Time limits in a two-tier unemployment benefit scheme under involuntary unemployment

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Abstract

The consequences of introducing or tightening time limits on receiving high unemployment benefits are studied in a shirking model. Stricter time limits have an ambiguous impact on the net wage, and changes of utility levels of employed workers and recipients of high unemployment benefits have the same sign as the variation in the net wage. The utility differential between the two groups of unemployed shrinks. The relative income position of skilled workers moves in the same direction as the net wage of unskilled workers. When access to high benefits is denied for caught shirkers, stricter time limits may decrease employment.

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1 Introduction

In many countries, unemployment benefits are organized as a two-tier scheme. If a worker is dismissed, he has access to unemployment insurance benefits for an eligibility period which may depend on individual characteristics. When the time limit is reached, the unemployed is moved to another program, such as unemployment assistance or social assistance, and benefits are reduced. A similar structure can sometimes be found in pure welfare schemes. The most prominent example is the Personal Responsibility and Work Opportunity Reconciliation Act in the United States. It states that, starting from 1996, nobody is eligible for receiving welfare payments based on the federal Temporary Assistance for Needy Families program for more than five years in lifetime and two years per spell. Should time run out, there is no further access to federal cash benefits, but only to some state-financed food stamp program in order to guarantee physical subsistence.

This paper focuses on the impact of introducing time limits on unemployment benefits on employment, wages, profits, and utility levels of both employed and unemployed workers. In contrast to the bulk of the empirical literature, which mainly addresses incentives for labor supply (see Blank, 2002, Lalive et al., 2004), a framework with involuntary unemployment is chosen. The contribution can be seen as complementing earlier analyses in which the success of welfare reform depends on changes of the behavior of recipients. If jobs are easily available, the reason for potential increases in employment is straightforward. Recipients of unemployment benefits will reduce their reservation wages when faced with a drop in their income. Under involuntary unemployment, labor demand will respond to changes in the incentive structure of the employed who see unemployment as a more severe threat now. New job opportunities are created affecting the well-being of forward-looking welfare recipients. These general equilibrium effects may compensate them for the time limits imposed.

Some evidence that the decline in welfare caseloads is actually driven by labor demand can be found in the empirical literature. For example, Ziliak et al. (2000) estimate that
about two thirds of the caseload decline in the Aid for Families With Dependent Children program observed between 1993 and 1996 is explained by macroeconomic factors represented, e.g., by lower unemployment rates. Only one third can be attributed to several regional welfare reforms adopted in various states. Similarly, Arulampalam and Stewart (1995) find for the UK that the effect of unemployment income on the individual hazard rate to exit from unemployment is substantially lower in periods of high unemployment, while demand-side factors captured by the local unemployment rate have a strong negative effect on the exit probability. Hence, an analysis of the effects of a time-limits reform is needed for a framework in which labor demand plays a decisive role while search effort is of secondary importance. In the light of our approach, the fall in unemployment in the U.S. should not be viewed as indicating some exogenous business cycle phenomenon. It may at least partially be traced back to stricter welfare eligibility rules that enable firms to cut wages.

We analyze an efficiency wage model where workers may shirk. The two levels of benefits for the unemployed are called unemployment benefits and social assistance. As we are mainly interested in studying the effects of changing time limits, we take the relevant benefit levels as given. Both types of benefits are financed by a proportional income tax. In the basic model, all individuals are identical with respect to ability and preferences.

It is shown that imposing a stricter time limit on receiving unemployment benefits increases employment. Since unemployment is made less comfortable, employers can cut gross wages and raise employment without having to fear that workers lose their incentive to exert effort. With a smaller number of unemployed, the tax burden tends to fall. However, since the average duration of unemployment falls with an increasing employment level, the share of short-term unemployed receiving full benefits may go in either direction. In some special cases the increase in short-term unemployment might be more costly than the decrease in long-term unemployment and hence lead to an increase in
overall benefit payments. Apart from this special case, the tax rate is generally expected to fall if the time limit is shortened. This enables firms to lower gross wages even further and to hire even more workers.

Imposing a stricter time limit will generally raise net profits due to falling gross wages and a lower tax rate. The impact on net wages is ambiguous. Expected lifetime utility levels of employed workers and unemployment benefit recipients will move in the same direction as net wages. The utility differential between the two groups remains constant because it is determined by the structure of incentives for employed workers. Recipients of unemployment benefits are compensated for the risk of losing parts of the welfare benefit by improved job opportunities. Those who would be on social assistance anyway may gain in utility even if the net wage declines because their prospects of getting a job are improved. The result indicates that imposing stricter time limits can even lead to a Pareto improvement.

Extending the analysis to a richer skill structure of the workforce, skilled workers will earn a higher wage and face a lower unemployment rate. We consider a benchmark scenario in which the two types of workers are perfect substitutes. Although skilled workers subsidize benefits for unskilled workers, the impact on welfare of the two groups is almost symmetric, and confirms the results from the basic model. The utility differential between skilled and unskilled workers of a given employment status moves in the same direction as the net wage of unskilled workers.

Finally, we analyze a variation in which shirkers who are dismissed do not have access to unemployment benefits. As a stricter time limit implies a higher probability for non-shirkers to lose the high-level unemployment benefits and drop down to the lower benefit level of social assistance, the value of not shirking in the workplace is reduced. Thus, tightening the time limit makes shirking more and not less attractive, which tends to require a higher wage associated with lower employment. At the same time, the smaller benefit per unemployed allows a cut in the tax rate which tends to lower the gross wage
and to increase employment. The overall employment effect is ambiguous. It turns out that the change in employment is decisive for the impact on utility levels of employed workers and social assistance recipients. Compared to these two groups, the relative utility position of short-term unemployed definitely deteriorates.

