worker Rankings				
1	5	2	3	2	2
2	2	5	1	5	5
3	4	4	2	4	4
4	3	3	5	3	3
5	1	1	4	1	1

Ranking	Worker 1	Worker 2	Worker 3	Worker 4	Worker 5
	Job Rankings				
1	3	2	3	2	2
2	5	5	2	4	4
3	2	4	5	5	5
4	4	1	4	1	1
5	1	3	1	3	3

The actual matching process according to Gale-Shapley begins on the basis of the preference lists developed with the minimum process. The sequence of steps is shown in Overview 1 below. Job 1 proposes becoming a pair to Worker 5 in line with its preference list. This results in a temporary pair since it is not yet known whether Worker 5, for whom Job 1 is ranked in fourth (next to last) place in his/her preference list, may eventually receive a more favorable offer. Meanwhile, Job 2 proposes pairing to Worker 2, and Job 3 to Worker 3, in line with these jobs' first choices on their preference lists with temporary pairs ensuing. In the next step, Job 4 proposes to its most preferred candidate, Worker 2. This worker, however, is temporarily "engaged" to Job 2. Worker 2 checks whether he/she should accept the new proposal or remain in the current engagement. The current pairing checks out positive since Job 2 is in the top position on Worker 2's preference list (whereas Job 4 is in third place). After this "rejection" Job 4 now turns to the second-ranked worker on its list, which is Worker 5, and then forms a temporary pair with him/her. This continues on through 12 steps until all candidates have been assigned into pairs. The result is stable since there is no other possible pairing for any of the participants with a partner on the other side which would be more beneficial for both partners.

Overview 1: Sequence of steps in the matching process according to Gale-Shapley for the numerical example

Step	Proposing Job	Responding Worker	Temp. Pair	Worker Moves To Job No.	Final Match
1	1	5	1 & 5	4	
2	2	2	2 & 2		2 & 2
3	3	3	3 & 3		3 & 3
4	4	2			
5	4	5	4 & 5		4 & 5
6	1	2			
7	1	4	1 & 4	5	
8	5	2	-		
9	5	5	-		
10	5	4	5 & 4		5 & 4
11	1	3	-		
12	1	1	1 & 1		1 & 1

The configuration of results with full details (i.e., profiles and respective minima) is presented in Table 4.

Table 4: Matching process result configuration

	Trait No.				Sum
	1	2	3	4	
First Pair					
Job No. 2	2.1	5.2	6.2	9.8	23.2
Worker No. 2	2.7	6.2	4.7	9.4	23.1
Minimum	2.1	5.2	4.7	9.4	**21.4**
Second Pair					
Job No. 3	6.3	8.7	4.3	1.3	20.6
Worker No. 3	5.8	9.3	4.8	3.3	23.1
Minimum	5.8	8.7	4.3	1.3	**20.1**
Third Pair					
Job No. 4	1.6	5.3	4.6	9.7	21.2
Worker No. 5	2.4	4.8	4.6	9.5	21.3
Minimum	1.6	4.8	4.6	9.5	**20.6**
Fourth Pair					
Job No. 5	2.8	6.0	3.2	9.7	21.7
Worker No. 4	1.3	5.0	4.9	9.3	20.5
Minimum	1.3	5.0	3.2	9.3	**18.8**
Fifth Pair					
Job No. 1	0.0	1.8	3.5	10.0	15.3
Worker No. 1	5.9	9.0	3.6	1.7	20.3
Minimum	0.0	1.8	3.5	1.7	**7.0**
Sum of Minima					**87.8**

