# In-Work Benefits in Search Equilibrium*

Ann-Sofie Kolm† and Mirco Tonin‡

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**Abstract**

In-work benefits are becoming an increasingly relevant labour market policy, gradually expanding in scope and geographical coverage. This paper investigates the equilibrium impact of in-work benefits and contrasts it with the traditional partial equilibrium analysis. We find under which conditions accounting for equilibrium wage adjustments amplifies the impact of benefits on search intensity, participation, employment, and unemployment, compared to a framework in which wages are fixed. We also account for the financing of benefits and determine the level of benefits necessary to achieve efficiency in a labour market characterized by search externalities.

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## 1 Introduction

In-work benefits are becoming an increasingly relevant labour market policy. Programmes including some type of benefit or tax credit conditioned on labour income have been introduced or are in the "policy pipeline" in several countries (e.g. Belgium, Canada, Finland, France, Ireland, the Netherlands, New Zealand and Sweden). Yet other countries have progressively extended the scope of existing programmes, which were originally targeted at a very small section of the labour force. For instance, the Earned Income Tax Credit (EITC) in

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†Department of Economics, Stockholm University, S-106 91 Stockholm, Ph. +46 8 163547. Fax +46 8 161425, E-mail address: ann-sofie.kolm@ne.su.se

‡Economics Division, School of Social Sciences, University of Southampton, Southampton, SO17 1BJ UK, Ph. +44 23 8059 2519, E-mail address: m.tonin@soton.ac.uk

the US, which was introduced more than 30 years ago, is now the largest cash transfer programme for low income families at the federal level and, in 2003, about twenty million families received a total of $34 billion in benefits from it[1]. Moreover, the United Kingdom has a more than 25-year history of in-work benefits and has seen a gradual increase in their scope.

The expansion of this type of programmes makes it increasingly relevant to account for their equilibrium impact on the labour market. Moreover, since a number of less market oriented economies have recently followed the US and the UK in introducing various kinds of in-work benefits with the aim of decreasing unemployment and increasing labour force participation, it is particularly important to take involuntary unemployment and search effort into consideration.

The aim of this paper is to study the equilibrium impact of in-work benefits in a simple analytical framework displaying involuntary unemployment. Using a search model a la Pissarides (2000), we show that the introduction of in-work benefits reduces equilibrium unemployment, moderates wages and boosts participation and search effort. Total employment increases as a result. We also show under which conditions accounting for their equilibrium impact on wages in-work benefits actually reinforces the effects on the labor market outcomes. Another contribution of the paper is to account for the impact of financing, as with the expansion of benefit programmes, the resources needed to finance them are not negligible. We also determine the level of benefits necessary to achieve (constrained) efficiency in a labour market characterized by search externalities.

Research has almost exclusively been concerned with the supply-side effects of in-work benefits. On the empirical side, the expansions of the programmes in the US and the UK have been used to evaluate the effect of in-work benefits on labour supply. These evaluations show that benefits have been quite successful in terms of increasing labour supply. Eissa and Liebman (1996) compare the labour supply responses of single women with children to the responses of single women with no children when the earned income tax credit expanded in 1986. They show that between 1984-1986 and 1988-1990, single women with children increased their relative labour force participation by up to 2.8 percentage points. Meyer and Rosenbaum (2001) estimate that 63 percent of the increase in labour

[1] See Eissa and Hoynes (2005).

force participation of single families in the US between 1984 and 1996 can be credited to the expansion of the EITC. Moreover, Fang and Keane (2004) estimate the most important explanation for the 11 percentage point increase in labour force participation in the US between 1993-2002 to be the EITC. In addition, the evaluations show that it is the participation decision rather than the hour decision that is mostly affected by the EITC.[2]

The theoretical research on the impact of EITC policies is also supply-side oriented. A standard labour supply models serves as the basis for predicting the effects of the EITC on work hours (See Meyer, 2002, Eissa and Hoynes, 2005). In addition, a number of papers have also accounted for labour supply responses on the extensive (participation) margin when considering the effects of EITC policies; see, for example, Saez (2002).

Considering that an important aim of an EITC type of policy is to increase employment, which is an equilibrium outcome involving both supply-side and demand-side factors, the limited number of studies that have accounted for the demand side of the market might be surprising. Some recent empirical papers have raised the question of how the EITC is likely to affect wages, and have tried to estimate the incidence of the EITC on wages in different ways. Leigh (2004) uses variations in US state EITCs to examine the effect of the policy on pre-tax wages. The study by Rothstein (2007) uses the federal expansion of the EITC in the mid-1990s to estimate the effects on wages of the policy. Leigh (2004) finds that wages are significantly reduced by the state EITC and Rothstein (2007) finds that women at the lower end of the skill distribution face lower wages than they would have faced without the federal expansion of the EITC.

Some recent model analyses of in-work benefits incorporate unemployment. Boone and Bovenberg (2004) stress the importance of in-work benefits in order to alleviate distortions in terms of an inefficiently low search effort among the unemployed. Moreover, the study by Boone and Bovenberg (2006) explains why in-work benefits can be demanded for both in countries with generous welfare

[2] Moreover, the evaluations of the Working Family Tax Credit (WFTC) in the UK show that the programme has had positive net-effects on labour supply (see Brewer and Browne, 2006, and Blundell, 2006).

benefits (such as many European countries) and countries with low welfare benefits (such as the US). In countries with relatively low levels of social assistance, in-work benefits are aimed at poverty alleviation. In contrast, countries with generous social assistance need in-work benefits in order to maintain workers in the labour force. Although these two studies account for unemployment in their models, unemployment is exogenously imposed. Thus, when investigating the impact of an in-work benefit, there will be no effect on wages and unemployment as they are fixed by assumption[3].

Two studies that account for adjustments in wages while allowing for unemployment to be endogenously determined are Boeter et al (2006) and Lise et al (2005). Boeter et al (2006) simulate the general equilibrium effects of a social assistance reform in Germany. They use a union wage bargaining framework and find that a cut in the minimum income guarantee for those able to work, combined with a reduction in effective marginal tax rates at the lower end of the income distribution, entails a decrease in unemployment. Accounting for the general equilibrium wage reactions mitigates labour supply effects. Lise et al (2005) simulate the general equilibrium effects of the Self Sufficiency Project (SSP) in Canada, using a search framework to model the specific institutional details. Their simulation results also imply that accounting for equilibrium effects reduces, or actually reverses, the impact of the policy. For instance, unemployment increases and employment decreases following the introduction of SSP and the cost-benefit analysis changes from a net gain from the programme to a net cost once the equilibrium impact is accounted for.

In this paper, we account for involuntary unemployment and wage adjustment using a model with search frictions and worker-firm wage bargains (see

[3] Another feature that may be of potential importance for the success of an EITC policy is a country's degree of wage compression. The study by Immervoll et al (2007) considers the potential effects of in-work benefits in European countries using a micro simulation model. They consider both the effect of such a reform on work hours and labour force participation, accounting for the fact that the earnings distribution may be more or less compressed in different countries. They show that in-work benefits will be less desirable in countries with a compressed earnings distribution. This follows as a given redistribution when earnings are equal induces larger deadweight losses. The labour market is treated as perfectly competitive in their analysis.

Pissarides, 2000) and show analytically that the introduction of an in-work benefit moderates wages, boosts participation and search effort, thus reducing equilibrium unemployment and increasing total employment. We derive the conditions under which accounting for the impact of the policy on labour market equilibrium through wage adjustment actually boosts its effects on labour market variables. In particular, when unemployment is too high compared to the socially efficient level, accounting for wage adjustments actually reinforces the impact of benefits. This is due to job creation and underlines the importance of taking the demand side into consideration. One contribution of this paper is to show under which conditions accounting for equilibrium wage adjustment actually reinforces the impact of an in-work benefit *per se*. Interactions with other labour market institutions or other feedbacks due to specific institutional details in the actual implementation of the benefit may act as a counterbalance to this direct effect. For instance, the presence of a minimum wage and of a specific time threshold to qualify for the SSP play a major role in explaining the results in Lise et al (2005).

Another contribution of the paper is to account for the financing of the benefit programme, an issue that cannot be overlooked for programmes applying to a non negligible part of the workforce. In particular, the size of the benefit necessary to achieve constrained efficiency is derived.

The results are derived in a simple and stylized model in sections 2 and 3. In section 4, we contrast the analysis done in an equilibrium model of the labour market to partial equilibrium analysis, where wages do not adjust. The next section considers the case when the in-work benefit is financed with payroll taxes or proportional income taxes. Section 6 simulates the model to decompose and quantify the effects of an in-work benefit on labour market outcomes. The last section concludes.

## 2 The Model

The economy consists of a population that is fixed in size which is, without loss of generality, normalized to unity. The size of the labour force is endogenous. An individual chooses to participate in the labour force if the return of participation exceeds the return of non-participation. Individuals are heterogeneous with respect to the value of leisure that they enjoy when not participating. A worker who decides to participate in the labour force is either employed or searching for a job.

The economy is characterized by trading frictions due to the costly and time-consuming matching of workers and firms. The matching process of vacancies and unemployed job searchers is captured by a concave and constant-returns-to-scale matching function, $X = h(v, su)$, where $v$ is the vacancy rate and $u$ is the unemployment rate. The rates are defined as the number of vacancies and the number of unemployed workers relative to the labour force. The search intensity by an average worker is denoted by $s$. $su$ defines the number of job searching workers in terms of efficiency units.

The rate at which a specific unemployed worker finds a job depends on the individual search effort, $s_i$, in relation to the average search effort of the unemployed, $s$. Thus, the transition rate of the unemployed individual $i$ into employment is given by $s_i X/su = s_i h(\theta, 1) = s_i \lambda(\theta)$, where $\theta = v/su$ denotes labour market tightness. Firms fill vacancies at the rate $X/v = h(1, 1/\theta) = q(\theta)$. Higher labour market tightness $\theta$ increases workers' probability of finding a job, but reduces the probability of a firm finding a worker, i.e., $\lambda'(\theta) > 0$ and $q'(\theta) < 0$, where $\eta(\theta) = -\frac{q'}{q}\theta$ is the elasticity of the expected duration of a vacancy with respect to tightness.