Our contribution is to some extent related to the theory of optimum unemployment insurance. A basic proposition of this literature states that payments should stay constant over time if the unemployed cannot influence their chances of gaining a job. In contrast, if the re-employment opportunities are determined by unobservable search efforts, expected utility of the beneficiaries is maximized by a declining benefit schedule that converges to zero (Shavell and Weiss, 1979). Hopenhayn and Nicolini (1997) argue that welfare can be further increased in such a moral hazard scenario if the personal wage tax increases with the duration of unemployment. However, Cahuc and Lehmann (2000) show that declining unemployment benefits may even lead to a higher unemployment rate, since insiders will drive up their wage demands when expecting a shorter period of unemployment upon losing their job. Fredriksson and Holmlund (2001) demonstrate within an equilibrium search model that the optimum time limit for receiving the higher benefit in a two-tier unemployment insurance system is always positive and finite. The optimum time limit exceeds zero because the search effort of those receiving the smaller benefit increases in the duration of the full benefit. Hassler and Rodriguez Mora (2002) argue that unemployment benefits should even increase over time if the insurer cannot observe consumption and savings. Individuals then prefer to finance short spells of unemployment by precautionary savings. Wang and Williamson (1996, 2002) add job-retention efforts of employed workers as a second source of moral hazard. They derive non-monotonic optimum benefit schedules and stress that the optimum scheme depends on the worker’s employment history.

Rather than deriving an optimum benefit schedule, we take as fixed the benefit levels in a system with at most two tiers, focusing on the distributional and welfare consequences of varying a stochastic time limit. This approach is justified in our analysis of a shirking
model based on the result that, even with highly risk averse agents, the optimum scheme
may simply consist of a minimum level of benefits, say zero, from the very beginning (Fath
and Fuest, 2005). Nevertheless, it is interesting to look at the distributional consequences
of a labor market reform that changes only the time limit that is relevant for a substantial
reduction in benefit entitlements.

The remainder of the paper is organized as follows. The basic model is introduced
in Section 2. Section 3 analyzes problems of existence and stability of equilibria. Com-
parative static results are derived in section 4. Section 5 introduces heterogeneity in
productivity across agents into our basic framework, and section 6 deals with a struc-
ture where shirkers who are dismissed are not entitled to receive unemployment benefits.
Section 7 concludes, discusses the findings and indicates directions for future research.

2 The basic model

The model is based on Shapiro and Stiglitz (1984). We consider \( N \) identical workers whose
preferences are described by the utility function \( U(\omega, e) = \omega - e \), where \( \omega \) denotes the
monetary compensation and \( e \) is the effort exerted at the workplace. With probability \( b \)
per unit of time, an employment relationship breaks down for exogenous reasons. Workers
are infinitely lived and maximize \( W = E \int_0^\infty U(\omega(s), e(s)) \exp(-rs)ds \), where \( s \) denotes
time, \( r > 0 \) is the discount rate, and \( E \) represents the expectations operator. Employed
workers can either shirk \((e = 0)\) or choose the required effort \((e = 1)\). Workers who
are shirking are detected with probability \( q \) per unit of time. Detected shirkers are fired
immediately. All individuals are identical with respect to ability and preferences. We
ignore all issues arising from savings and means tests to qualify for a welfare program.

The unemployed first receive an unemployment benefit \( \bar{w} \) until the time limit is ex-
hausted. The others get \( w \), henceforth called social assistance, where \( \bar{w} > w \geq 0 \). In many
existing two-tier schemes of unemployment compensation, the upper tier is represented
by unemployment insurance benefits, while the lower tier is often some welfare program.
Reinterpreted for an American-style welfare system, the higher benefit is meant to provide some minimum level of income above the physical subsistence level, while the lower benefit may represent a food-stamp program. Initially, both shirkers and non-shirkers have access to high unemployment benefits. In the basic model we ignore the fact that in many unemployment compensation schemes access to high benefits is denied in some cases of behavioral misconduct, a point stressed by Atkinson (1995). The benefits are financed by a proportional tax on wages and profits, the tax rate being \( t \).

Let \( V^\sigma_\varepsilon \), \( V^\rho_\varepsilon \), and \( V_u \) denote expected lifetime utility of employed shirkers, employed non-shirkers, and unemployed individuals receiving the full amount of unemployment benefits, respectively. The asset equations for shirkers and non-shirkers are given by

\[
rv^\sigma_\varepsilon = (1-t)w + (b + q)(V_u - v^\sigma_\varepsilon) \tag{1}
\]

and

\[
rv^\rho_\varepsilon = (1-t)w - e + b(V_u - v^\rho_\varepsilon) \tag{2}
\]

with \( w \) denoting the gross wage.

These asset equations have the structure that the return in a given period is equal to the flow benefits plus the expected change of the value of the asset. An employed worker will not shirk if \( v^\sigma_\varepsilon \leq v^\rho_\varepsilon \), which is equivalent to

\[
(1-t)w \geq rv_u + \frac{r + b + q}{q}e, \tag{3}
\]

the no-shirking condition. If workers were risk averse, a lower wage as the one given in (3) would be sufficient to induce effort. Such a modification would, however, not lead to substantially different results.

Firms are operating under decreasing returns. Output of the representative firm is given by \( Q = F(L) \) where \( L \) denotes effective labor, i.e. the number of employed workers who are not shirking. The production function satisfies \( F'(L) > 0, F''(L) < 0 \) and \( F'(N) > e \). The last assumption implies that full employment would be efficient.
An unemployed worker receiving benefit $\bar{w}$ will get a job with probability $a$ per unit of time. Rather than introducing a fixed time limit, the loss of the full benefit is modeled in a stochastic fashion. In a given period, access to regular benefits is lost with probability $h$. The hazard rate $h$ corresponds to an expected time limit and is seen as a policy variable. If $h = 0$, there is no time limit imposed on unemployment benefits. The asset equation of an unemployed worker receiving regular benefits is

$$rV_u = \bar{w} + a(V_u - V_u) + h(V_z - V_u),$$  \hspace{1cm} (4)$$

where $V_u = \max \{V_u^r, V_u^p\}$, and $V_z$ denotes expected lifetime utility of a social assistance recipient. We ignore the possibilities that firms are reluctant to hire long-term unemployed or previously detected shirkers. Therefore, the job acquisition rate is independent of the unemployment status. The asset equation of a welfare recipient reads

$$rV_z = \bar{w} + a(V_u - V_z).$$  \hspace{1cm} (5)$$

If not shirking is optimal, (2), (4), and (5) can be solved. Combining (4) and (5) yields

$$(r + h + a)(V_u - V_z) = \bar{w} - \bar{w},$$  \hspace{1cm} (6)$$

and subtracting (4) from (2) leads to

$$(r + b + a)(V_u - V_z) = (1 - t)w - e - \bar{w} + h(V_u - V_z).$$  \hspace{1cm} (7)$$