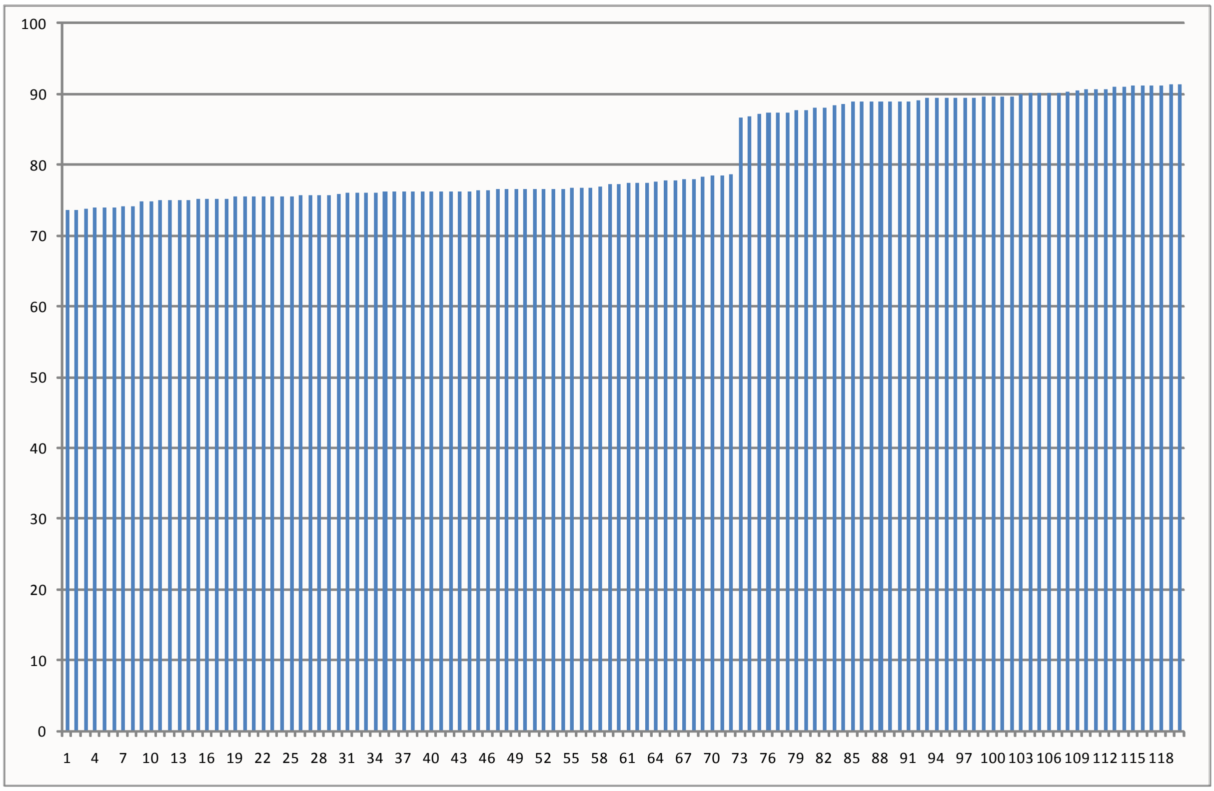
Because the profiles consist of cardinally measured traits both the value per pair (the pair's productivity) as well as the sum of all pairs can be presented. The summed value for the pairs presents the "macroeconomic" result on the basis of the formulated preferences and matching process.

There are a further 119 combinations in addition to the one presented above, yielding a total of 120 possible different combinations with five pairs each (respectively one job and one worker). Figure 1 shows the "economic" results of the combinations possible from the numerical example. The arithmetic mean of the possible combinations amounts to 81.5, the minimum comes to 73.7, and the maximum is 91.4. For the "economy" comprising these five workers and jobs, the maximum would be the most beneficial outcome. However, with the described decentralized procedure the stable outcome has a value of 87.8. This is markedly less favorable than the maximum.

The major gap between the stable and the macroeconomically maximal outcome suggests that an attempt should be made to seek possibilities for redesigning the process such that the stable outcome comes closer to the maximum outcome. Here, for example, thought is given to introducing compensation options among the pairs, or to altering the form that the available information takes. As the loss of information can be reversed which accompanies the translation of the sum of the minima into priorities and the associated loss in dimension from cardinal to ordinal scaling, potential is created for macroeconomically better and – with the inclusion of compensation elements – at the same time stable configurations. Consideration can also be given to the design of an internal company labor market in terms of how to formulate information about vacant positions and their requirement profiles, and correspondingly about change-seeking workers and their attribute profiles, so that from the company's perspective the maximal result occurs. In the context presented here this would actually mean reducing the available information. Both changes to the design would (presumably) make sense only from a centralized perspective, and not from the decentralized perspective pursued here.

From the decentralized view preferred and pursued here it is clear that the matching method according to Gale-Shapley quickly reaches the "natural" limits set by the number of participants. These limits result from the time entailed until all parties have formulated their preference lists, and which is required until the "courtship" among the created and dissolved pairs comes to a final conclusion. For the macroeconomic picture, which could comprise many hundreds of thousands or even millions of job-seekers and open positions, more conceivable would be many individual micro-markets which are operating concurrently and parallel to each other. If one incorporates the total of all existing pairs in the labor market (i.e., filled jobs and employed workers) into the picture then one could imagine a wide range of rather "quiet" areas in which no search or matching activities are occurring during the observed time-frame, as well as individual "hot" areas with many parallel activities.

