

# A Dynamic Stochastic Model of Programme Participation\*

Preliminary - please do not quote.

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## Abstract

Since 1994 unemployed workers in the Danish labour market have participated in active labour market programmes on a large scale. This paper contributes with structural estimates of the effects of these programmes. An economic model is set up in which workers act according to an optimal solution to a dynamic optimization problem sequentially choosing between three mutually exclusive states; working, participating in a labour market programme, and home production. The focus is on Danish unskilled males. The data set used is register-based and follows the sample from 1995 to 2000. Parameters from the dynamic programming model are found via maximum likelihood.

**Keywords:** Active labour market programmes, Programme Evaluation, Dynamic Programming.

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# 1 Introduction

This paper presents a fully structural approach to evaluating the effects of active labour market programmes and as such seeks to satisfy a growing demand for a more structural or economic modelling approach in this branch of applied economics, see Heckman, LaLonde & Smith (1999) for a discussion. The previous studies of the effects of programme participation on labour market outcomes have all focused on programme participation in a 'treatment' framework and have therefore all been using methods adapted from statistics or bio-statistics in particular, most often matching or duration analysis. However, these studies all battle with the problem of identifying the counterfactual outcome variable of interest, having to make more or less arbitrary non-testable assumptions which seldom can find grounds in economic theory.

In contrast, this paper presents a fully structural framework that imposes the restrictions put up by the theory and as such provides parameter estimates that can be interpreted rigorously. Firstly, parameters are estimated that are highly relevant for policy makers. The model quantifies both the effects of active labour market programmes while they are taking place as well as their subsequent importance. More importantly, however, since an explicit optimization problem is solved in this paper and explicit decision rules are derived, effects on decisions of altering specific policy variables in the model can be quantified. For example, the benefits received while in a programme could be altered as an 'experiment' and thus it can be assess how lowering benefits while in a programme would affect participation rates. Moreover, since the decision to participate in a programme is interrelated with subsequent labour market status, one can estimate the effect of a policy change, here lowering benefits, on subsequent labour market status or outcomes. This is possible in a structural model, which can most often generate more accurate predictions of the impacts of policy changes than the reduced form models mentioned above<sup>1</sup>. Finally, all previous work in this area has been silent about welfare gains or losses of these programmes. Since the issue of evaluating programmes is addressed in a structural framework, one can perform welfare analysis and therefore calculate distributional consequences of interventions on lifetime wealth and utility, see Keane & Wolpin (1997).

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<sup>1</sup>Reduced form econometric techniques can in general be thought of as uncovering the form of an agent's *historical* decision rules, and hence the resulting estimates can only be used to predict future behaviour given that the environment is stationary - removing the possibility of policy experimentation.

The model set up in this paper is a step towards building a model that encompasses the stylized facts concerning participation in programmes as found in Heckman & Smith (1999). Here it is found that Ashenfelter's dip is more a result of changes in labour force status, rather than a decline in wages or hours worked among those who work. So *changes* in labour force status, not changes in earnings, characterize participation in programmes. That the labour force status histories are important is not likely to be surprising given that many employment and training services are directly targeted to lead to immediate employment, e.g. job search assistance and on-the-job-training. Heckman et al. (1999) suggest a model where the participation in training is driven by changes in labour force states. They formulate a search model where persons select among the options available to them using the Gittens index. However, if, say, on-the-job-training in a private sector job is thought to have effects on outcomes in the public sector or if participation in formal classroom training could affect productivity in any sector in the economy, then their suggestion is inapplicable, see e.g. Keane & Wolpin (1994). Since a large part of the labour market programmes in especially Europe are aiming at augmenting the human capital base of the unemployed, such an assumption of independence would be overly strong and assuming away important parts of the intended effects of the policies under consideration. Therefore this paper sticks to a more or less strict human capital interpretation of the selection into labour market programmes but accommodate the findings that labour force status is a main driving force in determining programme participation.

The goals of the Danish active labour market policies are threefold manifested through a 'right and duty' principle:

The principle of right and duty gives the unemployed persons stronger incentives to finding ordinary employment as quickly as possible and constitutes an efficient test of the availability of the unemployed. At the same time, the skills and qualifications of the long-term unemployed are up-graded during periods where they cannot find ordinary employment [cited from [www.am.dk](http://www.am.dk) - the Danish Ministry of Employment].

Hence, any structural model attempting to replicate the environment faced by the unemployed would at least have to produce the following factors: since one aim of the programmes is to directly upgrade the skills and qualifications of its participants, a subsequent effect has to be present in future earnings, given the human capital interpretation.

These effects are meant to exist on both formal education and in the job placement activities. Moreover, and as importantly, the programmes also have the goal of making it less attractive to remain unemployed and receive benefits through 'taxation of leisure'. Hence, a potential effect needed to be present in the theoretical model below is a *non-pecuniary* cost of having to participate in these programmes. However, nothing *ex ante* should restrict any parameters to attain a negative value here, since participation in, say, formal educational programmes could also be considered to have a pure consumption value in much the same way as Heckman (1976).