Solving the last two equations for the lifetime utility differentials, it turns out that

$$V_u - V_u = \frac{(1 - t)w - e - (r + a)\bar{w} + hw}{r + a + b},$$  \hspace{1cm} (8)$$

$$V_u - V_z = \frac{\bar{w} - \bar{w}}{r + h + a}.$$  \hspace{1cm} (9)$$

This implies

$$rV_u = \frac{a}{r + a + b} [(1 - t)w - e] + \frac{r + b}{r + b + a} \frac{(r + a)\bar{w} + hw}{r + h + a},$$  \hspace{1cm} (10)$$
\[ rV_e = \frac{r + a}{r + a + b} [(1 - t)w - e] + \frac{b}{r + b + a} \frac{(r + a)\bar{w} + hw}{r + h + a}, \]  
\[ rV_z = \frac{rw + arV_e}{r + a}. \]

Inserting (10) into the no-shirking condition yields

\[ (1 - t)w \geq \frac{r + a + b + q}{q} e + \frac{(r + a)\bar{w} + hw}{r + h + a}. \]

Inducing workers not to shirk requires a higher wage \( w \) if either type of benefits, \( \bar{w} \) or \( w \), rises, the rate of exogenous splits \( b \) increases, the probability of finding a new job \( a \) goes up, the tax rate \( t \) increases, the rate of time preference \( r \) rises, or the quality of monitoring effort, measured by \( q \), falls. Equations (8) and (13) imply that employed workers will enjoy a higher expected remaining lifetime utility than those being unemployed at any given point in time. Thus, unemployment is involuntary. Compared to those receiving full benefits \( \bar{w} \), employed workers earn the information rent \( e/q \). The rent arises due to the fact that the monitoring technology is imperfect, that is, \( q \) is finite.

In equilibrium, the number of entries into unemployment must be equal to the number of exits,

\[ a(N - L) = bL. \]  

Similarly, the number of entries into social assistance has to be equal to the number of exits,

\[ a(N - L - U) = hU. \]

While \( U \) individuals receive unemployment benefits, \( N - L - U \) individuals participate in the welfare program. Last, the number of entrants into unemployment benefits is equal to the number of leavers,

\[ bL = (a + h)U. \]

As \( a = b \frac{L}{N - L} \), substituting for \( a \) from (14) into (13) leads to

\[ (1 - t)w \geq \frac{(r + q)(N - L) + bN}{q(N - L)} e + \frac{(r(N - L) + bL)\bar{w} + h(N - L)w}{(r + h)(N - L) + bL}. \]
Unemployment benefits are financed through a proportional income tax \( t \) on profits and labour income. The tax base is, therefore, equal to total output \( F(L) \), implying that the government budget equation reads

\[
tF(L) = \overline{w}U + \underline{w}(N - L - U). \tag{18}
\]

If workers do not shirk, the representative firm will set its labor input to the point where the marginal product of labor is equal to the gross wage, that is, where \( \underline{w} = F'(L) \).

Utilizing this relationship and building on (14) and (16), \( U = \frac{bL(N - L)}{bL + h(N - L)} \) implies that feasible allocations require

\[
(1 - t)\underline{w} = F'(L) \left[ 1 - \frac{bL\overline{w} + h(N - L)\underline{w}(N - L)}{bL + h(N - L)} \frac{(N - L)}{F(L)} \right]. \tag{19}
\]

All relevant decisions are taken simultaneously. The government always adjusts the income tax rate instantaneously so as to balance its budget. Firms generally take as given both the wage and the tax rate and choose employment in order to maximize their profits. They are willing to accept underbidding by unemployed workers as long as net wages are higher than necessary to satisfy the no-shirking constraint. Conversely, should the net wage be too low to prevent workers from shirking, firms will increase the gross wage. Taking as given wages, policy variables and the unemployment rate, employed workers choose whether or not to shirk.

3 **Equilibria and stability**

An equilibrium is described by a level of employment that satisfies both the no-shirking condition (17), now with equality, and the feasibility condition (19). The right-hand side of the no-shirking condition (17) is equal to \( \frac{r + b + q}{q} e + \frac{\overline{w} + h\underline{w}}{r + h} > 0 \) at \( L = 0 \). It increases in \( L \) and tends to infinity if \( L \to N \). Note that the right-hand side of the feasibility condition (19) will be equal to \( F'(N) > 0 \) if \( L = N \). Moreover, provided that
$F(0) = 0$, an employment level $L_0 \in (0, N)$ exists which satisfies
\[
\left[ 1 - \frac{bL \bar{w} + h (N - L) \bar{w} (N - L)}{bL + h (N - L) F(L)} \right] = 0.
\]

Figure 1 illustrates the equilibrium conditions. The no-shirking condition holds on and above the NSC curve, while the feasibility curve $G$ represents the budget constraint of the government combined with the marginal productivity rule of pay. If the two curves intersect, and if we neglect the possibility of a tangent point, at least two equilibria exist.

Insert Figure 1 about here

Rewriting (17) (with equality) and (19) shows that an equilibrium has to satisfy
\[
f_1((1-t)w, L) = (1-t)w - \frac{r + q e}{q} - \frac{bN}{q (N - L)}e - \frac{(r (N - L) + bL) \bar{w} + h (N - L) \bar{w}}{(r + h) (N - L) + bL},
\]
\[
f_2((1-t)w, L) = (1-t)w - F'(L) \left[ 1 - \frac{bL \bar{w} + h (N - L) \bar{w} N - L}{bL + h (N - L) F(L)} \right].
\]

where $f_1 = f_2 = 0$. The dynamic evolution of the two key variables is given by $[(1 - t)w] = \Phi_1(f_1((1-t)w, L))$ and $\dot{L} = \Phi_2(f_2((1-t)w, L))$ with $\Phi_1(0) = \Phi_2(0) = 0$, $\Phi'_1 < 0$, and $\Phi'_2 < 0$. The first differential equation expresses that wages will be cut if the no-shirking
constraint is not binding, while there is a wage increase when the no-shirking condition is not satisfied. The second equation of motion implies that employment will be reduced if the gross wage exceeds the marginal product of labor, and vice versa. For a locally stable equilibrium, it is necessary that the determinant of the Jacobian of the system of differential equations is non-negative at the equilibrium, and that its trace is non-positive. These conditions translate into \( \frac{\partial f_2}{\partial L} - \frac{\partial f_1}{\partial L} > 0 \) and \( \Phi_1'(0) + \Phi_2'(0) \frac{\partial f_2}{\partial L} < 0 \). The former condition requires that the NSC curve cuts the G curve from below.