Figure 1: The economic values of all possible combinations of five pairs each



It would be of interest to examine from the decentralized view what kind of change in one of the participating profiles in the micro-market would be sufficient for (a) affecting a change in the preference lists, and (b) a change in the stable outcome. In other words, an investigation would address what kind of change in the initial conditions leads to a changed outcome, and what role the re-sorting operations prompted by changed preference lists would have in this.

These lines of questioning will not be pursued further here. Instead, the investigation in the following will address the results from applying other distance measures and from applying other matching processes. In this way the peculiarities and advantages of the approach proposed here will be further elaborated.

5. Comparison with the job search theory

The theory deals with the job search of those seeking work[13] and represents the core of the currently dominant labor market model, that of Mortensen and Pissarides. As with the approach proposed here, the theory assumes imperfect information on both the parts of those seeking work as well as on the parts of companies. This differentiates it from the classical theory calling for perfect information. Job-seekers are not aware of the entire spectrum of open positions and their respective wages, and know only their own characteristics and the general labor market situation. On the basis of this information a job-seeker decides on a minimum, or reservation wage. At irregular intervals he/she receives job offers. The job offers (open positions) provide a fixed income which is independent of the person filling them. The companies are not aware of the whole spectrum of job-seekers and their minimum wage requirements. The optimal strategy for the job-seekers is in taking the first job they are offered whose pay is higher than their minimum acceptable wage. A job once taken remains in existence until the position is eliminated (for whatever reasons, e.g., price-related competitiveness issues or technical change). The companies are passive in filling positions as they accept the first willing applicants. It is assumed that a “willing” applicant is capable of completely fulfilling the requirements of the position. Waiting by companies for a possibly better fitting job-seeker is not foreseen.

Each element of each side (job-seekers and jobs) possesses only one trait. The trait of the job-seekers is the reservation wage, and the trait of the jobs is the offered wage. The meeting of job-seekers and open positions takes place “in the dark” as no one knows the number or characteristics of the other participants.

The procedure developed in this paper exhibits the following differences to this:

- Both the job-seekers and the companies (open positions) know about all elements of the “opposing side”.
- The elements of both sides have multiple traits.
- The values of the traits determine the preference lists.
- Selection behavior is governed by the ordinal preference list, and not by the categorical assignment to the class of the allowable partner.
- There is no lower limit for the measure of alignment between the two profiles. Unlike in the job search model, in the first step a job-seeker accepts every job offered. Only when a more favorable job is offered can he/she switch to the better option[14].

In the following the procedure of the job search model will be replicated with the values of the numerical example from Chapter 4. For simplifying the comparison of the processes, it is assumed here that the traits “reservation wage” and “offered wage” have a fixed relationship to the profiles. They should be the simple sum of the trait values. Matching correspondingly

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13 The presentation is based on Cahuc and Zylberberg (2004), Chapter 3 “Job Search”.

14 This aspect can be taken into consideration in the model proposed here in that lower limits are introduced for the measure of alignment between the two profiles (the sum of the minima). Both sides can determine the lower limits.

orients itself around two traits which provide some indication about the form of the profiles but with the details concealed. The result is shown in the configuration discussed below.

Table 5: Restrictions on the matching process with lower limits (reservation wages) on the parts of job-seekers

Job No.	**Offered Wage**	**Worker No.**	**Reser-vation Wage**	Job No.	Worker No.				
					1	**2**	**3**	**4**	**5**
1	15.3	1	20.3	1					
2	23.2	2	23.1	2	X	X	X	X	X
3	20.6	3	23.1	3	X			X	
4	21.2	4	20.5	4	X			X	
5	21.7	5	21.3	5	X			X	X

Job 1 remains unfilled because the wage offered there is below the reservation wage of all the job-seekers. Which worker connects with which job, and which worker remains without a job first depends on which options the workers have at their disposal (jobs with a wage > reservation wage).

The available options are marked with an “X” in the right-hand side of Table 5 above. Which of the available options is realized is random and dependent on the order in which jobs and workers encounter each other. In the numerical example 96 sequences of encounters (drawing without discarding) produces four pairs, while all other sequences end with three or two pairs. The reason for this is the fact that for Jobs 3 and 4 only two workers are available for a potential pairing, namely Workers 1 and 4. If these workers have already encountered the other two vacancies, Jobs 2 and 5, then Jobs 3 and 4 will go unfilled.