In non-structural analyses of the same policies, see Jespersen, Munch & Skipper (2004) and Munch & Skipper (2004), the programmes were found to be non-successful in raising the hourly wage rate of participants but the employability of participants was found to increase as a consequence of participation leading to an increase in earnings.

In this paper the focus is solemnly on unskilled workers. By selecting this group only I avoid having to model the endogenous choice of education in the ordinary schooling system making subsequent choices conditional on having taken a basic education only. Also, much debate has focused on this particular 'weak' group and the need to upgrade their skill base in a globalizing economy making it a highly policy relevant selection. Moreover, to limit the action space further, women are excluded avoiding having to model e.g. maternity leave as well. More importantly, however, is the fact that currently different labour market programmes are pooled into one state, a 'programme participation'-state, and as such no effect of heterogeneity between different programmes is investigated thus far. Programmes with an aim towards weaker groups of unemployed are excluded though, leaving only job training courses and ordinary class room training in the analysis.

[The estimated model suggests that the programmes are effective in 'taxing' leisure without destroying the possibility of consumption smoothing. However, in estimating the model numerical difficulties were encountered suggesting that the basic model in fact does not represent reality adequately. The within-sample fit of the distribution of choices is close to the observed. However, the basic model fails to capture the persistency of choices found in the data. An extension of the model is therefore suggested.]

The rest of the paper is organized as follows. The next section describes the institutional settings in Denmark. Section 3 describes the data. Section 4 presents the basic behavioural model. Section 5 contains the estimation results and evaluation of model fit

of the basic model. In section 6 extensions of the basic model are put forward. Section 7 presents the conclusion.

## 2 Active Labour Market Policies in Denmark

The essence of the Danish active labour market policies (ALMP) is to induce unemployed people<sup>2</sup> to work by improving their qualifications in fields where there is a strong demand for labour. The policies include a variety of activation measures such as on-the-job-training in both public and private firms, education, and financial support of small business enterprises. Prior to a reform in 1994 the "active" part of the Danish unemployment system had as focus the possibility of unemployed to re-earn their right to unemployment subsidies through governmental supported work. After 1994 the possibility for unemployed to renew eligibility for benefit periods by participating in these programmes was abolished and a 7 year period of support was introduced.

To qualify for benefits a UI-member has to have been member for at least one year as well as held a job for at least 26 weeks during the last three years. This later requirement was increased to 52 weeks as of January 1997. Some groups, those just finishing their education or apprenticeships, are exempt from these rules and the only requirement they have to meet is a minimum of one month of membership (and no employment requirements).

In the 1994 reform an initial passive period was set to four years. Once this period of unconditional benefits has expired the unemployed must participate in programmes during 75% of further time spent in unemployment<sup>3</sup>. Up until now, there has been some reforms of this system; gradually, the passive period has been cut back such that as of January 1999 the period ends after one year for those above the age of 25 and ends for those below<sup>4</sup> after six months. More importantly, however, is the fact that during the *entire* spell of unemployment the individual will receive offers to participate in programmes, and

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<sup>2</sup>The focus of this paper is on people with an unemployment insurance, since social security recipients besides being unemployed often also are disabled or are faced with social problems. Around 80 % of the labour force are members of UI funds. See Parsons, Tranæs & Lilleør (2003) for further insight into the Danish voluntary public unemployment insurance system.

<sup>3</sup>After seven years the unemployed receives the lower means tested social assistance, which is 80% or 60% of maximum UI benefits depending on whether or not the unemployed is a sole provider.

<sup>4</sup>A special youth programme was introduced in 1996, implying earlier activation and cuts in benefits. For more details and effects of this particular programme, see Jensen, Svarer & Rosholm (2003).

only after showing up at the start of an offered programme can he or she formally object and argue not to participate<sup>5</sup>. Should the individual refuse to participate in the offered programme the rights to receive UI benefits are cancelled and the lower social assistance is received instead.

The benefits received in the UI system are based on previous wage. The replacement rate is set to 90 per cent with a ceiling that causes the majority of unemployed individuals to have a replacement rate effectively lower. Individuals in both private and public OJT receive the minimum wage set by collective bargaining in the given sector. Furthermore, the number of working hours in public job training is set such that the wage income does not surpass the maximum UI benefit level. This is not the case for people in private OJT. Hence, people on private OJT can effectively have an income higher than the UI ceiling. Unemployed participating in an educational programme will receive the same amount of UI-benefits as they did prior to entering.

The fraction of unemployed participating in programmes has more than doubled since the first reform in 1994. This is partly due to the strengthening of active measures, and partly due to the fact that the reforms also entailed a forward shift in the active period such that more people are affected by the requirements of the activation. In 1994 more than 80,000 yearly fulltime UI fund members participated in some ALMP, and since then this number settled at a level between 45,000 and 55,000 with a decline to around 42,000 in 2001<sup>6</sup>. When comparing these numbers to the corresponding numbers of unemployed (288,000 in 1994 and 145,000 in 2001) it becomes clear that the scale of the Danish system of ALMPs today is massive, and this has led Kluge & Schmidt (2002) to highlight Denmark as the prime example among European countries performing the transition from a benefit system of passive measures to one of active measures.