In Figure 1, the equilibrium at the employment level \( L_1 \) is a saddle point, and therefore unstable. If a point on the G curve between \( L_1 \) and \( L_2 \) is realized, the firm is willing to accept underbidding by unemployed workers. As a consequence, employment will increase and the gross wage rate will fall. Underbidding will no longer be accepted at \( L_2 \) since the no-shirking condition would then be violated. The equilibrium at \( L_2 \) is always a locally stable focus if the G curve is downward sloping at this point. It is still a locally stable focus with an upward sloping G curve if the condition on the trace is met.

4 Changing the benefit loss rate

Proposition 1 summarizes the effects of an increasing risk of losing the full unemployment benefit and having to rely on social assistance.

**Proposition 1** Employment \( L \) increases and the gross wage \( w \) decreases with a higher benefit loss rate (that is, a tighter time limit) \( h \). The lifetime utility differential between employed workers and short-term unemployed, \( V_e - V_u \), remains constant. The lifetime utility differential between unemployment benefit recipients and social assistance recipients, \( V_u - V_z \), shrinks. Lifetime utility levels of employed workers and short-term unemployed move in the same direction as the net wage.

**Proof.** See Appendix A.

\( \square \)
The comparative statics can be interpreted as follows. A higher benefit loss rate is equivalent to a stricter time limit of receiving unemployment benefits. As a consequence, the threat of unemployment becomes more severe. The minimum wage needed to induce effort at the workplace decreases, which corresponds to a downward shift of the NSC curve. Due to a higher share of food stamp recipients at a given level of unemployment, the tax burden decreases. The feasibility curve $G$ shifts upwards for any positive unemployment level. Any given gross wage now corresponds to a higher net wage. The tax reduction thus represents a second channel allowing to cut gross wages and increase employment.

The unemployed are affected by the loss in expected benefits. At the same time, their job opportunities are becoming better. Moreover, their net wage changes once they re-enter employment. For employed workers, the threat of becoming unemployed is more serious now due to a stricter time limit for receiving the full benefit. At the same time, the increasing opportunities of regaining employment work in the opposite direction. In any case, employed workers are also confronted with a variation in their net wage. The lifetime utility differential between employed workers and those receiving unemployment benefits remains unchanged. This result holds because the no-shirking condition implies that the utility differential is exclusively determined by the level of effort exerted at the workplace and the quality of the detection technology. The net impact on per-period utility for each of these groups is determined by the variation in net wages. By contrast, the impact of the stricter time limit on social assistance recipients is more likely to be positive. As it takes two transitions, into and out of employment, before the time limit can bite, current social assistance recipients are least affected by a tighter time limit. They benefit from better opportunities to leave unemployment and, as forward-looking agents, also take into account the change in net wages. In terms of absolute utility differentials, their utility position compared to the other two groups of workers improves.
Tedious computations provided in Appendix B show the response of the equilibrium tax rate to a rising benefit loss rate. The outcome is not obvious because the fall in unemployment will be associated with a smaller share of social assistance recipients among the unemployed. While being quite an implausible scenario, an increase in total unemployment benefits via a rising number of recipients of high-level benefits cannot be ruled out. It can be demonstrated that the equilibrium tax rate is going to fall if either the discount rate or the benefit loss rate are sufficiently small. Therefore, we will ignore the possibility of a perverse tax reaction in the following.

Firms take advantage of the lower gross wage and the lower tax rate. Their net profits must increase according to

\[
\frac{\partial \pi_n}{\partial h} = - [F(L) - LF'(L)] \frac{\partial t}{\partial h} - (1-t)LF''(L) \frac{\partial L}{\partial h}.
\]  

(22)

The impact on the net wage,

\[
\frac{\partial [(1-t)w]}{\partial h} = \frac{\partial w}{\partial h} (1-t) - \frac{\partial t}{\partial h} w,
\]  

(23)

is ambiguous in general and mainly depends on the properties of the production function and the level of unemployment benefits. If the marginal product of labor responds to a higher labor input in an unelastic fashion, the tax reduction dominates the reduction in gross wages, implying a rise in net wages. Conversely, if the change in the marginal product of labor is stronger, while unemployment benefits are relatively small, the overall effect will go in the opposite direction.

Interestingly, the share of social assistance recipients among the unemployed does not necessarily increase. According to equation (15), the ratio between individuals with regular benefits and social assistance recipients is equal to \( \frac{U}{N - L - U} = \frac{a}{h} \). While a stricter time limit (higher \( h \)) directly induces a higher share of welfare recipients, the resulting increase in the employment level is associated with a rising job acquisition rate \( a \). The latter effect reduces the number and the share of those living on social assistance. Hence, if the increase in employment is so strong that the elasticity \( \eta_{a,h} := \frac{h \frac{\partial a}{\partial h}}{a \frac{\partial h}{\partial h}} \) exceeds
unity, a smaller share of welfare recipients among the unemployed will be the result. This is illustrated by an example presented in Appendix C.

Should net wages fall, it may still be the case that introducing time limits wins a political majority. First, the residual income, which can be interpreted as capital income, increases. Provided that there is a sufficiently even distribution of wealth, losses in workers’ expected utility may be offset by gains in capital income. Second, workers may take into account that there is a higher probability to be among the employed under the new framework. A worker taking decisions behind a veil of ignorance – that is, not knowing the realization of his employment status – may therefore opt for the stricter time limit even if this is associated with a utility reduction under all possible employment states. In fact, the higher total production outweighs the additional effort of the workers. With risk-neutral agents deciding under a veil of ignorance, this property effectively calls for abolishing unemployment benefits. At the same time, however, the probability of being faced with the least fortunate state of a social assistance recipient will often increase. If workers are risk averse, it is thus conceivable that a utilitarian government will not simply set the time limit to zero. On the other hand, Fath and Fuest (2005) argue that the impact on the incentives for the employed could be so strong that abolishing all unemployment benefits will still be optimal if agents are extremely risk averse.