The procedure of the job search model can gradually be converged towards the procedure proposed in this paper. The matched pairs change thereby, as does the total result in the aggregate for the matched pairs. The encounter situation between jobs and applicants is “better illuminated” to the extent that the participants can recognize the presence of the others and their traits (offered wage and reservation wage). The participants are hence put in the position of being able to formulate their respective preference lists, which are then used to operate the matching process as per Gale-Shapley.

The result is the following set of preference lists. As before, Job 1 finds no job-seeker who is willing to work for the wage it is offering. The jobs formulate their rankings in line with the guideline that the job-seekers are more desirable the higher their reservation wage is (the reservation wage signals a certain competence level, and is less than the offered wage which in any event will be paid). The workers formulate their rankings according to the guideline that the higher the wage, the more attractive the job.

Overview 2: Preference lists in the “illuminated” job search model

Ranking	Job No.				
	1	**2**	**3**	**4**	**5**
	Worker Rankings				
1	n.a.	2	4	4	5
2	n.a.	3	1	1	4
3	n.a.	5			1
4	n.a.	4			
5	n.a.	1			

Ranking	Worker No.				
	1	**2**	**3**	**4**	**5**
	Job Rankings				
1	2	2	2	2	2
2	5			5	5
3	4			4	
4	3			3	

The matching process occurs on the basis of the rankings according to Gale-Shapley, and a stable outcome results (Overview 3).

After the pairs have been formed there is sufficient information available for determining the value of the pairs according to the minimum principle. This amounts to 81.0 for the stable outcome. This value is not directly comparable with the outcome encompassing the five pairs in Chapter 4.

If, however, one uses the minimum process to calculate the “inner” values of all 96 possible configurations with four pairs which can occur “in the dark”, and which are all equally possible, there then results for them an expected value = mean value across the 96 configurations of 70.3. The difference from the value of the stable outcome (81.0) indicates the value of the additional information made available by “illuminating” the scenery.

Overview 3: Sequence of steps in the matching process according to Gale-Shapley for the illuminated Job Search Model

Step	Proposing Job	Responding Worker	Temp. Pair (Job & Worker)	Worker Moves To Job No.	Final Pair
1	2	2	2 & 2		2 & 2
2	3	4	3 & 4	4	
3	4	4	4 & 4		4 & 4
4	3	1	3 & 1		3 & 1
5	5	5	5 & 5		5 & 5

The job search model with its assumptions about the amount of available information and its use is apparently too restrictive. The assumption of a random encounter between job and worker can be abandoned.

6. Comparison with the matching model (the urn and ball model)[15]

The urn and ball model is an attempt to establish the microeconomic foundation of the so-called matching model. Cahuc and Zylberberg characterize the matching model as follows: “ …a simple model of the labor market in which transaction costs explain the simultaneous existence of vacant jobs and unemployed persons. Wage formation is here described by a bargaining process between employers and workers; in other words, the hypothesis of

15 14 The presentation is based on Cahuc and Zylberberg (2004), Chapter 9.3 “The Matching Model”. For Germany see Fahr and Sunde (2001), Sunde (2002), and Falk, Huffman and Sunde (2006).

competitive wages is dropped. The model is structured around the concept of a matching function, which sums up, at the aggregate level, the outcomes of encounters between persons in search of a job and firms with positions vacant".

They go on to describe the use of the urn and ball model for establishing a microeconomic foundation of the matching model as follows: "A simple but not truly realistic way of obtaining an aggregate matching function consists of comparing vacant jobs to "urns" and job applications to "balls" tossed at the urns by job-seekers (Pissarides, 1979; Blanchard and Diamond, 1994). A match occurs when a ball goes into an urn. The inefficiency of the job search process is reflected in the greater or lesser precision with which the balls are tossed in the direction of the urns. … D and V will denote respectively the number of job-seekers and the number of vacant jobs at a given date. Let us assume that job-seekers know the locations of all vacant jobs, and that a particular job-seeker, whom we shall call Mr. i, simultaneously sends e_i applications out randomly among the V jobs vacant. Parameter e_i is an indicator of the effort that Mr. i puts into his job search. When more than one application is received for the same vacant job, a random draw determines who will get it, and the other applications go into the wastepaper basket. Let us further suppose that there is no coordination among the job-seekers. That being so, it is possible that one vacant job will receive a heap of applications while another will not receive any".