## 2.1 Workers and Firms

Let $E$, $U$, and $N$ denote the expected present values of employment, unemployment, and non participation. The flow value functions for an individual worker can be written as:

$$rE_i = w_i + IWB - \phi(E_i - U_i), \tag{1}$$

$$rU_i = -\sigma(s_i) + s_i \lambda(\theta)(E - U_i), \tag{2}$$

$$rN_i = l_i, \tag{3}$$

where $r$ is the exogenous discount rate, $w$ is the wage, and $\phi$ the exogenous separation rate. $\sigma(s)$ captures the search costs of the unemployed, where

$\sigma_s(.), \sigma_{ss}(.) > 0$. The term $IWB$ represents the in-work benefit which is received only when employed. $l$ is the per period real value of leisure if not participating in the labour force which is assumed to be distributed in the population according to the cumulative distribution function $F(l)$.

The unemployed worker chooses search effort, $s_i$, so as to maximize the discounted value of unemployment, $U_i$, taking search effort by other unemployed workers, $s$, as well as other market variables, as given. This yields:

$$\sigma_{s_i}(.) = \lambda(\theta)(E - U_i). \tag{4}$$

Thus, the unemployed worker chooses search effort so as to equalize the marginal return of search with the marginal cost of search.

The economy consists of a large number of small firms that employ one worker only. Let $J$ and $V$ denote the expected present values of an occupied and a vacant job, respectively. The asset equations of a specific occupied job and a vacant job can be written as:

$$rJ_i = y - w_i - \phi(J_i - V), \tag{5}$$

$$rV = -k + q(\theta)(J - V), \tag{6}$$

where $y$ is worker productivity and the vacancy cost is denoted by $k$.

## 2.2 Wage determination

Matching frictions create quasi-rents for any matched pair providing a scope for bilateral bargaining after a worker and an employer meet. The baseline wage specification assumption found in the literature on search equilibrium is the generalized axiomatic Nash bilateral bargaining outcome with a 'threat point' equal to the option of looking for an alternative partner. The threatpoint for the worker is given by the value of unemployment. Note that the value of unemployment is at least as high as the value of non participation for workers in the labour force. Thus, employed workers do not consider the option of dropping out of the labour force as a threat when bargaining over wages.

Assuming that the worker has bargaining power $\beta$, the solution to the Nash bargaining problem satisfies the following first-order condition:

$$\frac{\beta}{1-\beta}J = E - U, \tag{7}$$

where we have imposed a symmetric equilibrium and used the free-entry condition $V = 0$. From (7) and the flow value functions in (1)-(6) and the free-entry condition, we get the wage rule:

$$w = \beta\left(y + ks\theta\right) - \left(1-\beta\right)\left[IWB + \sigma\left(s\right)\right]. \tag{8}$$

With free entry, we derive the job creation curve from (5) and (6)

$$\frac{k}{q\left(\theta\right)} = \frac{y-w}{r+\phi}, \tag{9}$$

Using (8) to substitute for the wage, we get tightness conditional on search effort. Similarly, search effort in equilibrium is derived conditional on tightness by imposing $s_i = s$ in (4) and using the free-entry condition $V = 0$ in (6) together with (7). This yields the following two equations determining search effort and tightness in equilibrium:

$$\frac{k\left(r+\phi\right)}{q\left(\theta\right)} = \left(1-\beta\right)\left[y + IWB + \sigma\left(s\right)\right] - \beta sk\theta, \tag{10}$$

$$\sigma_s\left(s\right) = \frac{\beta k\theta}{1-\beta}. \tag{11}$$

### 2.3 Labour force participation

A worker enters the labour force into the state of unemployment by choosing to conduct search. It will be worthwhile to enter the labour force if the return from entering exceeds the return from not entering. In equilibrium, the following condition determines the value of leisure of the worker who is indifferent between entering and not entering the labour force:

$$rU = rN\left(\hat{l}\right),$$

where $\hat{l}$ denotes the value of leisure of the marginal worker. Workers with a value of leisure higher than $\hat{l}$, i.e., $l_i > \hat{l}$, will choose non-participation, whereas workers with a value of leisure lower than $\hat{l}$, i.e., $l_i \leq \hat{l}$, will choose participation. The participation condition can be written as $s\lambda(\theta)(E-U)-\sigma(s)=\hat{l}$ by using the flow equations in (2) and (3) in symmetric equilibrium. Using the free-entry condition $V = 0$, together with equations (6) and (7) and the cumulative distribution function for leisure, we have the labour force given by:

$$LF = F\left(\frac{s\beta k\theta}{1-\beta} - \sigma(s)\right). \tag{12}$$

### 2.4 Employment

In equilibrium, the flow into unemployment equals the flow out of unemployment, i.e., $\phi(1-u)LF = s\lambda(\theta)uLF$. The equilibrium unemployment rate is then given by:

$$u = \frac{\phi}{\phi + s\lambda(\theta)}, \tag{13}$$

which depends positively on the separation rate and negatively on tightness and search intensity. The total number of employed workers is given by:

$$Employment = (1-u)LF. \tag{14}$$

## 3 Effects of in-work benefits

This section derives the effects of in-work benefits on wage formation, search effort, unemployment and employment in equilibrium. Section 5 will deal with the generalization of these results when proportional income or payroll taxation is used to finance the in-work benefit. We summarize the results in the following proposition:

**Proposition 1** *An in-work benefit will reduce wages and increase tightness and search effort. Moreover, the equilibrium rate of unemployment falls, and labour force participation and employment increase, with an in-work benefit.*

**Proof.** See appendix. ■

An in-work benefit which, by definition, is conditioned on work, makes it relatively more attractive to have a job, so it tends to reduce wage demands. As wage demands fall, it becomes more profitable to open vacancies in relation to the number of efficient job searchers in the unemployment pool, which induces tightness to increase. As the expected unemployment spells become shorter, the return to job search increases, which induces unemployed workers to devote more time to search. The equilibrium rate of unemployment falls both because unemployed workers search more intensively for a job and because there are more posted vacancies relative to the number of efficient job searchers. An in-work benefit will also induce more workers to choose participation instead of non-participation. The shorter expected unemployment spells simply increase the return to participation. Consequently, total employment increases both because the equilibrium rate of unemployment falls and because more workers choose to participate in the labour market.

The role of job creation becomes even more pronounced if we account for unemployment benefits in the analysis. Including a fixed level of unemployment benefits, $B$, in the present model will not modify the results in the proposition, nor will the assumption of unemployment benefits that are indexed to the wage, i.e. $B = bw$. However, when benefits are indexed to the wage, an increase in the in-work benefit ($IWB$) tends to have a larger effect on wage demands. This follows as the wage moderation entails a reduction in unemployment benefits, which further reduces the wage demands. In fact, the take home pay when employed, $w + IWB$, may fall in this case. However, despite the fact that labour income may fall with an increase in the in-work benefit, search effort and participation increase as the expected unemployment spell becomes shorter. This illustrates a case when the employment increase caused by an in-work benefit is solely driven by job creation.

# 4 Fixed wages

In this section we analyze the impact of introducing an in-work benefit under the assumption of fixed wages. By assuming a fixed wage, and thus no wage adjustments following a policy change, the traditional partial equilibrium labour supply story can be told. An in-work benefit will increases search effort and labour force participation as the take-home pay increases. As supply creates its own demand, also employment increases.

The derivation in the fixed wage case is straightforward. With wages fixed at the pre-benefit level $\tilde{w}$, tightness is the same as in the pre-benefit equilibrium, given by

$$\frac{k\left(r+\phi\right)}{q\left(\theta\right)}=y-\tilde{w}.$$

The behavioral equation giving search effort is still (4). However, (7) no longer holds. Combining (1) and (2), we obtain

$$E-U=\frac{\tilde{w}+IWB+\sigma\left(s\right)}{r+\phi+s\lambda\left(\theta\right)},$$

so that, using (4), search effort is determined by

$$\sigma_{s}\left(.\right)=\lambda\left(\theta\right)\frac{\tilde{w}+IWB+\sigma\left(s\right)}{r+\phi+s\lambda\left(\theta\right)}. \tag{15}$$

Conversely, labour force participation is given by

$$LF=F\left(s\lambda\left(\theta\right)\frac{\tilde{w}+IWB+\sigma\left(s\right)}{r+\phi+s\lambda\left(\theta\right)}-\sigma\left(s\right)\right), \tag{16}$$

while the expressions for unemployment and employment are unchanged. Under fixed wages, as it is the case under flexible wages, search effort, labour force participation, and employment increase with the introduction of an in-work benefit, while unemployment decreases. If we contrast the labour market outcomes in the two cases, we find the following results:

**Proposition 2** *An in-work benefit will have a larger positive impact on search effort and labour force participation in equilibrium, when wage adjustments are accounted for, than in partial equilibrium when wages are assumed to be fixed, if and only if* $\beta\geq\eta\left(\theta\right)$*. The same condition is a sufficient condition for an in-work benefit to have a larger impact on employment and unemployment when wages adjust compared to the case when they are fixed.*

**Proof.** See appendix. ■

An increase in the in-work benefit will induce larger positive responses in both search effort and labour force participation if wages are flexible in comparison to if they are assumed to be fixed, provided that $\beta>\eta\left(\theta\right)$. These larger responses in search effort and labour force participation when wages are flexible, will, in turn, reinforce the fall in the equilibrium rate of unemployment and the increase in employment. However, there is also a direct negative impact on the equilibrium rate of unemployment when wages are flexible as the reduced wages increases the transition rate into employment. Thus, the condition $\beta \geq \eta(\theta)$ is only sufficient, not necessary when it comes to the impact of in-work benefits on the unemployment rate and employment. We know that because of trading externalities, equilibrium search intensity and participation are generally too low from the point of view of society when $\beta > \eta(\theta)$, wages are simply set too high and tightness too low from a social point of view (Pissarides, 2000). Under these circumstances, the positive effect on search effort due to the fact that job offers arrive more frequent will dominate the negative effect on search effort due to the fact that lower wages reduce the pay off from work. This holds also for the participation decision which is concerned with weighting the effects on the take-home pay against a higher job offer arrival rate for the unemployed.

# 5 Financing of the in-work benefit

In this section, we study the effects of in-work benefits when their financing through proportional income taxation is taken into account. In particular, wages are taxed at the proportional rate, $t$ [4]. The flow value function for employment in (1) becomes

$$rE_i = w_i(1-t) + IWB - \phi(E_i - U_i),$$

while (2), (3), (5), and (6) remain unchanged. The first-order condition for wage determination in (7) becomes

$$(1-t)\frac{\beta}{1-\beta}J = E - U, \tag{17}$$

and the wage rule corresponding to (8) becomes

$$w = \beta(y + ks\theta) - \frac{1-\beta}{1-t}\left[IWB + \sigma(s)\right]. \tag{18}$$

[4] The IWB being financed by payroll taxation would yield the same results.