With the veil of ignorance removed, the outcome can be reversed. A Pareto improvement may even not be achieved by taxing capital on a lump-sum basis and equally redistributing the proceeds among the workers. Note that this type of redistribution would not affect incentives in the model. Yet, as the share of workers enjoying the highest utility level increases, capital owners and both employed and unemployed workers may lose after redistribution has taken place in such a setting.
5 Heterogeneous labor

Introducing heterogeneous types of labor represents an extension of the model that is potentially useful to shed more light on the distributional implications of tightening eligibility rules for welfare recipients. In particular, skilled workers will typically subsidize unskilled workers through the unemployment compensation system. Due to the higher productivity of skilled workers, the shirking model implies that their unemployment rate is smaller than the unemployment rate of unskilled workers. Further, at a given proportional tax rate, the higher wage per worker is reflected in higher tax payments. A fall in unemployment reduces the subsidy per employed skilled worker in favor of the unskilled unemployed. Another effect arises if skilled and unskilled labor are complements in production. In this case, any reduction in unemployment of one type of labor raises the productivity of the other type of labor, implying a positive impact on the welfare of the other group.

Assume now that firms use skilled labor $S$ and unskilled labor $L$ as inputs in production. Part of the skilled labor force $M$ as well as the stock of unskilled labor $N$ remain unemployed in order to preserve the incentives to deliver effort at the workplace.

We restrict our attention to a benchmark scenario in which skilled and unskilled labor are perfect substitutes. One unit of skilled labor equals $\alpha > 1$ efficiency units of unskilled labor. The production function can be written as $F(L + \alpha S)$ with $F' > 0$ and $F'' < 0$. The gross wage of skilled workers is given by $w_s = \alpha w$ with $w$ denoting the gross wage of unskilled workers. Let eligibility rules for benefits, the monitoring technology, and the exogenous separation rate be independent of qualification.

Denoting the unemployment rate of skilled workers by $\theta := \frac{M - S}{M}$, the no-shirking conditions for unskilled and skilled workers are given by (17) and

$$ (1 - t)\alpha w \geq \bar{w} + e + \frac{r}{q} e + \frac{b}{q} \frac{1}{\theta} - \frac{h(w - w)}{r + h + b \frac{1}{\theta}} $$

respectively. At any given net wage $(1-t)w$, the two inequalities determine the maximum
employment that induces workers not to shirk.

The feasibility condition now reads

\[(1 - t)w = F'(L + \alpha S) \left[1 - \frac{(w - \bar{w})(U + U^S) + w(N + M - L - S)}{F(L + \alpha S)}\right]\]

\[\frac{w}{b(1 - u) + hu} \cdot \frac{N - L}{F(L + \alpha S)}\]

where \(U^S\) is the number of skilled social assistance recipients. At given employment levels \((S, L)\), equation (25) determines the gross wage \(w\) that satisfies the input rule and the tax rate \(t\) that balances the government’s budget.

Any equilibrium \(((1 - t)w, S, L)\) is determined by the three equations

\[g_1 = (1 - t)w - \frac{r}{q} e - \frac{b}{q} \frac{1}{u} + \frac{h(\bar{w} - w)}{r + h + b \frac{1 - u}{u}},\]  

\[g_2 = (1 - t)w - \frac{1}{\alpha} \left[e + \frac{r}{q} e + \frac{b}{q} \frac{1}{\theta} - \frac{h(\bar{w} - w)}{r + h + b \frac{1 - \theta}{\theta}}\right],\]  

\[g_3 = (1 - t)w - F'(L + \alpha S) \left[1 - \frac{w}{b(1 - u) + hu} \cdot \frac{N - L}{F(L + \alpha S)}\right],\]

with \(g_1 = g_2 = g_3 = 0\).

We restrict our attention to interior solutions where both skilled and unskilled workers will be employed. Then, the two no-shirking conditions (26) and (27) immediately imply that \(u > \theta\), that is, the unemployment rate among skilled workers will be smaller than the corresponding rate of unskilled workers. This result is easily understood. The wage differential between skilled and unskilled workers reflects differences in their productivities. Since unemployment benefits are independent of qualification, a smaller level
of equilibrium unemployment is sufficient to restore work incentives for skilled workers. With a varying net wage, equations (26) and (27) also imply that changes in group-specific unemployment rates will always move in the same direction.

It can easily be checked that all the results stated in Proposition 1 regarding the impact of changes in the benefit loss rate on employment, wages, utility levels and utility differentials within each skill group can be replicated here. Hence, a higher benefit loss rate will lead to a reduction in unemployment for each skill group and a fall in gross wages. For each skill group, the lifetime utility differential between employed workers and short-term unemployed remains constant, whereas the utility differential between the short-term unemployed and the long-term unemployed is shrinking. All utility levels of employed workers and short-term unemployed move in the same direction as the net wage of the unskilled. Proposition 2 summarizes the distributional effects across skill groups of an increasing risk of losing the full unemployment benefit and having to rely on social assistance.

**Proposition 2** With a higher benefit loss rate (that is, a tighter time limit) $h$, the lifetime utility differential between skilled employed workers and unskilled employed workers, $V^S_{e} - V_{e}$, moves in the same direction as the net wage of unskilled workers. The same holds for the lifetime utility differential between skilled and unskilled recipients of unemployment benefits, $V^S_{u} - V_{u}$. Lifetime utility levels of employed workers and unemployment benefit recipients move in the same direction as the net wage.

Proof. See Appendix D. □

Proposition 2 states that increasing the benefit loss rate will generally have distributional consequences across skill groups. Both skilled and unskilled workers will benefit from lower unemployment, and both groups experience a falling gross wage. The reduction of the unemployment rate for the unskilled will typically exceed the one for the skilled workers. On the other hand, the subsidies that skilled workers have to pay to the unskilled via the unemployment compensation scheme will go down.
It turns out that the change in the net wage is crucial for determining the relative winners and losers. If the net wage falls, the utility differential between skilled and unskilled workers in a given employment status is shrinking. Conversely, an increasing net wage is associated with an increasing skill premium in terms of higher lifetime utility.