There has also been a shift in the composition of how the different types of programmes have been used. The most frequently used programmes are ordinary class room training, private job training, and public job training. In 1995 30% of all participants were activated in ordinary educational programmes, while this percentage has risen to 65 in 2000. At

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<sup>5</sup>If the cause for the complaint is then found to be valid, the individual is of course exempt from participation. In Munch & Skipper (2004) it is found that many very short programme spells exist and that these most likely are a consequence of these 'misplacings'.

<sup>6</sup>In the same period the number of yearly fulltime social security recipients participating in active measures rose steadily from somewhat more than 20,000 in 1994 to around 36,000 in 2001.

the same time the fraction of those participating in private OJT was more than halved from 14% to 6%, while the share of participants in public OJT fell from 31 to 15%, see Munch & Skipper (2004) for a more detailed discussion. The duration of private job training programmes are on average shorter than those in the public sector with an average duration of 22 weeks for private OJT and 39 weeks for public OJT in 1996. Ordinary CT lasted on average 28 weeks that year, again see Munch & Skipper (2004) for further details.

### 3 Economic Model

The simple idea of the model is the following: each individual has a finite decision horizon beginning at age 16 when the unskilled individual leaves school and enters the labour market and ending at age 60 at which time the individual retires.<sup>7</sup> To make the model computationally feasible, a period will be defined as half a year, hence we get the following time index  $t \in \{0, 1, 2, \dots, 87\}$ . Each period an individual chooses among 3 mutually exclusive and exhaustive alternatives: either work ( $W$ ), engage in home production ( $H$ ), or participate in ALMP ( $A$ ). Let  $d_t$  represent this *control variable*, ( $d_t \in \{W, H, A\}$ ) at each discrete point in time  $t$ . The utility accruing to the agent each period is measured in units of DKK. The specifications below all assume that utility is linear in income and agents will therefore be solving a working life time wealth maximization problem in the same way as agents in Keane & Wolpin (1997).<sup>8</sup> Following Rust (1994) I let the *state variables*,  $\mathbf{s}_t$  be partitioned into  $(\mathbf{x}_t, \boldsymbol{\varepsilon}_t)$ , where  $\mathbf{x}$  is a vector of observed state variables, and  $\boldsymbol{\varepsilon}$  is a vector observed only by the individual. Furthermore it is assumed that the two vectors are conditionally independent, see Rust (1994).

It is assumed that the single period utility function is additive and given by  $u_t = u_t(\mathbf{x}_t, d_t, \boldsymbol{\theta}_u) + \varepsilon_t(d_t)$ , where  $\boldsymbol{\theta}_u$  is a vector of parameters to be estimated but known to the individuals, and  $\varepsilon_t(d_t)$  are *i.i.d.* Gumbel or Extreme value type I distributed. The value function is given by

$$V_t(\mathbf{x}_t, \boldsymbol{\varepsilon}_t, \boldsymbol{\theta}) = \max_{d_t \in D(\mathbf{x}_t)} [v_t(\mathbf{x}_t, d_t, \boldsymbol{\theta}) + \varepsilon_t(d_t)],$$

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<sup>7</sup>This upper bound is found empirically to be the relevant retirement age for this group, see Pedersen (1998).

<sup>8</sup>A previous model assumed non-separability between consumption and leisure, but it was unidentified in the data, see appendix for results and discussion.

where  $D(\mathbf{x}_t)$  is the choice set available to the individual in state  $\mathbf{x}_t$ , and  $v_t(\cdot)$  is the *expected value function* defined recursively as

$$v_t(\mathbf{x}_t, d_t, \boldsymbol{\theta}) = u_t(\mathbf{x}_t, d_t, \boldsymbol{\theta}_u) + \delta \left\{ \log \sum_{d_{t+1} \in D(\mathbf{x}_t)} \exp[v_{t+1}(\mathbf{x}_{t+1}, d_{t+1}, \boldsymbol{\theta})] \right\}, \quad (1)$$

for  $t < 87$  and

$$v_{87}(\mathbf{x}_{87}, d_{87}, \boldsymbol{\theta}) = u_{87}(\mathbf{x}_{87}, d_{87}, \boldsymbol{\theta}),$$

where  $\delta$  is the discount factor. This is set to  $\sqrt{.95}$  in what follows below.

In each period,  $t$ , the individual is faced with some constraints that depend on the choice made in the previous period,  $d_{t-1}$ <sup>9</sup>. These constraints are introduced as simple 'offer probabilities'  $\pi$ ; i.e. there exist exogenous probabilities of actually receiving either wage / job offers or offers to participate in some programme. Hence, someone choosing to work in period  $t$  will have a different probability of receiving a wage offer in period  $t+1$  compared to someone engaging in home production,  $\pi_{WW} \neq \pi_{HW}$ , or participating in a programme,  $\pi_{WW} \neq \pi_{AW}$ . Equivalently, being unemployed in period  $t$  will result in a different probability of receiving an offer to participate in a labour market programme in  $t+1$  than had the individual instead been employed in  $t$  possibly coming from the fact that case workers might help the individual in finding available openings once an initial 'open' period of unemployment is passed. This will introduce a constraint in the optimization problem. Also, these constraints accommodate the already mentioned findings of Heckman & Smith (1999) that changes in labour market states drives programme participation.