It can be noted that a higher tax rate will have a direct negative effect on wage demands given by (18). The reason for this is that $IWB$ and $\sigma(s)$ is not taxed and the marginal value of an additional unit of wage is $(1-t)$. Thus, a higher tax rate works as an increase in the $IWB$ when formulating (gross) wage demands.

In-work benefits are financed by taxing wages. We study two cases. First, we derive analytical results for the case when benefits are fully financed by taxing the beneficiaries. Then, we deal with the case when the whole workforce is taxed to finance benefits for which only part of the population is eligible. For this case, labelled "partial financing", we here derive the main equations, while the simulation results are discussed in section 6.3. Also, in line with the analysis done in the previous sections, we present simulation results comparing the full financing case when wages can adjust and when they are instead fixed. Notice that when in-work benefits are fully financed by taxing the beneficiaries, wages fixed at the pre-benefit level $\tilde{w}$ imply that the income of a worker when employed, $\tilde{w}(1-t)+IWB$, always equals $\tilde{w}$, so that the equilibrium with or without in-work benefits is the same.

## 5.1 Full financing

As only employed workers receive the benefits, a balanced budget implies [5]

$$IWB = tw. \tag{19}$$

Substituting (19) into (18) and rearranging, we get the wage as an expression of the tax rate

$$w = \frac{\beta(1-t)}{1-\beta t}(y + ks\theta) - \frac{1-\beta}{1-\beta t}\sigma(s). \tag{20}$$

[5] When unemployment benefits are also accounted for, the analysis of financing becomes more complex, as the tax rate necessary to finance a given level of in-work benefits and unemployment benefits (or a given replacement rate) depends on the equilibrium level of unemployment. In this case, an increase of in-work benefits is likely to be partly financed by reduced unemployment benefits and, if unemployment benefits are also taxed, by higher tax revenues from unemployed. If we also consider some kind of social assistance available to non participants, also the size of the labour force is of importance.

Substituting (20) into the job creation curve (9), we get the expression for equilibrium tightness corresponding to (10):

$$\frac{k\left(r+\phi\right)}{q\left(\theta\right)}=\frac{1-\beta}{1-\beta t}\left[y+\sigma\left(s\right)\right]-\frac{\beta\left(1-t\right)}{1-\beta t}ks\theta. \tag{21}$$

In the $(\theta, w)$ space, increasing the tax rate shifts the wage curve (20) downward and clockwise while leaving the job creation curve (9) unchanged, thus clearly reducing the equilibrium wage and increasing tightness, i.e.

$$\frac{\partial w}{\partial t}<0, \frac{\partial \theta}{\partial t}>0.$$

Note that changes in $t$ working through $s$ will have no effect on these expressions as $s$ is optimally chosen. Thus, we can state that an increase in proportional taxes used to finance in-work benefits reduces wages and increases tightness. It is also straightforward to formally verify this by differentiating (21) and (20) with respect to $t$, $\theta$, and $w$ [6].

The relationship between the tax rate and the in-work benefits may not be monotonic. For a given wage, an increase in $t$ increases $IWB$. However, in equilibrium the tax rate has a moderating impact on wages, with a higher $t$ corresponding to a lower $w$. Thus, the effect of an increase in the tax rate on tax revenues, i.e. on in-work benefits, may be dominated by the reduction in the tax base, i.e. the reduction in wages due to a tax hike[7]. There may thus be some sort of "Laffer curve", but as far as the economy is on the side of the curve where an increase in the tax rate increases total revenues, i.e. $\frac{\partial IWB}{\partial t}>0$, the derivatives w.r.t. $t$ have the same sign as the derivatives w.r.t. $IWB$, thus

$$\frac{\partial w}{\partial IWB}<0, \frac{\partial \theta}{\partial IWB}>0.$$

[6] Differentiating (21) with respect to $t$ and $\theta$ yields $\frac{\partial \theta}{\partial t}=\frac{(1-\beta)[y+ks\theta+\sigma(s)]}{(1-\beta t)sk(1-t)[1+z]}>0$, where $z=-\frac{(r+\phi)q'}{q^2}\frac{(1-\beta t)}{(1-t)s\beta}>0$. Then, differentiating (20) with respect to $w$ and $t$ accounting for $\theta$ being affected by $t$, yields: $\frac{\partial w}{\partial t}=-\frac{\beta(1-\beta)[y+ks\theta+\sigma(s)]}{(1-\beta t)^2}\left[1-\frac{1}{1+z}\right]<0$. Once more, note that changes in $t$ working through $s$ will have no effect on these expressions as $s$ is optimally chosen by the individuals.

[7] Using (20) in (19) and differentiating wrt, $t$ we get $\frac{\partial IWB}{\partial t}=\frac{\left(\beta t^2-2t+1\right)\beta[y+ks\theta+\sigma(s)]}{(1-\beta t)^2}+\frac{\beta kst(1-t)}{1-\beta t}\frac{\partial \theta}{\partial t}-\sigma\left(s\right)$. The first term is positive iff $t\in\left[0, \frac{1-\sqrt[2]{1-\beta}}{\beta}\right]\supset[0, \frac{1}{2}]$. The second term is always positive as $\frac{\partial \theta}{\partial t}>0$. So, for $\sigma\left(s\right)$ small enough and $t$ not too high $\frac{IWB}{\partial t}>0$. Substituting the expression for $\frac{\partial \theta}{\partial t}$ we get $\frac{\partial IWB}{\partial t}=\frac{\beta[y+ks\theta+\sigma(s)]}{1-\beta t}\left[(1-t)-\frac{t(1-\beta)}{(1-\beta t)}\left(1-\frac{1}{1+z}\right)\right]-\sigma\left(s\right)$. Notice that at $t=0$, $\frac{\partial IWB}{\partial t}=w>0$.

Search intensity is given by (4). Using the free-entry condition $V = 0$ in (6) together with (17), we get

$$\sigma_s(s) = (1-t)\frac{\beta k\theta}{1-\beta}. \tag{22}$$

For search intensity to grow as the tax rate increases, we need the following condition to hold:

$$(1-t)\frac{\partial\theta}{\partial t} - \theta > 0. \tag{23}$$

The labour force is given by

$$LF = F\left((1-t)\frac{s\beta k\theta}{1-\beta} - \sigma(s)\right), \tag{24}$$

which increases with $t$ iff $(1-t)\frac{\partial\theta}{\partial t} - \theta > 0$. Unemployment is given by (13). If search intensity increases with $t$, then unemployment certainly decreases with $t$. Employment is given by (14). If (23) holds, then employment also increases with $t$. Thus, (23) is a sufficient, but not necessary, condition for unemployment and employment to increase with the tax rate.

When is it the case that $(1-t)\frac{\partial\theta}{\partial t} - \theta > 0$? Substituting the expression for $\frac{\partial\theta}{\partial t}$ into (23), the condition is equivalent to

$$\frac{(1-\beta)\left[y + ks\theta + \sigma(s)\right]}{1-\beta t} > \theta sk\left[1 - \frac{(r+\phi)\,q'}{q^2}\frac{1-\beta t}{(1-t)\,s\beta}\right].$$

Using the equilibrium expression for tightness (21) and rearranging, we get

$$\eta(\theta) < \frac{1-t}{1-\beta t}\beta, \tag{25}$$

With $t = 0$ the condition is $\eta(\theta) < \beta$. We know that because of trading externalities, equilibrium search intensity and participation are generally too low from the point of view of society and, when $\beta > \eta(\theta)$, equilibrium unemployment is above the socially efficient rate (Pissarides, 2000). What we show is that under these circumstances, there is room for in-work benefits to improve labour market efficiency by increasing search intensity, labour force participation, employment, and reducing unemployment, even when financing is taken into account.

**Proposition 3** *Proposition 1 holds also when the in-work benefits are financed through proportional taxes on wages, provided that the tax rate is such that a higher tax rate implies higher fiscal revenues and that* $\eta(\theta) < \frac{1-t}{1-\beta t}\beta$.

The intuition behind this result is the following. Equilibrium tightness and search when in-work benefits are financed through proportional taxation at the rate $t$ are given by equations (21) and (22), while the wage is given by equation (20). We get exactly the same expressions when substituting $\beta$ with

$$\beta' \equiv \frac{\beta\left(1-t\right)}{1-\beta t} < \beta,$$

and $IWB = 0$ into equations (10), (11), and (8) that characterize the equilibrium when financing of benefits is not taken into account. This means that the equilibrium of a model with in-work benefits financed through a proportional tax on wages $t$ and with workers' bargaining power $\beta$ is isomorphic to the equilibrium of a model without in-work benefits and with workers' bargaining power $\beta' < \beta$. Thus, an increase in the tax rate used to finance in-work benefits is equivalent to reducing the "effective" bargaining power of the worker. In a search model as that used here, (constrained) efficiency is reached when workers' bargaining power equals the elasticity of the expected duration of a vacancy with respect to tightness. If instead $\beta > \eta\left(\theta\right)$, then a marginal increase in taxation moves the labour market toward efficiency, thus increasing search intensity and participation and reducing unemployment. This goes on until $\frac{\beta(1-t)}{1-\beta t} = \eta\left(\theta\right)$, after which a further increase in taxation to finance in-work benefits moves the economy away from efficiency, reducing search intensity and participation, while the effect on unemployment is ambiguous.

$>$From (25), we can calculate the tax rate that gives efficiency as the solution to the system formed by equations (21) and (22) and by

$$t = \frac{\beta - \eta\left(\theta\right)}{\beta\left(1-\eta\left(\theta\right)\right)}, \tag{26}$$

which is easy to calculate in case of a Cobb-Douglas matching function as $t^* = \frac{\beta-\eta}{\beta(1-\eta)}$, where $\eta$ then constant. This provides a simple condition for the level of fully financed in-work benefits needed to achieve (constrained) efficiency in a labour market characterized by search externalities.