6 Restricted benefit access for shirkers

Let us finally consider an environment in which shirkers cannot claim high benefits. In fact, immediate access to unemployment insurance benefits is in many countries denied for workers who quit voluntarily or who are dismissed due to industrial misconduct (Atkinson, 1995). In this section, taking these rulings as given and enforceable, we modify our model as follows. Dismissed shirkers do not have access to high unemployment benefits, but will immediately receive social assistance. By contrast, those who quit their jobs due to exogenous separations still qualify for unemployment benefits. The asset equation for shirkers then reads

\[ rV^\sigma_e = (1 - t)w + b(V_u - V^\sigma_e) + q(V_z - V^\sigma_z). \]  

(29)

With this alternative rule, the no-shirking condition changes to

\[ (1 - t)w \geq \frac{r + b + q}{q} e + rV_z - b(V_u - V_z). \]  

(30)

Hence, the minimum net wage necessary to induce effort at the workplace increases with a higher level of the value attached to receiving social assistance and falls with an increasing utility differential between the two states of being unemployed. While the former property is quite intuitive and parallel to the basic model, the latter needs some explanation. Although hazard rates of exogenous separation are identical for shirkers and non-shirkers, there is a higher probability of being in the state of receiving high benefits for non-shirkers after some short (finite) period of time. Therefore, a higher utility differential between the two states of unemployment discourages shirking.
The other asset equations are not affected by the modified treatment of shirkers, implying that equations (8) and (9) still represent the utility differentials \( V_e - V_u \) and \( V_u - V_z \). However, as can easily be shown by comparing the asset equations of shirkers and non-shirkers, if the no-shirking condition is satisfied with equality, the utility differential between employed workers and social assistance recipients is now fixed,

\[
V_e - V_z = \frac{e}{q}.
\] (31)

Inserting the solutions of the utility differentials, starting from

\[
(1-t)w - e - \frac{(r+a)\bar{w} + hw}{r + h + a} + \frac{\bar{w} - w}{r + h + a} \geq \frac{e}{q},
\] (32)
the no-shirking condition can be rewritten as

\[
(1-t)w \geq \frac{r + a + b + q}{q} e - \frac{b\bar{w} - (r + a + b + h)w}{r + h + a}.
\] (33)

Using the conditions on flow equilibria, this leads to the aggregate no-shirking condition,

\[
(1-t)w \geq \frac{(r + q)(N-L) + bN}{q(N-L)} e - \frac{[b\bar{w} - (r + h)w](N-L) - bNw}{(r + h)(N-L) + bL}.
\] (34)

At the same time, there is no change of the feasibility equation. Thus the equilibrium vector of \((1-t)w, L\) is determined by the equations (19) and (34).

The results regarding the impact of changes in the benefit loss rate \( h \) on utility levels and differentials are summarized in Proposition 3:

**Proposition 3** With a higher benefit loss rate (that is, a tighter time limit) \( h \), the lifetime utility differential between employed workers and social assistance recipients, \( V_e - V_z \), remains unchanged. The respective utility levels move in the same direction as employment. The lifetime utility differential between the two groups of unemployed, \( V_u - V_z \), shrinks if \( \frac{\partial a}{\partial h} > -1 \), that is, if employment does not decrease too much.
Proof. See Appendix E.

A variation in the benefit loss rate has no impact on the utility differential between employed workers and social assistance recipients, as this depends only on the effort exerted at the workplace and the shirking detection rate. Given the fixed utility differential, the change in lifetime utility of social assistance recipients is exclusively determined by the impact on employment opportunities. Those living on unemployment benefits are affected by the impact on the job acquisition rate in a similar fashion. In addition, they suffer from the expectation of losing their benefits earlier. Hence, it is unsurprising that unemployment benefit recipients tend to be the losers of the reform in terms of utility differentials.

Comparative static results can be derived in the usual way. Rewriting the equilibrium conditions (34), with equality, and (19) as

\[
\varphi_1((1-t)w, L) = (1-t)w - \frac{r + q}{q}e - \frac{bN}{q(N-L)}e
\]
\[
+ \frac{[b\bar{w} - (r + h)\bar{w}] (N-L) - bNw}{(r + h)(N-L) + bL} = 0,
\]
\[
\varphi_2((1-t)w, L) = (1-t)w - F'(L) \left[ 1 - \frac{bL\bar{w} + h(N-L)\bar{w} N - L}{bL + h(N-L) F(L)} \right] = 0,
\]

the stability condition \( \varphi_{11}\varphi_{22} - \varphi_{12}\varphi_{21} > 0 \) implies

\[
sgn \left( \frac{\partial L}{\partial h} \right) = sgn \left[ \varphi_{1h} - \varphi_{2h} \right]
\]

and

\[
sgn \left( \frac{\partial (1-t)w}{\partial h} \right) = sgn \left[ \varphi_{2h}\varphi_{1L} - \varphi_{1h}\varphi_{2L} \right]
\]

according to the implicit function theorem.

Evaluating (37) yields

\[
\varphi_{1h} - \varphi_{2h} = -\frac{b(N-L)^2(\bar{w} - \bar{w})}{[r + h)(N-L) + bL]^2} + \frac{F'(L)(N-L) bL (N-L) (\bar{w} - \bar{w})}{F(L)}\left[ h(N-L) + bL \right]^2
\]
\[
= \frac{b(N-L)^2(\bar{w} - \bar{w})}{[r + h)(N-L) + bL]^2} \left[ F'(L) \left[ \frac{(r + h)(N-L) + bL}{h(N-L) + bL} \right]^2 - 1 \right].
\]
As the wage share \( \frac{F'(L)L}{F(L)} < 1 \) can be arbitrarily close to unity and the interest rate \( r \)
may be close to zero, the sign of \( \frac{\partial L}{\partial h} \) is clearly indeterminate.

Further, it turns out that

\[
\varphi_{2h}\varphi_{1L} - \varphi_{1h}\varphi_{2L} = -\frac{F''(L)(N - L) bL (N - L) (\bar{w} - \bar{w})}{F(L)} \frac{bN}{q (N - L)^2} e^{-\frac{b^2 N (\bar{w} - \bar{w})}{[(r + h) (N - L) + bL]^2}} \\
+ \frac{b (N - L)^2 (\bar{w} - \bar{w})}{[(r + h) (N - L) + bL]^2} \\
\cdot \left[ -F''(L)(1 - t) + F'(L) \frac{\partial t}{\partial L} \right].
\]

This term is positive provided that \( \frac{\partial t}{\partial L} \) is not too negative.