The following  $3 \times 3$  offer probability matrix specifies the constraints faces by the individuals

$$\pi_t(d_t | d_{t-1}, \boldsymbol{\theta}_p) = \begin{pmatrix} \pi_{WW} & 1 & \pi_{WA} \\ \pi_{HW} & 1 & \pi_{HA} \\ \pi_{AW} & 1 & \pi_{AA} \end{pmatrix},$$

where  $0 < \pi_{ij} \leq 1$  and the second column assumes that home production is always an option, regardless of prior choice.

The deterministic part of the single period utility function takes on the following form

$$u_t(\mathbf{x}_t, \boldsymbol{\theta}_u) = y_t + \alpha_W \mathbf{1}(d_t = W) + \alpha_H \mathbf{1}(d_{t-1} \neq W \wedge d_t = H) + \alpha_A \mathbf{1}(d_t = A).$$

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<sup>9</sup>Of course it is very likely that current opportunities will be affected by choices made further into the past as well. The assumption made here that only the previous period's choice matters greatly simplifies the model solution.



$y_t$  is income, which is assumed to be equal to labour earnings,  $w_t$ , if the individual works,  $d_t = W$ , and equal to the unemployment insurance,  $UI$ , if the individual either engages in home production,  $d_t = H$  when no offer to participate in a programme has been received, or participates in a labour market programme,  $d_t = A$ . If the unemployed has received an offer to participate in a programme but chooses to stay at home,  $y_t$  is assumed to be equal to 60 % of  $UI$ .  $\alpha_W$  measures a non-wage aspect of employment,  $d_t = W$ . This additive parameter is added to the utility function reflecting the net monetary equivalent value of working conditions, indirect compensation, or fixed costs of working. A similar effect arises from programme participation,  $\alpha_A$ , which might be interpreted as an effort cost of the programme<sup>10</sup>.  $\alpha_H$  is the cost of entering a second 'non-working' period,  $d_{t-1} \neq W$ , and still remain at home,  $d_t = H$ . This cost captures a potential psychological stress cost of having to show up at meetings with a caseworker arguing *not* to participate in a programme. Finally, note that there is no possibility of saving in the model.

The wage is set equal to the marginal product of the worker. This marginal product is the product of the rental price times the number of skill units possessed by the individual. Following standard human capital formulation augmented to take account of skills obtained while in a labour market programme, the skill accumulated up to any point in time depends on regular work experience,  $e_t^W$ , which takes on the typically quadratic form, Mincer (1958). Let  $e_t^A$  denote the skill units acquired from labour market program participation. Furthermore, the skill technology function is augmented to allow for skill depreciation by adding a dummy variable for whether or not the individual worked in the previous period:

$$\begin{aligned} w_t &= \exp \left( \beta_0 + \beta_1 e_t^W - \beta_2 (e_t^W)^2 + \beta_3 e_t^A - \beta_4 (e_t^A)^2 - \beta_5 \mathbf{1}(d_{t-1} \neq W) \right), & (2) \\ t &= 1, \dots, 87. \end{aligned}$$

This leads to a standard log wage equation, where the constant term  $\beta_0$  is the log rental price. Errors in observed wages are assumed to be log-normally distributed.

The model consists of 3 different state variables in addition to time  $t$ . The experience vector  $\mathbf{e}_t \equiv (e_t^W, e_t^A)'$ , where  $e_{t+1}^i = e_t^i + .5 \cdot \mathbf{1}(d_t = i)$  is experience accumulated at the beginning of period  $t + 1$  in years, for  $i = \{W, A\}$ .  $d_{t-1}$  will also appear as a state variable in the model, since last periods choice is assumed to affect current periods opportunities<sup>11</sup>.

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<sup>10</sup>83% of the programmes are full time equivalent, 37 hours a week, with the remaining 17% around 33 and 34 hours.

<sup>11</sup>Again, for simplicity it is assumed that only the previous period's choice matters as this will ease

The solution of the dynamic programme serves as input into estimating the parameters of the model. The parameters, 16 in all, are estimated using maximum likelihood. The contribution to the likelihood depends on observed choices. Consider first the probability of observing someone working in period  $t$  receiving wage  $w_t$  :

$$\begin{aligned} \Pr(w_t, d_t = W | \boldsymbol{\theta}, \mathbf{x}_t) &= f(w_t | \mathbf{x}_t, \boldsymbol{\beta}) * \pi(W | d_{t-1}, \boldsymbol{\theta}_p) * \\ &\quad \{ \pi(A | d_{t-1}, \boldsymbol{\theta}_p) * \Pr(V_W > V'_U, V_W > V_A | w_t, \mathbf{x}_t, \boldsymbol{\theta}) \\ &\quad + (1 - \pi(A | d_{t-1}, \boldsymbol{\theta}_p)) * \Pr(V_W > V_U | w_t, \mathbf{x}_t, \boldsymbol{\theta}) \}, \end{aligned}$$