## 5.2 Partial Financing

Here, we study the case when only part of the population is entitled to benefits, which are financed by the whole workforce. We assume that there are two types of agents in the population. One type, representing a share $\rho$ of the total population, is entitled to in-work benefits, while the other type is not. This may be due to the fact that the two types have different productivities or that they differ in some other relevant dimension, like having children or not. To simplify the analysis and focus on the fiscal aspects of in-work benefits, we assume that these two types of agents are active in separate labour markets. Thus, they are solely linked through the fiscal system. In particular, all agents are subject to a tax on wages at rate $t$, used to finance an in-work benefit to which only a part of the population is eligible. Moreover, all structural parameters, except possibly productivity, are the same in the two labour markets. First, we characterize the equilibrium labour market outcome for the part of the population that is non-eligible to benefits, then for the eligible part. Simulation results are discussed in section 6.3. Subscripts "$n$" and "$e$" are used to indicate the two groups. The labour market outcome for the economy as a whole is determined as a weighted average of the corresponding variables for the two groups, in which weights reflect their relative size (see the Appendix for details).

**Non-eligible Workers** Workers of this type have their wage taxed at tax rate $t$, but in-work benefits are not available to them. Substituting in (9) the wage equation given by (18) with $IWB = 0$**,** we get the expression characterizing tightness in this labour market

$$\frac{k\left(r+\phi\right)}{q\left(\theta_n\right)} = \left(1-\beta\right) y_n - \beta k s_n \theta_n + \frac{1-\beta}{1-t}\sigma\left(s_n\right).$$

Search intensity $s_n$ is given by expression (22), while the participation rate, the unemployment rate and the employment rate are given by expressions (13), (14), and (24), respectively. To get the absolute number of participants and employed, we need to account for the fact that these agents represent a fraction $(1-\rho)$ of the total population. The total fiscal resources collected from this group of workers are given by

$$b = \left(1-\rho\right) e_n t w_n,$$

where $e_n$ is the employment rate and $w_n$ the equilibrium wage.

**Eligible Workers** This group of workers has the wage taxed at rate $t$ and is eligible to an in-work benefit. The analysis is similar to the case with full financing, where the per capita amount of benefits implied by a balanced budget is given by

$$IWB = tw_e + \frac{b}{\rho e_e}. \tag{27}$$

The first term, $tw_e$, is the "self-financing" part, while the second term represents the part financed by ineligible workers, which depends on the total fiscal resources collected, $b$, and the number of eligible workers among which these resources must be split, $\rho e_e$. Substituting (27) into the wage equation given by (18) we get

$$w_e = \frac{\beta(1-t)}{1-\beta t}(y_e + ks_e\theta_e) - \frac{1-\beta}{1-\beta t}\left[\frac{b}{\rho e_e} + \sigma(s_e)\right],$$

which substituted in (9) gives

$$\frac{k(r+\phi)}{q(\theta_e)} = \left(\frac{1-\beta}{1-\beta t}\right)\left[y_e + \frac{b}{\rho e_e} + \sigma(s_e)\right] - \frac{\beta(1-t)}{1-\beta t}ks_e\theta_e,$$

where $e_e$ depends on $\theta_e$. Search intensity, participation rate, unemployment rate and employment rate are given by expressions (22), (13), (14), and (24), respectively.

Next we turn to a calibrated version of the model in order to provide some numerical examples of the magnitude of the effects on labour market performance.

# 6 Numerical simulations

In this section we calibrate the model to gauge insights on the magnitudes involved. First, we compare the impact of benefits with and without wage adjustment when financing is not accounted for. Then, we look at the model with financing, both full and partial.

## 6.1 Calibration

To calibrate the model, we assume the matching function to be Cobb-Douglas, so that

$$X = h(v, su) = mv^{1-\eta}(su)^{\eta} \quad \text{where} \;\; m > 0; \eta \in (0,1). \tag{28}$$

The convex search cost function is assumed to be a power function and therefore

$$\sigma(s) = s^{\alpha}, \text{ where } \; \alpha > 1. \tag{29}$$

The month is the basic time unit. Productivity $y$ is normalized to 1. Worker bargaining power $\beta$ is set to the standard value in the literature of 0.5, while the real interest rate $r$ is 0.005. Following Christensen et al. (2005), parameter $\alpha$ equals 2, implying a quadratic search cost[8]. In the baseline specification, $\eta$ equals 0.4, while parameters $k$, $\phi$, and $m$ are set to replicate an unemployment rate of 0.06, an average duration of unemployment of three months, and an average duration of a vacancy of one month in the absence of in-work benefits, giving $k = 4.5616$, $\phi = 0.0213$, and $m = 0.6807$. Finally, we assume the per period value of leisure to be distributed according to an exponential function with parameter $\mu$, calibrated so that the participation rate without in-work benefits equals 0.7. See the Appendix for details. The table below summarizes the baseline parametrization.

| $y$ | $\beta$ | $k$ | $r$ | $\phi$ | $m$ | $\eta$ | $\alpha$ | $\mu$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.5 | 4.5616 | 0.005 | 0.0213 | 0.6807 | 0.4 | 2 | 0.631 |

## 6.2 Numerical results with flexible and fixed wages

The theory implies that the introduction of in-work benefits entails an increase in tightness and a fall in wages when wages are flexible, while these two quantities do not move when wages are fixed. In both cases search effort, labour force participation, and employment increase, while unemployment declines. The

[8] Christensen et al. (2005) structurally estimate a model with on-the-job search using Danish microdata. A quadratic function is also the preferred specification in Yashiv (2000), who structurally estimates a model with search only by the unemployed using Israeli aggregate time-series data.

conditions under which the impact is greater with or without wage adjustment have been derived in proposition 2. Here, we explore the quantitative impact of benefits in both cases.

The simulation results show that the quantitative impact on unemployment and employment is significantly stronger when the effect of benefits on wages is taken into account. Figure 1 describes the effects on the main labour market variables of introducing in-work benefits up to the equivalent of half of labour productivity. The continuous line represents the case where wages are flexible, while the dotted line represents the case with fixed wages. Compared to an unemployment rate of 6% without in-work benefits, the introduction of benefits equivalent to 40% of productivity implies a decline in unemployment to 4.97% when wages are fixed and to 4.41% when they are flexible, while employment increases by an additional 0.62% with flexible wages as compared to the case with fixed ones. Moreover, the impact on search intensity and labour force participation is stronger when wages are allowed to move but quantitatively, the difference is very small. The fall in wages makes employment less attractive and so partly, but not entirely, offsets the increase in search effort and participation due to the increase in labour market tightness.

Thus, accounting for the equilibrium impact of benefits actually reinforces their positive effect on the labour market. Thus, the extension of benefits to larger portions of the workforce does not entail, in itself, a decline in their effectiveness or, worse, a reversal of their effect. In the next section we look at another issue that needs to be taken into account when the scope of benefit programmes is increased to comprise a non negligible share of the workforce: their financing.

## 6.3 Numerical results with financing

Here we use the same parametrization, with the only difference that when simulating the model with partial financing, we also consider the case where the productivity of non-eligible workers is double the productivity of eligible ones.

First, we compare the effects on the main labour market variables of introducing fully-financed benefits up to the equivalent of half of labour productivity[9] when wages can adjust and when they are instead fixed. We also compare the impact of benefits when their financing is accounted for to the impact when the issue of financing is disregarded.

In Figure 2, the continuous line represents the case when wages can adjust, while the dotted line represents the case with fixed wages. As previously stated, when benefits are fully financed by taxing beneficiaries and wages are downward rigid, in-work benefits do not have any effect. When wages can adjust, tightness increases and gross wage decreases. Notice that in this setting, gross wage is equivalent to total income, as fiscal revenues are entirely used to finance benefits. The comparison of figures 1 and 2 reveals that both tightness and wages respond more strongly when benefits are financed through taxation on beneficiaries' wages as compared to the case when an identical amount of in-work benefits is a "windfall", financed through other sources. This is due to the additional wage moderation stemming from taxation. As predicted by the theory, with full financing the response of search intensity and labour force participation is hump-shaped, initially increasing with the level of benefits (and taxes) and then declining. In the baseline parametrization, the tax rate at which both quantities reach their peak is, from (26), $t = 1/3$, corresponding to $IWB \approx 0.28$, at which (constrained) efficiency is achieved. Further increases in fully financed benefits take the labour market away from efficiency. However, search intensity and participation stay above the level they have when no benefits are paid until $IWB \approx 0.47$ ($t \approx 56\%$). Unemployment declines in the whole range, falling, for instance, from 6% to 4.15% when benefits are equivalent to 40% of productivity. Total employment increases, reaching approximately 67.2% of the population when $IWB = 0.4$, as compared to 65.8% with no benefits.

We look at three scenarios in the "partial financing" case, where the share of the population eligible for benefits financed by the whole workforce is 25%, 50%, and 75%, respectively. To make the comparison easier, we focus on the case when both eligible and non-eligible workers have the same productivity.

[9] The tax rate corresponding to $IWB = 0.5$ is approximately 60%. In the baseline parametrization, the maximum attainable amount of benefits with wage flexibility is 0.64, achieved at a tax rate of 88%.

However, the case with non-eligible workers having higher productivity is also investigated.

Figure 3 reports the main labour market indicators for eligible workers as a function of in-work benefits in the three scenarios. For comparison, indicators with "no financing" and "full financing" are also depicted. As could be expected, the "partial financing" cases lie between the two polar ones, moving toward the "full financing" equilibrium as the share of eligible workers increases. The corresponding figure for non-eligible workers is 4. For this group of workers, given the share of eligibles in the population, an increase in benefits just represents an increase in taxation. An increase in the share of eligible workers, given a level of benefits, also represents an increase in taxation. Thus, increasing benefits or increasing eligibility reduce wages, search intensity, labour force participation, and employment of non-eligible workers, while tightness and unemployment increase.

The impact of benefits on the labour market as a whole is presented in figure 5, that includes the "full financing" case for reference, and in table 1. Unemployment decreases with the introduction of benefits, and the impact on it is stronger as benefits increase and as the share of eligible workers increases, with the equilibrium smoothly converging to the "full financing" case. The behavior of labour force participation and employment is more complex. Their response to benefits is hump-shaped, first increasing and then decreasing as benefits increase. The response to an increase in eligibility is also non-linear. For a given level of benefits, labour force participation and employment may decrease with the share of the population eligible for benefits rising from 25% to 50%, but then bounce back with a further increase to 75%. While improving labour market conditions for eligible workers, the negative impact of increased taxation on non-eligible ones implies that in-work benefits above a relatively low level do not improve participation and employment in the labour market as a whole. This no longer happens if the productivity of non-eligible workers is set to double the productivity of eligible ones (see table 2). In this case, labour market conditions improve with the introduction of benefits even at relatively high levels. The analysis of the partial financing case done here is just preliminary, but indicates the importance of accounting for the financing of benefits when evaluating their impact on the labour market as a whole.