Increasing the benefit loss rate in the modified model raises the incentives to shirk, as the utility differential between the two states of unemployment is shrinking. Thus, the no-shirking curve shifts upwards. This effect tends to decrease employment. Taken in isolation, the net wage will fall if the feasibility curve is upward sloping at the stable equilibrium, and it will rise if the feasibility curve is downward sloping there. At the same time, with a smaller cost per unemployed worker, the feasibility curve shifts upwards. As more people can be profitably employed at a given net wage, this effect tends to increase employment and also the net wage. Clearly, employment can move in either direction. The net wage will always rise if unemployment does not increase.

7 Concluding discussion

The main message of this paper is that changes in the net wage are of particular importance when evaluating the welfare consequences of introducing or tightening time limits in unemployment benefit or welfare systems. Lifetime utilities of employed and short-term unemployed workers move in the same direction as the net wage, while the long-term
unemployed may gain in expected lifetime utility even if their prospective net wage falls.

Although unemployment is generally reduced, the structure of unemployment may in extreme cases evolve in an unexpected fashion. In itself, the stricter time limit induces a reduction in the share of those receiving full benefits. At the same time, the rising number of jobs reduces the average duration of unemployment, such that the resulting higher share of short-term unemployed could offset the direct effect of the stricter time limit.

Evaluating the overall welfare consequences of introducing or tightening time limits on benefit receipt remains difficult due to the distributional implications. While the expected increase in total output involves the potential for a Pareto improvement, a higher share of low benefit recipients is certain when the time limit is introduced, and it will often turn out when the time limit is tightened. On the other hand, our analysis indicates that recipients of low benefits are the winners of the reform in terms of changes in utility differentials when compared to other types of workers.

Allowing for a heterogeneous workforce does not change the results substantially. It is still true that changes in net wages drive the effects for utility levels of employed workers and short-term unemployed. The skill premium in terms of utility is an increasing function of the net wage of unskilled workers. This last result will presumably no longer hold if the different types of labor are complements rather than substitutes. As the ratio of unskilled workers to skilled workers is expected to go up when the time limit is tightened, the skill premium will probably rise.

The consequences of stricter time limits are completely different if legal rules are actually enforced that deny access for dismissed shirkers to the more generous tier of the unemployment compensation scheme. In this case, stricter time limits increase the incentive to shirk because the reform hurts non-shirkers more than shirkers. The reason is that the unemployed who are not dismissed because of shirking receive unemployment benefits for a shorter period of time. Employment gains may still arise due to the falling cost of an
unemployed at a given level of total employment. It turns out that the change in employment is crucial for assessing the gains of employed workers and long-term unemployed. The short-term unemployed tend to be the losers of stricter time limits.

Obviously, a possible extension of our analysis would be to investigate the structure of an optimal unemployment scheme in a framework with risk averse agents, where both the benefit levels and the benefit loss rate can be chosen. However, as Fath and Fuest (2005) have shown, repercussions on the incentive structure of the employed that arise in shirking models will generally imply that there is no demand at all for unemployment insurance. Hence, even with strong risk aversion the optimum level of benefits will typically be equal to zero.
Appendix

A: Proof of Proposition 1

Utilizing the implicit function theorem, it follows for any variable $x \in \{1-t\}w, L \}$ that $\frac{dx}{dh} = -\frac{\Delta_{xh}}{\Delta}$, where $\Delta$ is the determinant of the Jacobian of (20) and (21), and $\Delta_{xh}$ represents the determinant of the corresponding Jacobian in which the column vector $(f_1, f_2)$ is replaced by $(f_{1h}, f_{2h})$. Taking into account the sufficient stability condition $\Delta > 0$, and ignoring the case that only the necessary condition is satisfied, it follows that $\text{sgn} \left[ \frac{\partial x}{\partial h} \right] = -\text{sgn} [\Delta_{xh}]$. Evaluating the derivatives reveals that

$$\text{sgn} [\Delta_{Lh}] = \text{sgn} [f_{2h} - f_{1h}]$$

$$= \text{sgn} \left[ -\frac{F'(L)(N-L)^2(\overline{w}-w)}{F(L)} \frac{bL}{(h(N-L) + bL)^2} \right. \left. - \frac{(N-L)(\overline{w}-w)(r(N-L) + bL)}{(r+h)(N-L) + bL)^2} \right] < 0,$$

$$\frac{\partial w}{\partial h} = F''(L) \frac{\partial L}{\partial h} < 0, \quad (42)$$

$$\frac{\partial [V_z - V_u]}{\partial h} = 0, \quad (43)$$

$$\frac{\partial [V_u - V_z]}{\partial h} = -\frac{(\overline{w} - w) \left( (N-L)^2 + bN \frac{\partial L}{\partial h} \right)}{(r+h)(N-L) + bL)^2} < 0. \quad (44)$$

Since $\frac{\partial [V_z - V_u]}{\partial h} = 0$, it follows from (3) that $r \frac{\partial V_z}{\partial h} = \frac{\partial (1-t)w}{\partial h}$.

B: Impact on the tax rate

Calculating the impact of a higher benefit loss rate on the tax rate from equation (19), and taking into account (14), yields

$$\frac{\partial t}{\partial h} = \frac{a}{[a+h]^2} \frac{N-L}{F(L)} + \frac{h(\overline{w} - w)(b+a)}{[a+h]^2} \frac{1}{F(L)} \frac{\partial L}{\partial h}$$

$$- \frac{a\overline{w} + hw}{a+h} \frac{F(L)}{[F(L)]^2} \frac{(N-L)F'(L) \partial L}{\partial h}.$$
The first term on the right-hand side reflects the smaller expenditure level due to the higher share of social assistance recipients at a given unemployment rate. The second term mirrors the shift in the structure of unemployment towards a higher share of those receiving unemployment benefits with a fall in total unemployment. Finally, the third term indicates the increase of the tax base. Taking into account $\frac{\partial L}{\partial h} = -f_{2h} - f_{1h}$ leads to

$$\text{sgn} \left( \frac{\partial t}{\partial h} \right) = \text{sgn} \left[ -\left( \frac{\overline{w} - \overline{w}}{a + h} \right) \frac{a}{[a + h]^2} \frac{N - L}{F(L)} (f_{2L} - f_{1L}) \right. + \left[ \frac{a\overline{w} + hw}{a + h} \frac{F(L) + (N - L)F'(L)}{[F(L)]^2} - \frac{h(\overline{w} - \overline{w})(b + a)}{[a + h]^2} \frac{1}{F(L)} \right] (f_{2h} - f_{1h}) \right].$$