where  $f(\cdot)$  is the lognormal density function and  $V'_U \neq V_U$  because of the lowered  $UI$  benefits received if the individual would choose to stay at home despite having received an offer to participate in a programme. The probability of observing someone engaging in home production in period  $t$  is similarly given as

$$\begin{aligned} \Pr(d_t = H | \boldsymbol{\theta}, \mathbf{x}_t) &= \pi(A | d_{t-1}, \boldsymbol{\theta}_p) * \pi(W | d_{t-1}, \boldsymbol{\theta}_p) * \Pr(V'_U > V_A, V'_U > V_W | \mathbf{x}_t, \boldsymbol{\theta}) \\ &\quad + (1 - \pi(A | d_{t-1}, \boldsymbol{\theta}_p)) * \pi(W | d_{t-1}, \boldsymbol{\theta}_p) * \Pr(V_U > V_W | \mathbf{x}_t, \boldsymbol{\theta}) \\ &\quad + \pi(A | d_{t-1}, \boldsymbol{\theta}_p) * (1 - \pi(W | d_{t-1}, \boldsymbol{\theta}_p)) * \Pr(V'_U > V_A | \mathbf{x}_t, \boldsymbol{\theta}) \\ &\quad + (1 - \pi(A | d_{t-1}, \boldsymbol{\theta}_p)) * (1 - \pi(W | d_{t-1}, \boldsymbol{\theta}_p)). \end{aligned}$$

Finally, the probability of observing someone participating in a programme is given by

$$\begin{aligned} \Pr(d_t = A | \boldsymbol{\theta}, \mathbf{x}_t) &= \pi(A | d_{t-1}, \boldsymbol{\theta}_p) * \\ &\quad \{ \pi(W | d_{t-1}, \boldsymbol{\theta}_p) * \Pr(V_A > V'_U, V_A > V_W | \mathbf{x}_t, \boldsymbol{\theta}) \\ &\quad + (1 - \pi(W | d_{t-1}, \boldsymbol{\theta}_p)) * \Pr(V_A > V'_U | \mathbf{x}_t, \boldsymbol{\theta}) \}. \end{aligned}$$

With the assumed distribution of unobservables we get the following types of conditional choice probabilities for the working alternatives<sup>12</sup>

$$\Pr(V_W > V'_U, V_W > V_A | w_t, \mathbf{x}_t, \boldsymbol{\theta}) = \frac{\exp(v_t(\mathbf{x}_t, W, \boldsymbol{\theta}, w_t))}{\sum_{i \in \{W, H, A\}} \exp(v_t(\mathbf{x}_t, i, \boldsymbol{\theta}))}, \quad (3)$$

and

$$\Pr(V_W > V_U | w_t, \mathbf{x}_t, \boldsymbol{\theta}) = \frac{\exp(v_t(\mathbf{x}_t, W, \boldsymbol{\theta}, w_t))}{\sum_{i \in \{W, H\}} \exp(v_t(\mathbf{x}_t, i, \boldsymbol{\theta}))}.$$

Hence, this dynamic logit model is benefitting from the computationally simplifications of the specification of the error terms *without* introducing the problem of independence from solving and estimating the model.

<sup>12</sup>To save space the remaining five conditional choice probabilities for  $d_t = \{H, A\}$  are left out.

irrelevant alternatives present in static logit models simply due to the future component of (1), see Rust (1994) for discussion.

The model is estimated in a way similar to Rust (1987) in that first the model is solved via backward induction and the solution to this is then used as inputs in the likelihood contributions<sup>13</sup>. This results in a new parameter vector  $\hat{\theta}^1$  which is used as arguments in solving the backward induction problem once again. This is repeated until convergence,  $\hat{\theta}^n \cong \hat{\theta}_u^{n+1}$ . Similar models such as Keane & Wolpin (1997) and Sauer (1998) have used Monte Carlo techniques in managing the computational difficulties in managing such models as opposed to solving them directly.

## 4 Data Description

The data set used in this paper consists of a 10 % random sample of the Danish population in the period between 1995 and 2000. The data is a longitudinal data set with accurate and detailed information on the individual’s labour market states along with information on individual socio-economic characteristics. The socio-economic variables are extracted from the integrated database for labour market research (IDA) and the income registers in Statistics Denmark. For the persons in this sample event histories have been created, such that it is possible to identify every persons labour market state at any week during the year. That is, it is known whether the individuals are employed, unemployed, participating in ALMPs or out of the labour force.

The sample used to estimate the model of participation in labour market programmes consists of all Danish males, UI fund members with only basic schooling as highest completed educational attainment and between 24 and 47 years of age in 1995<sup>14</sup>. Individuals being self-employed at any point in time through this period are excluded. I also exclude people receiving pensions in the form of early retirement and disability pensions etc. Finally, people having spells outside the labour market, non-participation and leaves, are excluded because of the computational complexity associated with solving the dynamic programming problem with these additional states.