## 7 Conclusions

In-work benefits are becoming increasingly popular among policy-makers due to their success in the American and British contexts. Whether they can be successfully adopted in other countries and help solve some of the problems characterizing their labour markets is an open issue. This paper represents a first step towards addressing this question. We analyze the impact of in-work benefits on some of the main labour market indicators in a search framework, taking into account the effects on labour market equilibrium . We find that introducing or increasing in-work benefits increases labour force participation, employment, and search intensity by unemployed, while wages and the unemployment rate decline. This result is robust to various extensions.

Considering in-work benefits in an equilibrium setting reveals that their impact on job creation is an important factor behind employment growth, in contrast to the existing literature that mainly looks at their impact on labour supply via a higher take-home pay. In fact, in-work benefits may even reduce the take-home pay as wage demands are moderated[10]. However, the lower wages boost job creation which reduces unemployment. The shorter expected unemployment spell, in turn, encourages job search and labour force participation which reinforces the increase in employment. Our model suggests that the job creation dimension should be taken into account in evaluating ex ante the impact of introducing such benefits in a European country. The risk is, otherwise, to miss a very important link.

The analysis of financing reveals the conditions under which benefits that are financed through proportional taxation on wages increase labour force participation, employment, and search intensity of the targeted group.

Both these aspects of in-work benefits, their impact on job creation and their financing, have mostly been overlooked by the existing literature, but become increasingly relevant as the scope of programmes including benefits or tax credits

[10] This was concluded in section 4, where unemployment benefits were indexed to the wage which induced additional wage moderation which could actually reduce the take-home pay.

conditioned on labour income is extended.

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# Appendix

## A1 Proofs of propositions

**Proposition 1**. Differentiation of (10) with respect to $\theta$ and $IWB$ yields $\frac{\partial\theta}{\partial IWB} = \frac{(1-\beta)}{s\beta k\left(1-k(r+\phi)\frac{q'}{s\beta kq^2}\right)} > 0$. To get the equilibrium effect on tightness, we need to account for the fact that $s$ is a function of $\theta$ through (11). However, as search is optimally determined by workers, the effects working through search effort in (10) will have no impact on tightness. Using how $IWB$ affects tightness and the fact that search is optimally determined, we can show the following for search effort, wage, income from work, labour force participation, the unemployment rate, and employment: $\frac{\partial s}{\partial IWB} = \frac{\beta k}{\sigma_{ss}(s)(1-\beta)}\frac{\partial\theta}{\partial IWB} > 0$ from (11), $\frac{\partial w}{\partial IWB} = -(1-\beta)\left[1-1/\left(1-k(r+\phi)\frac{q'}{s\beta kq^2}\right)\right] < 0$ from (8), $\frac{\partial(w+IWB)}{\partial IWB} = -(1-\beta)\left[1-1/\left(1-k(r+\phi)\frac{q'}{s\beta kq^2}\right)\right]+1 > 0$, $\frac{\partial LF}{\partial IWB} = F'(.)\frac{s\beta k}{(1-\beta)}\frac{\partial\theta}{\partial IWB} \geq 0$ from (12), $\frac{\partial u}{\partial IWB} = -\frac{\phi}{\phi+s(\theta)\lambda(\theta)}\left(\frac{\partial s}{\partial\theta}\lambda(\theta)+s\frac{\partial\lambda}{\partial\theta}\right)\frac{\partial\theta}{\partial IWB} < 0$ from (13), and $\frac{\partial Employment}{\partial IWB} = -\frac{\partial u}{\partial IWB}LF+(1-u)\frac{\partial LF}{\partial IWB} > 0$ from (14).

**Proposition 2**. Optimal search is determined by (15). Differentiation of (15) gives: $\frac{\partial s}{\partial IWB} = \frac{\lambda(\theta)}{N}+\frac{A}{N}$ where $N = \sigma_{ss}(s)(r+\phi+s\lambda(\theta))$ and $A = \lambda(\theta)\frac{\partial w}{\partial IWB}+\frac{\partial\lambda(\theta)}{\partial\theta}\frac{1}{\lambda(\theta)}\frac{\partial\theta}{\partial IWB}\sigma_s(.)(r+\phi)$. The first term captures the direct effect (and only effect if wages are fixed) and the second term captures the effects due to flexible wages. As the first term is the same in the fixed and flexible case, the effect on search due to wage adjustments depends on the sign of the second term. Using the expressions in the proof of proposition 1 and the fact that $\lambda(\theta) = q\theta$ and $\frac{\partial\lambda(\theta)}{\partial\theta} = q'\theta+q$, we have $A = \frac{(r+\phi)}{(s\beta+(r+\phi)\eta(\theta)/\lambda(\theta))}[\beta-\eta] > 0 \leftrightarrow \beta > \eta$. Use (15) to rewrite (16) as $LF = F(s\sigma_s(s)-\sigma(s))$.Differentiation yields $\frac{\partial LF}{\partial IWB} = F'(.)s\sigma_{ss}(s)\frac{\partial s}{\partial IWB}$ . Therefore, the condition for labour force participation to increase more with a marginal increase in IWB under flexible wages is the same as the one for search intensity. Moreover, $\frac{\partial u}{\partial IWB} = -\frac{\phi}{\phi+s(\theta)\lambda(\theta)}\left(\frac{\partial s}{\partial IWB}\lambda(\theta)+s\frac{\partial\lambda(\theta)}{\partial\theta}\frac{\partial\theta}{\partial IWB}\right)$. Thus the unemployment rate tends to fall by more when wages are flexible as the higher tightness increases the transition rate into employment irrespective of whether $\beta$ is larger or smaller than $\eta$. However, if $\beta > \eta(\theta)$, search increases by more if wages are flexible, and thus we have an additional negative effect on the unemployment rate, making $\beta > \eta(\theta)$ a sufficient but not necessary condition for unemployment to decline more when wages are flexible. This is also the case for employment, as $\frac{\partial Employment}{\partial IWB} = -\frac{\partial u}{\partial IWB} + (1-u)\frac{\partial LF}{\partial IWB}$.

## A2 Expressions and calibration

The expressions we used in the calibration are derived in this appendix.

**No financing** Using (29) in (11), we get

$$s = \left[\frac{\beta k\theta}{\alpha(1-\beta)}\right]^{\frac{1}{\alpha-1}}, \tag{30}$$

that, substituted into (10) and together with (28), gives

$$\frac{k(r+\phi)}{m\theta^{-\eta}} = (y + IWB)(1-\beta) - \left(1 - \frac{1}{\alpha}\right)\frac{(\beta k\theta)^{\frac{\alpha}{\alpha-1}}}{[\alpha(1-\beta)]^{\frac{1}{\alpha-1}}}, \tag{31}$$

which implicitly determines equilibrium tightness as a function of parameters. Equilibrium search is given by substituting equilibrium tightness into (30). Given that $\theta$ and $s$ are determined by (30) and (31), we can derive the wage, the unemployment rate, the labour force and employment in the following way. $>$From (8) we get the equilibrium wage

$$w = \beta(y + ks\theta) - (1-\beta)[IWB + s^{\alpha}],$$

and from (13) the equilibrium unemployment rate

$$u = \frac{\phi}{\phi + sm\theta^{1-\eta}}. \tag{32}$$

$>$From (12) and the assumption that the per period value of leisure is distributed according to an exponential function with parameter $\mu$, we get the labour force

$$LF = 1 - \exp\left(-\frac{(\alpha-1)}{\mu}\left[\frac{\beta k\theta}{\alpha(1-\beta)}\right]^{\frac{\alpha}{\alpha-1}}\right), \tag{33}$$

and, finally, from (14) we can derive the equilibrium employment.

**With financing** Using (29) in (22), we get

$$s=\left[\frac{(1-t)\beta k\theta}{\alpha\left(1-\beta\right)}\right]^{\frac{1}{\alpha-1}}, \tag{34}$$

which, substituted into (21) and together with (28), gives

$$\frac{k\left(r+\phi\right)}{m\theta^{-\eta}}=\frac{1-\beta}{1-\beta t}\left[y+(1-\alpha)\left(\frac{(1-t)\beta k\theta}{\alpha\left(1-\beta\right)}\right)^{\frac{\alpha}{\alpha-1}}\right], \tag{35}$$

that implicitly determines equilibrium tightness as a function of parameters. Equilibrium search is given by substituting equilibrium tightness into (34). Given that $\theta$ and $s$ are determined by (34) and (35), we can derive the wage, the unemployment rate, the labour force and employment in the following way. From (20), we get the equilibrium wage

$$w=\beta\left[y+ks\theta+s^{\alpha}\right]\frac{1-t}{1-\beta t}-s^{\alpha},$$

and from (19), the corresponding in-work benefits, $IWB$. Labour force participation is given by (24)

$$LF=1-\exp\left(-\frac{(\alpha-1)}{\mu}\left[\frac{(1-t)\beta k\theta}{\alpha\left(1-\beta\right)}\right]\right),$$

while the expressions for unemployment and unemployment are the same as in the case without financing.

**Partial Financing** For non-eligible workers, tightness is given by

$$\frac{k\left(r+\phi\right)}{m\theta_n^{-\eta}}=\left(1-\beta\right)y_n-\left(\alpha-1\right)\frac{1-\beta}{1-t}\left[\frac{(1-t)\beta k\theta_n}{\alpha\left(1-\beta\right)}\right]^{\frac{\alpha}{\alpha-1}},$$

and the wage is given by

$$w_n=\beta\left(y_n+ks_n\theta_n\right)-\frac{1-\beta}{1-t}s_n^{\alpha},$$

while the other expressions are the same as in the total financing case.