It follows that

$$\text{sgn} \left( \frac{\partial t}{\partial h} \right) = \text{sgn} \left[ -\left( \frac{\overline{w} - \overline{w}}{a + h} \right) \frac{a}{[a + h]^2} \right. \cdot \left. \frac{-F''(L)(1 - t) + \frac{beN}{q(N - L)^2} + \frac{h(\overline{w} - \overline{w})}{[r + h + a]^2} \frac{a + b}{N - L}}{F(L)} + \frac{F'(L)(\overline{w} - \overline{w})}{[h + a]^2} h(a + b) \right. \cdot \left. \frac{F'(L)}{a + h} \frac{F(L) + (N - L)F'(L)}{[F(L)]^2} \right] + \left[ \frac{(\overline{w} - \overline{w})}{[a + h]^2} \frac{h(a + b)}{a + h} - \frac{aw + hw}{a + h} \left[ 1 + \frac{(N - L)F'(L)}{F(L)} \right] \right] + \left[ \frac{F'(L)(N - L)(\overline{w} - \overline{w})}{a} \frac{(\overline{w} - \overline{w})(r + a)}{(r + h + a)^2} \right].$$

Simplifying this expression yields
\[
\text{sgn} \left[ \frac{\partial t}{\partial h} \right] = \text{sgn} \left[ -(N - L) \frac{a}{[a + h]^2} \left[ -F''(L)(1 - t) + \frac{beN}{q(N - L)^2} \right] \right.

- \frac{[a\bar{w} + hw]}{a + h} \left( N - L \right) \frac{F'(L)}{F(L)} \frac{(r + a)}{(r + h + a)^2} 

\left. - \frac{1}{(r + h + a)^2 [a + h]^2} \left[ (a + h)(a\bar{w} + hw)(r + a) - rh(a + b)(\bar{w} - w) \right] \right] 
\]

and

\[
\text{sgn} \left[ \frac{\partial t}{\partial h} \right] = \text{sgn} \left[ -(N - L) \frac{a}{[a + h]^2} \left[ -F''(L)(1 - t) + \frac{beN}{q(N - L)^2} \right] \right.

- \frac{[a\bar{w} + hw]}{a + h} \left( N - L \right) \frac{F'(L)}{F(L)} \frac{(r + a)}{(r + h + a)^2} 

\left. - \frac{1}{(r + h + a)^2 [a + h]^2} \left[ a(a + h)(a\bar{w} + hw) + r \left[ (a + h)^2 \bar{w} + (a^2 - hb)(\bar{w} - w) \right] \right] \right].
\]

It follows that \( \frac{\partial t}{\partial h} < 0 \) if either the discount rate \( r \) or the benefit loss rate \( h \) is sufficiently close to zero.

**C: Example with falling share of social assistance recipients**

Assume a production function with diminishing marginal returns \( pL^\alpha + 3L \) with the parameter values \( p = 10000 \) and \( \alpha = .001 \), a population of \( N = 1,000 \), an interest rate of \( r = .04 \), a required effort of \( e = .1 \), a separation rate of \( b = .01 \), a detection probability of \( q = .9 \), unemployment benefits of \( \bar{w} = 1 \) and social assistance benefits of \( w = .1 \). In Table 1, we compute the stable employment level and the ratio of unemployment benefit recipients to social assistance recipients for different levels of the benefit loss rate \( h \).
Table 1. Impact on the structure of unemployment benefit recipients.

<table>
<thead>
<tr>
<th>$h$</th>
<th>$L$</th>
<th>$a/h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>250.61</td>
<td>$3.3442 \times 10^{-3}$</td>
</tr>
<tr>
<td>0.8</td>
<td>211.23</td>
<td>$3.3475 \times 10^{-3}$</td>
</tr>
<tr>
<td>0.6</td>
<td>166.57</td>
<td>$3.3310 \times 10^{-3}$</td>
</tr>
<tr>
<td>0.4</td>
<td>117.50</td>
<td>$3.3286 \times 10^{-3}$</td>
</tr>
<tr>
<td>0.2</td>
<td>65.267</td>
<td>$3.4912 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

It turns out that the share of regular social assistance recipients falls when the benefit loss rate is increased from .2 to .4. The opposite reaction occurs when the rate is further increased from .4 to .8.

D: Proof of Proposition 2

Note that the no-shirking constraint for skilled workers is

$$(1 - t) \alpha w \geq rV_u^S + \frac{r + b + q}{q} e,$$  \hfill (45)$$

where $V_u^S$ denotes lifetime utility of a skilled social assistance recipient. Taking the no-shirking constraints to hold with equality and subtracting (3) from (45) yields

$$(1 - t)w(\alpha - 1) = r(V_u^S - V_u).$$  \hfill (46)$$

It immediately follows that

$$\frac{d(V_u^S - V_u)}{dh} = \frac{\alpha - 1 d(1 - t)w}{r}.$$  \hfill (47)$$

Since $V_e^S - V_u^S = V_e - V_u = e/q$, equation (2) and the corresponding version for skilled workers imply

$$\text{sgn} \left[ \frac{dV_e}{dh} \right] = \text{sgn} \left[ \frac{dV_e^S}{dh} \right] = \text{sgn} \left[ \frac{d(1 - t)w}{dh} \right].$$  \hfill (48)$$
E: Proof of Proposition 3

It is immediate from equation (31) that \( \frac{\partial (V_e - V_z)}{\partial h} = 0 \). Inspection of (5) and (14) reveals that
\[
\text{sgn} \left[ \frac{\partial V_z}{\partial h} \right] = \text{sgn} \left[ \frac{\partial a}{\partial h} \right] = \text{sgn} \left[ \frac{\partial L}{\partial h} \right].
\]
Finally, (9) shows that \( \frac{\partial a}{\partial h} > -1 \) is necessary and sufficient for \( \frac{\partial (V_a - V_z)}{\partial h} < 0 \).
References


Figure 1. Equilibria and dynamics.