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<sup>13</sup>Rust (1987) finds a fix point to the value function and does not solve the model by backward induction.

<sup>14</sup>The lower age bound is set due to the differences in requirements youths have to meet after April, 1996 (see note 4 above). The higher age bound is set due to issues on reliability of labour market experience in the data.

**TABLE 1**

SAMPLE SELECTION INFORMATION	
Selection Criteria	Remaining Sample Size
10 % sample of Danish population	385,918
Aged 24-47 years	170,080
and male	86,374
and Danish citizens	82,006
and full time UI fund member	57,896
and not self employed	52,692
and basic schooling only	12,350
and not receiving pensions	12,092
and having 'complete' event histories	10,595

Of the 10,595 unskilled males in the sample, 1,010 (9.5%) participate in one or more programmes in the six year period. 526 participate in only one programme and 1 individual participates in 19 distinct programmes. Figure 1 shows the distribution of programme lengths. Notice that 40 % of the sampled programmes have duration shorter than 2 months. This may be a result of people initially either being 'misplaced' into a programme or that people leave unemployment as soon as they are to participate in a programme (see however Munch & Skipper (2004) for a discussion of this latter effect).

In figure 2 mean annual earnings among unskilled Danish males is depicted. As seen from the figure the purchasing power of earnings has been increasing over the period, from DKK 225,000 to DKK 240,000 in 2000<sup>15</sup>. Also depicted is the maximum yearly UI benefit as a fraction of mean earnings among unskilled Danish males. This ratio has been steadily declining over the period resulting in a drop in relative income for those unemployed of more than 7 percentage points. I.e. in the same period as Denmark introduced labour market programmes on a large scale in part to reduce the incentives to stay unemployed, the economic incentives were declining as well. Unfortunately, the complexity involved in solving the model does not allow me to introduce varying levels of benefits. It will therefore be assumed below that agents expect to be receiving DKK 2555 a week in real terms in the future, see below. I.e. in what follows it will be assumed that the decline is

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<sup>15</sup>The wage is deflated using a consumer price index constructed from approximately 25,000 prices, with 1995 as the base year.

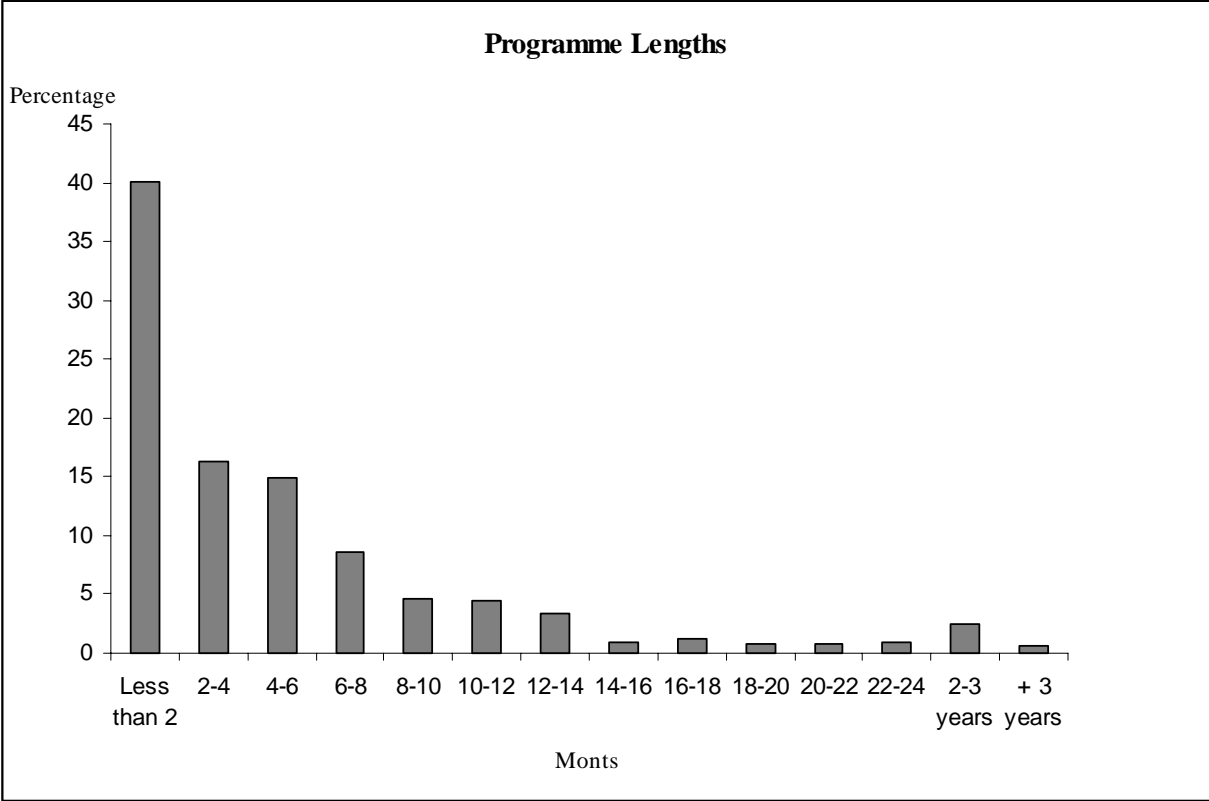


Figure 1: Length of programmes

of only transitory nature and that benefits will stay on the level of 60 per cent of mean earnings of workers. Moreover, it will be assumed that agents realize and act on this.