For eligible workers, tightness is given by

$$\frac{k\left(r+\phi\right)}{m\theta_e^{-\eta}}\left(\frac{1-\beta t}{1-\beta}\right)=\left(y_e+\frac{b}{\rho}\frac{1+\phi\left[\frac{(1-t)\beta k\theta_e}{\alpha(1-\beta)}\right]^{-\frac{1}{\alpha-1}}m^{-1}\theta_e^{\eta-1}}{1-\exp\left(-\frac{\alpha-1}{\mu}\left[\frac{(1-t)\beta k\theta_e}{\alpha(1-\beta)}\right]^{\frac{\alpha}{\alpha-1}}\right)}\right)+(1-\alpha)\left[\frac{(1-t)\beta k\theta_e}{\alpha\left(1-\beta\right)}\right]^{\frac{\alpha}{\alpha-1}}.$$

The wage is given by

$$w_e = \frac{\beta(1-t)}{1-\beta t}\left(y_e + ks_e\theta_e\right) - \frac{1-\beta}{1-\beta t}\left[\frac{b}{\rho e_e} + s_e^{\alpha}\right],$$

while the other expressions are the same as in the total financing case. The labour force participation rate for the economy as a whole is given by

$$LF = \rho LF_e + (1-\rho)\, LF_n,$$

while total employment is given by

$$E = \rho e_e + (1-\rho)\, e_n.$$

The unemployment rate for the economy as a whole is

$$u = \frac{\rho LF_e u_e + (1-\rho)\, LF_n u_n}{LF},$$

and the average wage is

$$w = \frac{\rho e_e w_e + (1-\rho)\, e_n w_n}{E}.$$

**Fixed wages** Expression (15) becomes

$$m\theta^{1-\eta}(\alpha-1)s^{\alpha} + \alpha s^{\alpha-1}(r+\phi) - m\theta^{1-\eta}\left(\tilde{w} + IWB\right) = 0,$$

while expression (16) is given by

$$LF = 1 - \exp\left(-\frac{1}{\mu} sm\theta^{1-\eta}\frac{\tilde{w} + IWB + s^{\alpha}}{r+\phi+sm\theta^{1-\eta}} + \frac{1}{\mu}s^{\alpha}\right).$$

**Calibration** Parameters $k, \phi, m$ are set to replicate an unemployment rate of $\bar{u}$, an average duration of unemployment of $d_u$ months, and an average duration of a vacancy of $d_v$ months in the absence of in-work benefits. Unemployment is given by (32), so that

$$\frac{\phi}{\phi + sm\theta^{1-\eta}} = \bar{u}.$$

Expected duration of unemployment is given by

$$\frac{1}{s\lambda(\theta)} = \frac{1}{sm\theta^{1-\eta}} = d_u.$$

Expected duration of a vacancy is given by

$$\frac{1}{q(\theta)} = \frac{1}{m\theta^{-\eta}} = d_v.$$

Substituting the value of the expected duration of unemployment in the expression for unemployment, we pin down the value of $\phi$:

$$\bar{u} = \frac{\phi}{\phi + \frac{1}{d_u}} \Longleftrightarrow \phi = \frac{\bar{u}}{d_u(1-\bar{u})}.$$

Taking the ratio of the expected duration of unemployment and of a vacancy we have

$$\frac{d_v}{d_u} = s\theta.$$

Substituting from (30) we get

$$\frac{d_v}{d_u} = \left[\frac{\beta k}{\alpha(1-\beta)}\right]^{\frac{1}{\alpha-1}} \theta^{\frac{\alpha}{\alpha-1}} \Longleftrightarrow \theta = \left(\frac{d_v}{d_u}\right)^{\frac{\alpha-1}{\alpha}} \left[\frac{\alpha(1-\beta)}{\beta k}\right]^{\frac{1}{\alpha}}.$$

Taking (31) with $IWB = 0$ and substituting we get

$$\theta = \frac{\alpha(1-\beta)^{\frac{1}{\alpha}}}{\beta k} \left[\frac{y(1-\beta) - k(r+\phi)d_v}{\alpha - 1}\right]^{\frac{\alpha-1}{\alpha}}.$$

The two expressions together imply

$$k = \frac{(1-\beta)}{\left(1-\frac{1}{\alpha}\right)\beta\left(\frac{d_v}{d_u}\right) + (r+\phi)d_v} y.$$

The corresponding tightness is given by substituting $k$ into one of the two above expressions, i.e.

$$\theta = \left(\frac{d_v}{d_u}\right)^{\frac{\alpha-1}{\alpha}} \left[\frac{(\alpha-1)\beta\left(\frac{d_v}{d_u}\right) + \alpha(r+\phi)d_v}{\beta y}\right]^{\frac{1}{\alpha}},$$

while $m$, the matching function scale parameter, is given by

$$\frac{1}{m\theta^{-\eta}} = d_v \Longleftrightarrow m = \frac{\theta^{\eta}}{d_v},$$

and $s$, search intensity, by

$$s = \frac{d_v}{\theta d_u}.$$

The chosen parameter values plus the calibration of an unemployment rate of 0.06, an average duration of unemployment of three months, and an average duration of a vacancy of one month in the case without in-work benefits imply a separation rate $\phi = 0.0213$ (equivalent to an annual separation rate of 0.255), a vacancy cost $k = 4.5616$, with the corresponding tightness $\theta = 0.3823$, the scale parameter of the matching function $m = 0.6807$, while search is given by $s = 0.8719$. Using (33), we get the distribution parameter as

$$\mu \ s.t. \ F\left(\frac{s\beta k\theta}{(1-\beta)} - s^{\alpha}; \mu\right) = \bar{L}$$

which, in case of an exponential distribution, is equivalent to

$$\mu = \frac{1}{\ln\left(\ 1-\bar{L}\right)}\left(\frac{s\beta k\theta}{(1-\beta)} - s^{\alpha}\right)$$

and gives a value of $\mu = 0.631$ for a labour force participation without in-work benefits equal to 0.7.

Figure 1: No financing (variables as a function of in-work benefit - solid line: flexible wage; dotted line: fixed wage)

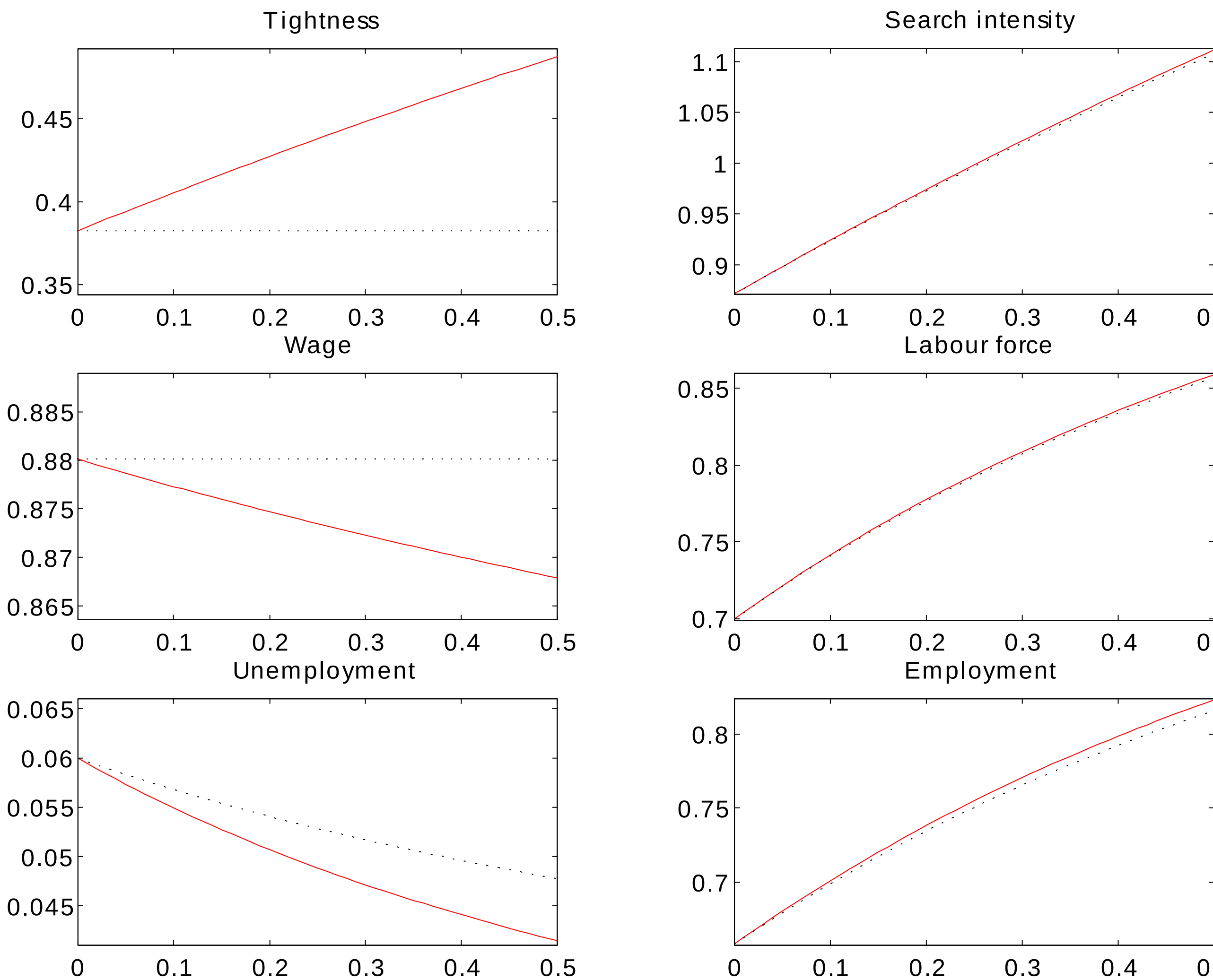

Figure 2: Full financing (variables as a function of in-work benefit - solid line: flexible wage; dotted line: fixed wage)