The model is quite similar to the one proposed here, as well as to the job search model covered above. Initially obvious, as with the job search model, is the general passivity and "blindness" of the businesses.

In the matching model the vacant jobs do not know the job-seekers. In comparison to the job search model in which both sides do not know of each other, one can characterize the matching model as "half-illuminated scenery" in that at least one market side is aware of objects on the other side of the market. In the model proposed here the job-seeker knows about the vacant jobs, and the vacant jobs know about the job-seekers.

To this extent both the "application" of the job-seeker as well as the "announcement" of an open job occur at a point in time before the process proposed here and are not its subject. The matching model, though, begins with a "getting to know you" (i.e., application) process. In the approach proposed here, the open positions have information about all the job-seekers and know that they are out looking and will react positively to offers (and vice versa).

The job search model and matching model can be further harmonized. This is done by assuming that in the matching model the job-seeker will only submit applications for those jobs which meet the conditions of the job search model, namely that the wage offered is above the respective reservation wage.

The difference between the two models still remains that in the job search model the first job-seeker applying will be accepted, while in the matching model the company will make a random choice from among the applicants. For the overall result, meaning the quality of the matched pairs, the same result occurs as shown above for the job search model. It is random and depends on either the sequence of the encounters (job search model) or on the sequence in which the vacant jobs arrive at their random choices. Every choice made reduces the options of the other vacant jobs.

7. Practical applications

7.1 Understanding skilled labor scarcity

Complaints about the scarcity of skilled labor constantly plague those responsible for education and labor market policy. While it was a lack of IT specialists which was bemoaned back in the 1990s, today it is engineers and natural scientists who are missing. Beginning in around 2015, hence a good five years from now, it can be expected that skilled labor scarcity will be a more widespread phenomenon and not just occur occasionally and in narrowly defined labor markets. The reason for this is that beginning at that time demographic trends will bring about both an ageing and in absolute terms decreasing amount of labor market potential. Even though in 2015 there will still be another 20 years or so until pervasive scarcity will set in with all labor market reserves having been exhausted in numerical terms (unemployment, hidden reserve, labor participation), scarcities in individual labor sub-markets will still occur more frequently and be more serious than today.

Despite the familiarity with the phenomenon of skilled labor scarcity there still remains a lack of appropriate conceptual and analytical instruments with which the phenomenon can be presented, its impacts qualitatively described and quantified, and strategies for combating it developed. Up to now we were not able to precisely describe which adaptation strategies businesses and wage-earners can use to react to bottlenecks, and we do not know how quickly and comprehensively "substitute solutions" can be developed and implemented. We also can not accurately quantify how great the "damage" is which is caused by the scarcity of skilled labor when we seek to account for the substitution and adaptation processes taking place in response. On one hand these processes serve to mitigate the damage, but on the other they are associated with costs which otherwise would not have been incurred. And what role do prices play (here: wages), which according to economic theory should bring about a balance between supply and demand? Scarcity-induced changes in relative prices should certainly not be considered to be "damage".

The impacts of skilled labor scarcity are determined by the extent to which the job requirements among the matched pairs are not being satisfied. They are measured by the "undercoverage" measure defined above. If the values of the attributes are measured cardinally, then with the same calculation rule the difference between actual productivity (the sum of the minima of the existing attribute values) and potential productivity (the sum of job requirements) can be determined. It then corresponds to the "damage" attributable to the scarcity of skilled labor.

7.2 Better organized job placement

Better placement results are macroeconomically growth-inducing because they bring about a better combination of people and jobs, even in cases where a job can first only be filled after it has been proven that an appropriate worker exists for it. A better combination of people and jobs increases value-added and yields more income.

Better placement is expressed in the framework of this model primarily in that the costs incurred by the reciprocal collection of information are reduced. This tends to increase the area across which a search can be conducted and across which the matching process takes place. This in turn tends to result in more and better-quality connections. On the other hand, the amount of reasonable search effort on both sides is dependent on the "difference" between a just barely satisfactory and an optimal search result. If this difference is small, then it is generally not worthwhile to continue searching once a "satisfactory" solution has been found. In this way the model can theoretically capture the importance and quantitative effect of improved placement. The impact of continuing education and qualification of labor is expressed in the framework of the model in that the area in which both profiles are aligned becomes larger.