In order to computationally manage the estimation below, some aggregation of the spells is necessary. The event histories are aggregated from being on weekly basis to being on half year basis. Hence, one decision period for the individuals is half a year. Because the data provide information on weekly basis and individuals may actually be in several alternative states in the six month period, any rules used to create bi-annual data on choices will be somewhat arbitrary. Mutually exclusive alternatives were assigned in a hierarchical fashion as follows.

1. *Programme participation* - since the aim of the paper is to estimate effects of labour market programmes, these have been given highest priority in the selection mechanism. An individual is considered to have participated in a programme during the six month period if the individual attended a given programme for *at least* 8 weeks within the relevant months.

2. *Work*. An individual is considered to have been working in the six month period if he

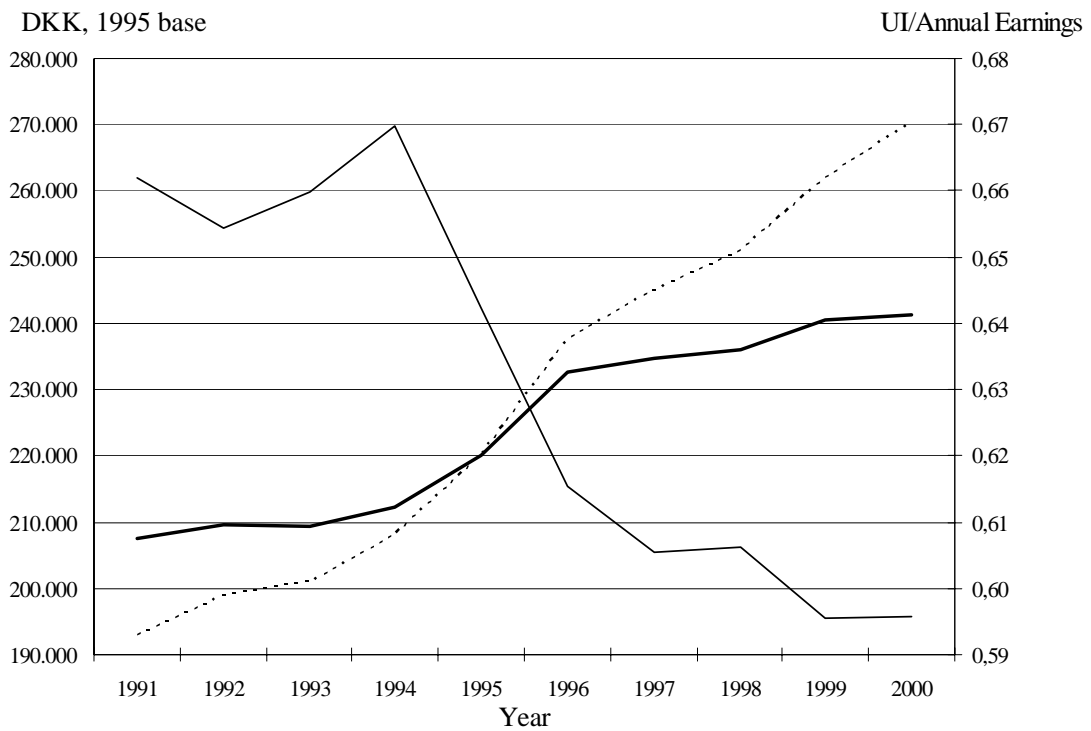


Figure 2: Average annual earnings of a Danish fulltime employed unskilled male worker. - - - nominel earnings, ——— CPI-adjusted earnings, ——— annual maximum UI benefits divided by nominel earnings.

was not assigned to a programme state and if he worked for at least half of the time within the period. *Real wages* - wages are constructed from annual earnings. Unfortunately, a disaggregation of the wage rate into six months periods has not been possible. This means that if I observe an individual working in both periods within a given calendar year, then he is observed to receive the same wage in both periods. To the wage are added compulsory contributions to pension schemes<sup>16</sup>. The wage is deflated using a GNP deflator, with 1995 as the base year<sup>17</sup>. A weekly wage rate is used assuming a 37 hours working week.