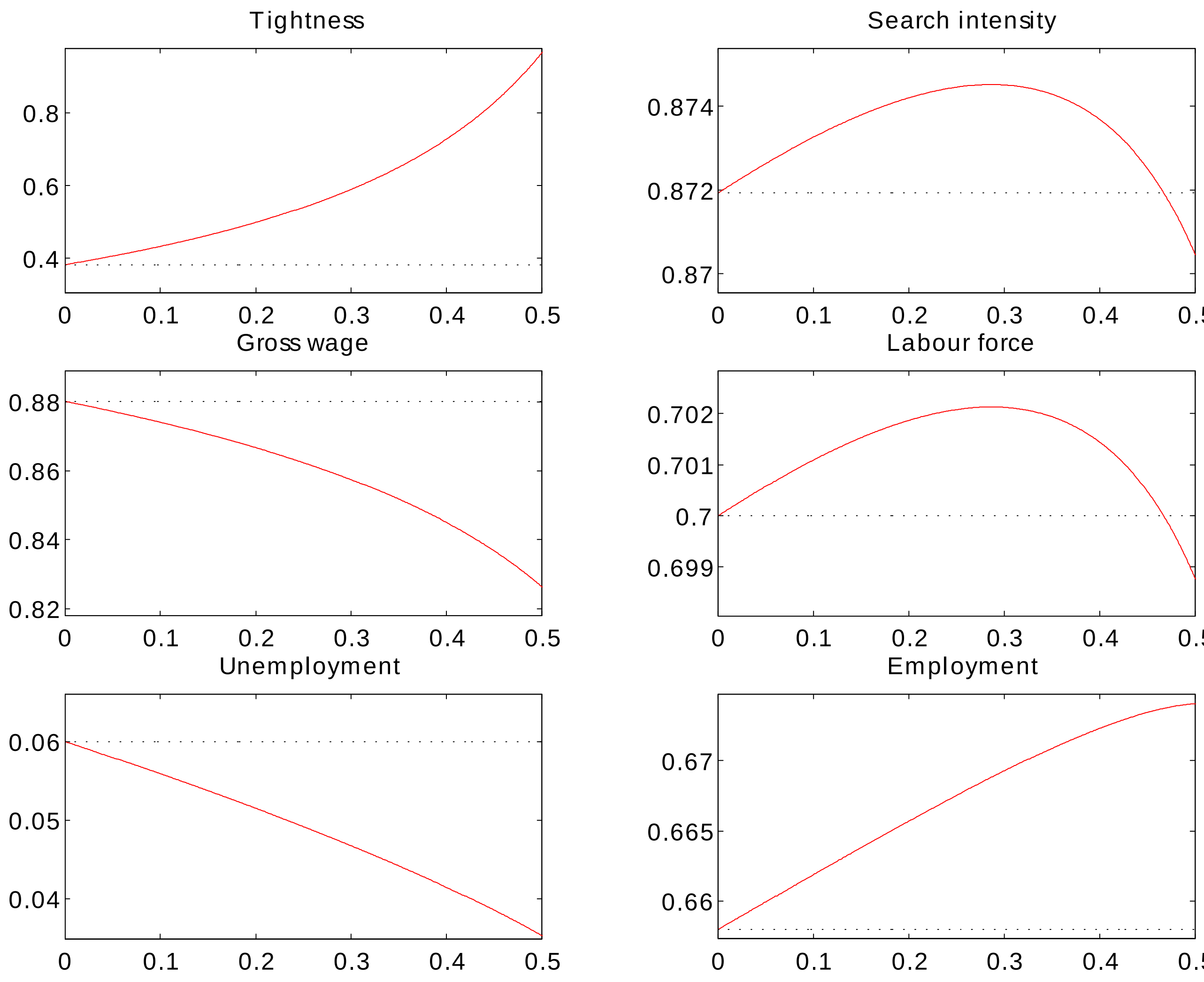

Figure 3: Partial financing - Eligible workers (variables as a function of IWB - dashed: $\rho$=0.25, dashdot: $\rho$=0.5, dotted: $\rho$=0.75, solid: no financing and full financing).

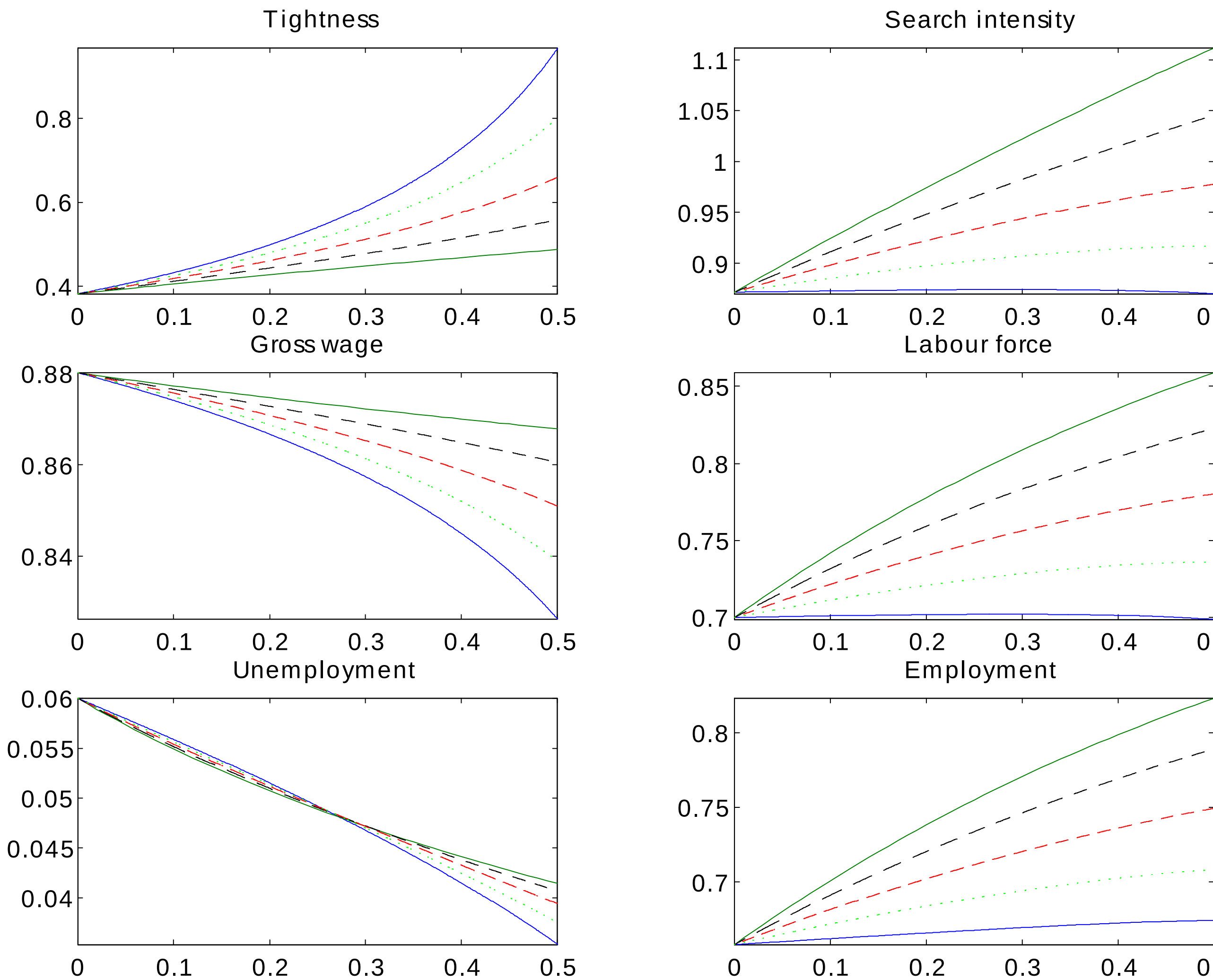

Figure 4: Partial financing - Non-eligible workers (variables as a function of IWB - dashed: $\rho$=0.25, dashdot: $\rho$=0.5, dotted: $\rho$=0.75)

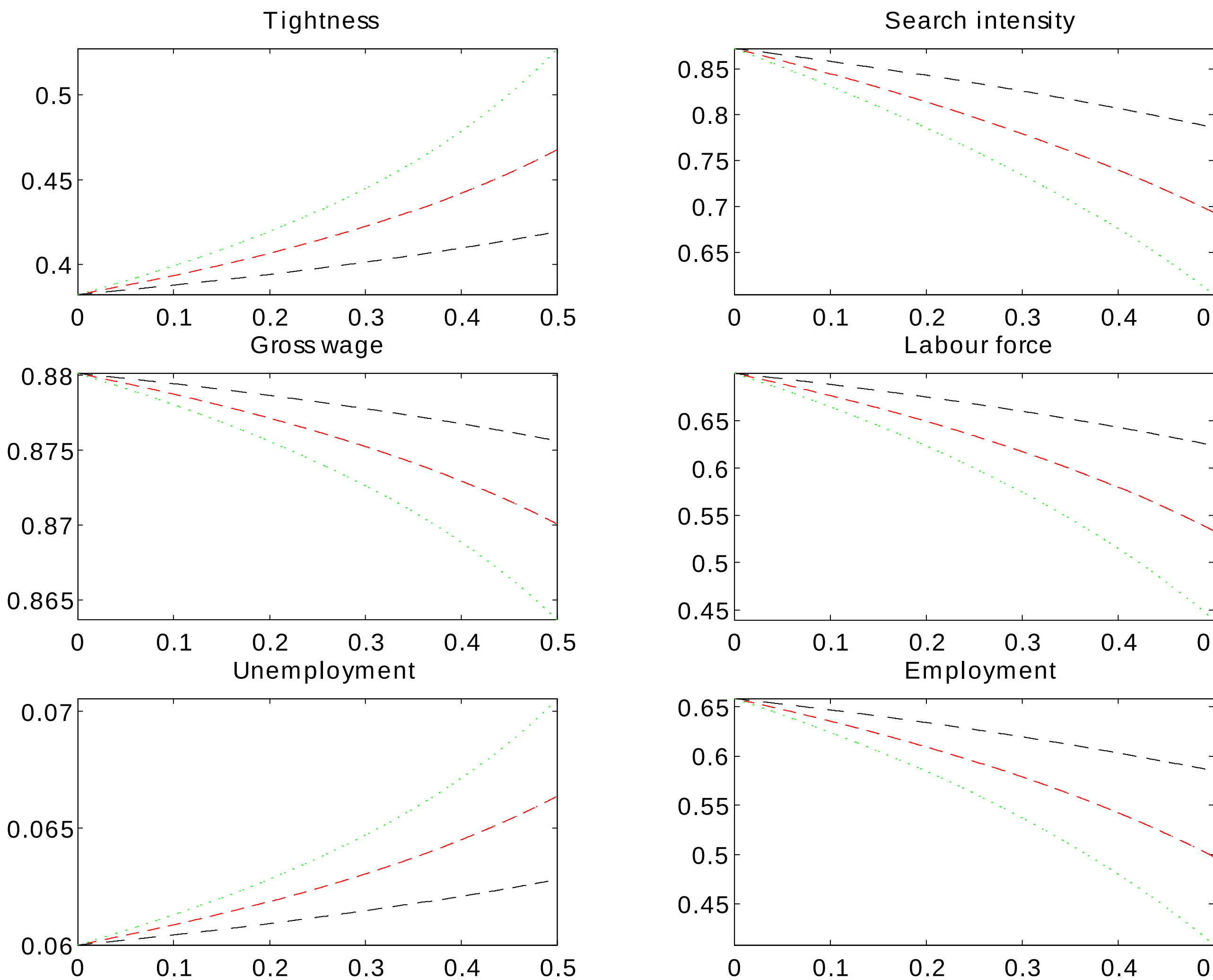

Figure 5: Partial financing - All workers (variables as a function of IWB - dashed: $\rho$=0.25, dashdot: $\rho$=0.5, dotted: $\rho$=0.75, solid: full financing)