7.3 Understanding the advantage of diversity

Just because the results of the matching process are stable does not mean that all participants are satisfied with the result. The sum of the rankings which both sides of the ultimately formed pairs have given each other in advance of the matching process can be used as a measure of satisfaction. In the ideal case two partners come together who have ranked each other as their respective top choices; in the least favorable case the pair consists of two partners who have both ranked each other lowest. Pairs with high dissatisfaction levels will possibly dissolve during the next round of matching and renew their search for a higher-valued partner. Caldarelli and Capocci (2001) show that with increasing ranking uniformity (i.e., all companies have put the applicants in largely similar rank orders, and all applicants have largely the same ranking for the positions) average satisfaction diminishes. Conversely, average satisfaction increases with increasing heterogeneity of rankings.

7.4 Understanding the tendency towards equilibrium

An ongoing matching process may possibly not achieve a stable condition because actors enter and exit the scene, or because the profiles of the actors change. This leads to the question regarding macroeconomic equilibrium in the labor market. It is distinguished by many simultaneously occurring movements: New jobs are created, each with its own demand profile, while existing jobs cease to exist; people graduate from the educational system to become part of labor market potential and contribute their attribute profiles, while others retire from the labor market pool due to age and other reasons.

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Appendix

Profile comparison of jobs and workers: minima and their sums

Worker	1	2	3	4	Sum		Job	1	2	3	4	Sum
Job No. 1							Worker No. 1					
5	0.0	1.8	3.5	9.5	14.8		3	5.9	8.7	3.6	1.3	19.5
2	0.0	1.8	3.5	9.4	14.7		5	2.8	6.0	3.2	1.7	13.7
4	0.0	1.8	3.5	9.3	14.6		2	2.1	5.2	3.6	1.7	12.6
3	0.0	1.8	3.5	3.3	8.6		4	1.6	5.3	3.6	1.7	12.2
1	0.0	1.8	3.5	1.7	7.0		1	0.0	1.8	3.5	1.7	7.0
Job No. 2							Worker No. 2					
2	2.1	5.2	4.7	9.4	21.4		2	2.1	5.2	4.7	9.4	21.4
5	2.1	4.8	4.6	9.5	21.0		5	2.7	6.0	3.2	9.4	21.3
4	1.3	5.0	4.9	9.3	20.5		4	1.6	5.3	4.6	9.4	21.0
3	2.1	5.2	4.8	3.3	15.4		1	0.0	1.8	3.5	9.4	14.7
1	2.1	5.2	3.6	1.7	12.6		3	2.7	6.2	4.3	1.3	14.5
Job No. 3							Worker No. 3					
3	5.8	8.7	4.3	1.3	20.1		3	5.8	8.7	4.3	1.3	20.1
1	5.9	8.7	3.6	1.3	19.5		2	2.1	5.2	4.8	3.3	15.4
2	2.7	6.2	4.3	1.3	14.5		5	2.8	6.0	3.2	3.3	15.3
5	2.4	4.8	4.3	1.3	12.8		4	1.6	5.3	4.6	3.3	14.8
4	1.3	5.0	4.3	1.3	12.0		1	0.0	1.8	3.5	3.3	8.6
Job No. 4							Worker No. 4					
2	1.6	5.3	4.6	9.4	21.0		2	1.3	5.0	4.9	9.3	20.5
5	1.6	4.8	4.6	9.5	20.6		4	1.3	5.0	4.6	9.3	20.2
4	1.3	5.0	4.6	9.3	20.2		5	1.3	5.0	3.2	9.3	18.8
3	1.6	5.3	4.6	3.3	14.8		1	0.0	1.8	3.5	9.3	14.6
1	1.6	5.3	3.6	1.7	12.2		3	1.3	5.0	4.3	1.3	12.0
Job No. 5							Worker No. 5					
2	2.7	6.0	3.2	9.4	21.3		2	2.1	4.8	4.6	9.5	21.0
5	2.4	4.8	3.2	9.5	19.9		4	1.6	4.8	4.6	9.5	20.6
4	1.3	5.0	3.2	9.3	18.8		5	2.4	4.8	3.2	9.5	19.9
3	2.8	6.0	3.2	3.3	15.3		1	0.0	1.8	3.5	9.5	14.8
1	2.8	6.0	3.2	1.7	13.7		3	2.4	4.8	4.3	1.3	12.8