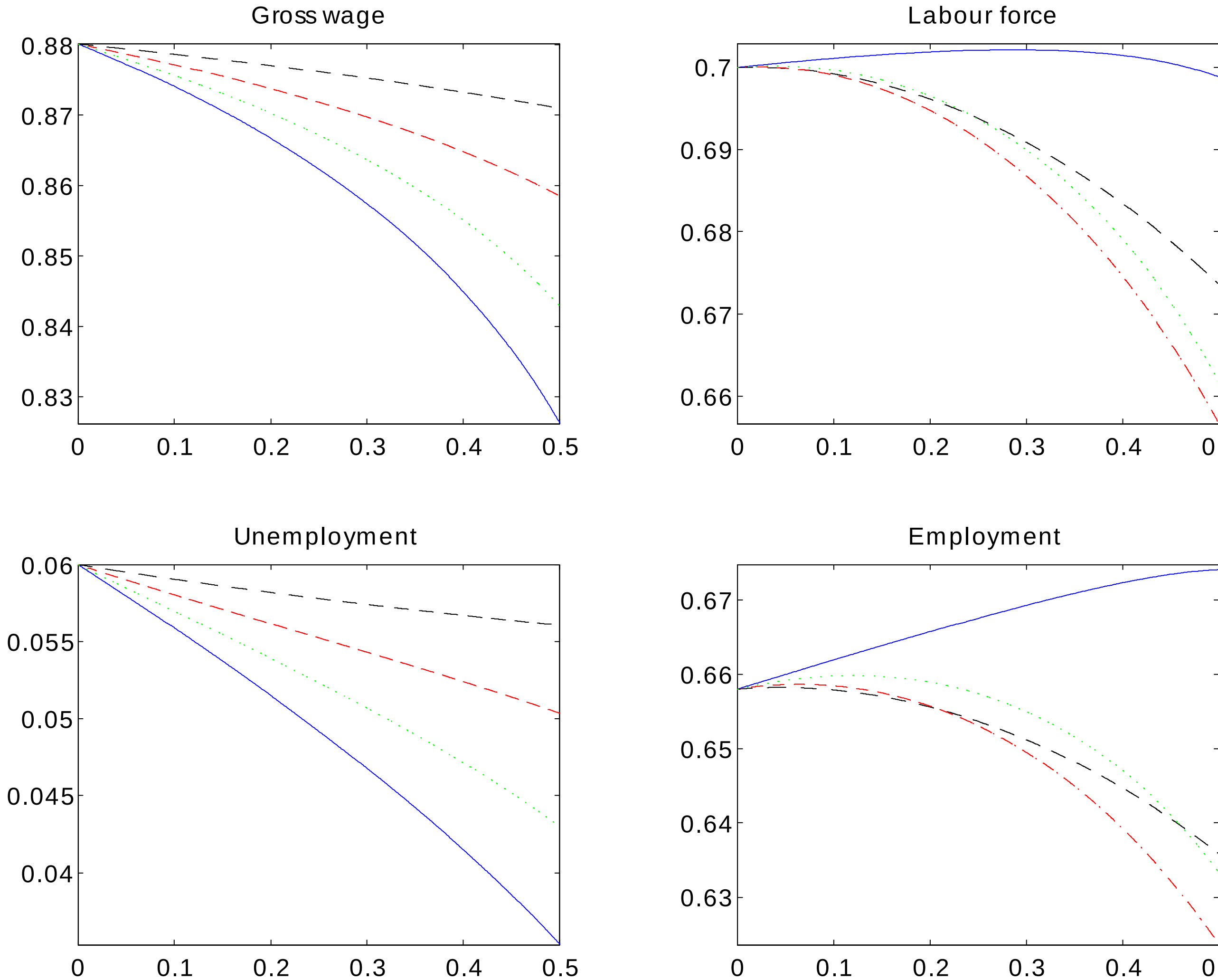

Table 1: Main labour market variables

| | | $t$ | $\theta$ | $s$ | $w$ | $LF$ | $u$ | $e$ |
|---|---|---|---|---|---|---|---|---|
| **IWB=0** | | | 0.382 | 0.872 | 0.880 | 70.0% | 6.00% | 65.8% |
| **IWB=0.2** | | | | | | | | |
| | no financing | | 0.427 | 0.974 | 0.875 | 77.8% | 5.07% | 73.8% |
| | full financing | 23.1% | 0.498 | 0.874 | 0.867 | 70.2% | 5.15% | 66.6% |
| **IWB=0.4** | | | | | | | | |
| | no financing | | 0.468 | 1.068 | 0.870 | 83.6% | 4.41% | 79.9% |
| | full financing | 47.3% | 0.727 | 0.874 | 0.845 | 70.1% | 4.15% | 67.2% |
| **Partial financing - $y_n = 1$** | | | | | | | | |
| **IWB=0.2** | | | | | | | | |
| | eligible | | 0.443 | 0.948 | 0.873 | 75.9% | 5.10% | 72.0% |
| $\rho = 0.25$ | non-eligible | 6.3% | 0.394 | 0.843 | 0.879 | 67.5% | 6.09% | 63.4% |
| | all | | – | – | 0.877 | 69.6% | 5.82% | 65.6% |
| | eligible | | 0.461 | 0.922 | 0.871 | 74.0% | 5.12% | 70.2% |
| $\rho = 0.5$ | non-eligible | 12.3% | 0.407 | 0.814 | 0.877 | 64.9% | 6.19% | 60.9% |
| | all | | – | – | 0.874 | 69.5% | 5.62% | 65.6% |
| | eligible | | 0.479 | 0.898 | 0.869 | 72.1% | 5.14% | 68.4% |
| $\rho = 0.75$ | non-eligible | 17.9% | 0.419 | 0.785 | 0.876 | 62.4% | 6.28% | 58.4% |
| | all | | – | – | 0.870 | 69.7% | 5.39% | 65.9% |
| **IWB=0.4** | | | | | | | | |
| | eligible | | 0.515 | 1.015 | 0.865 | 80.4% | 4.38% | 76.9% |
| $\rho = 0.25$ | non-eligible | 13.7% | 0.410 | 0.807 | 0.877 | 64.3% | 6.21% | 60.3% |
| | all | | – | – | 0.873 | 68.3% | 5.67% | 64.5% |
| | eligible | | 0.575 | 0.963 | 0.859 | 76.9% | 4.33% | 73.6% |
| $\rho = 0.5$ | non-eligible | 26.6% | 0.442 | 0.740 | 0.873 | 58.0% | 6.45% | 54.2% |
| | all | | – | – | 0.865 | 67.5% | 5.24% | 63.9% |
| | eligible | | 0.648 | 0.914 | 0.852 | 73.4% | 4.25% | 70.3% |
| $\rho = 0.75$ | non-eligible | 38.1% | 0.479 | 0.676 | 0.869 | 51.5% | 6.71% | 48.0% |
| | all | | – | – | 0.855 | 67.9% | 4.72% | 64.7% |

a. $y_e = 1$, $y_n = 1$.

Table 2: Main labour market variables - Partial financing - High productivity

| | | $t$ | $\theta$ | $s$ | $w$ | $LF$ | $u$ | $e$ |
|---|---|---|---|---|---|---|---|---|
| **IWB=0** | | | | | | | | |
| | eligible | | 0.382 | 0.872 | 0.880 | 70.0% | 6.00% | 65.8% |
| | non-eligible | | 0.575 | 1.311 | 1.859 | 93.4% | 3.22% | 90.4% |
| $\rho = 0.25$ | all | | | | 1.668 | 87.6% | 3.77% | 84.3% |
| $\rho = 0.5$ | all | | | | 1.447 | 81.7% | 4.41% | 78.1% |
| $\rho = 0.75$ | all | | | | 1.188 | 75.9% | 5.14% | 72.0% |
| **IWB=0.2** | | | | | | | | |
| | eligible | | 0.434 | 0.964 | 0.874 | 77.0% | 5.08% | 73.1% |
| $\rho = 0.25$ | non-eligible | 2.6% | 0.582 | 1.293 | 1.858 | 92.9% | 3.24% | 89.9% |
| | all | | - | - | 1.648 | 88.9% | 3.64% | 85.7% |
| | eligible | | 0.444 | 0.948 | 0.873 | 75.9% | 5.10% | 72.0% |
| $\rho = 0.5$ | non-eligible | 6.3% | 0.593 | 1.267 | 1.857 | 92.1% | 3.26% | 89.1% |
| | all | | - | - | 1.417 | 84.0% | 4.09% | 80.6% |
| | eligible | | 0.461 | 0.923 | 0.871 | 74.0% | 5.12% | 70.2% |
| $\rho = 0.75$ | non-eligible | 12.2% | 0.612 | 1.226 | 1.855 | 90.7% | 3.31% | 87.7% |
| | all | | - | - | 1.160 | 78.2% | 4.59% | 74.6% |
| **IWB=0.4** | | | | | | | | |
| | eligible | | 0.486 | 1.046 | 0.868 | 82.3% | 4.40% | 78.7% |
| $\rho = 0.25$ | non-eligible | 5.6% | 0.591 | 1.273 | 1.857 | 92.3% | 3.26% | 89.3% |
| | all | | - | - | 1.633 | 89.8% | 3.52% | 86.7% |
| | eligible | | 0.515 | 1.016 | 0.865 | 80.5% | 4.38% | 77.0% |
| $\rho = 0.5$ | non-eligible | 13.5% | 0.616 | 1.216 | 1.855 | 90.4% | 3.32% | 87.4% |
| | all | | - | - | 1.391 | 85.4% | 3.82% | 82.2% |
| | eligible | | 0.570 | 0.966 | 0.859 | 77.2% | 4.33% | 73.9% |
| $\rho = 0.75$ | non-eligible | 25.7% | 0.663 | 1.124 | 1.851 | 86.5% | 3.43% | 83.5% |
| | all | | - | - | 1.131 | 79.5% | 4.09% | 76.3% |

a. $y_e = 1$, $y_n = 2$.