3. *Home* - an individual is classified as being home during a six month period if the individual neither participated in a programme nor worked, according to the definitions above. *Unemployment Benefits*. - The way the selection criteria outlined in table 1 has been constructed excludes non-participants and people not eligible for UI benefits in general. Hence, everybody in the sample classified as being home will receive UI benefits. The benefits are deflated using the same index of GNP as used constructing the hourly wage rate and are also given in a weekly level-equivalent. The compensation received is a function of previous wages with both a minimum and a cap. This means that nobody in my sample receives more than DKK 2555 a week being unemployed in 1995. Even though the benefits are a function of previous wages I have chosen to disregard this fact in order not to introduce a continuous control variable, the realized / observed previous wage, in my dynamic programming problem below. More importantly, more than 97 % of my sample will receive the maximum compensation should they become unemployed, and having wages as a control variable for this problem alone will therefore be disproportionately burdensome. Hence, in what follows, people will receive a fixed weekly compensation treated as given from the individual's perspective. The same carries over to benefits received while participating in programmes. I.e. even though benefits received while in private OJT is potentially higher, they are in effect set to the UI level in what follows.<sup>18</sup>

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<sup>16</sup>Remember from the previous section, that the agents are working lifetime wealth maximizing.

<sup>17</sup>This is done in order to avoid having to solve the model for each age cohort.

<sup>18</sup>This is also done due to problems in the actual data of identifying additional compensation received while participating in a programme. Hence,  $\alpha_A$  will capture some of this increased income.

**TABLE 2**TRANSITION MATRIX: DANISH UNSKILLED MALES AGED 24-47

Choice ( $t - 1$ )	Choice ( $t$ )		
	Working	Home	ALMP
Working	97.1	2.9	0.1
Home	39.8	52.9	7.3
ALMP	22.2	15.8	62.0

Table 2 presents the transition matrix. The figures indicate a high degree of persistence in all states. More than half of the observations beginning at home are also at home the following period with the remaining 40 percent entering a job and little less than 10 percent entering an ALMP. Note also, that working as destination state after having participated in ALMP is only slightly more likely than staying home. Hardly anyone makes the transition from work to ALMP directly.

**TABLE 3**CHOICE DISTRIBUTION: DANISH MALES AGED 24-47

Age	Choice			Total
	Working	Home	ALMP	
24-29	15,271	1,426	327	17,024
	89.7	8.4	1.9	100.0
30-39	51,466	3,434	716	55,616
	92.5	6.2	1.3	100.0
40-49	53,881	3,415	744	58,040
	92.8	5.9	1.3	100.0
50-59	46,123	3,162	803	50,088
	92.1	6.3	1.6	100.0

Table 3 shows the choice distribution by age groups. As the table shows, the majority of the sample is working, from little less than 90% of the younger, to around 92% of the the older with the picture being stable across age groups.

## 5 Estimation Results



## 6 Conclusion

In this study a dynamic programming model is build to study the labour force dynamics of unskilled males in Denmark using register data from 1995 to 2000. The framework set forward is an augmented human capital model of Mincer (1958) in which agents receive utility from consumption and leisure. This paper is to the author's knowledge the first empirical paper, that incorporates active labour market policies in a general structural dynamic labour supply model with explicit optimization behaviour of agents.

The estimation of this model is performed within a nested algorithm procedure in which the model is solved explicitly by backward induction and the solution is used as input in a maximum likelihood algorithm iteratively. The basic model suggests that there are considerable non-pecuniary components of working and that ALMP is a successful mechanism of taxing leisure without destroying the possibility for consumption smoothing. It was also found that ALMP participation did not increase the earnings potentials contrary to one of the stated goals of these policies. The model was not capable of replicating the persistency found in choices which suggests that the optimization done by real agents might be done under restrictions not present in the basic model. An extension was suggested in which previous choices influence the possible action space available to the agent in a straight forward way done by introducing exogenous 'offer-probabilities'. The suggestion is currently being implemented.

Another desirable extension would be to allow for heterogeneity in both programmes and sector of employment. Empirically, job training is found to take place at both private and public employers. Non-structural estimates of the effects of these job training schemes suggest significant differences. It would therefore be attractive to extend the current model in this dimension allowing for selection into three distinct programmes, private and public OJT and ordinary class room training. Such a model would allow for differences in returns to training across public and private employers. However, this extension would increase the set of possible choices in each period from its current three to seven giving rise to an exponential increase in computational cost.

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## 7 Appendix: Estimation results on basic model

A basic model without the added constraints from the offer probabilities has been estimated. This basic model did not assume wealth maximization but had as per period utility function the following specification

$$u_t(\mathbf{x}_t, \boldsymbol{\theta}_u) = y_t^\gamma * (80 - \alpha_W \mathbf{1}(d_t = W) - \alpha_H \mathbf{1}(d_{t-1} \neq W \wedge d_t = H) - \alpha_A \mathbf{1}(d_t = A))^{1-\gamma}$$

I.e. nonseparability was assumed between leisure and consumption by a standard Cobb-Douglas specification. It was assumed further that people had potentially 80 hours of leisure available a week. Table A1 and A2 provides estimates of the parameters in this model as well as associated standard errors.

**TABLE A1**

PARAMETER ESTIMATES	
<u>PER-PERIOD UTILITY FCT.</u>	
Parameter	Estimate
	<b>0.761</b>
$\gamma$	(0.004)
	<b>-227.029</b>
$\alpha_1$	(95.42)
	<sup>1</sup>
$\alpha_2$	0
	.
	<sup>1</sup>
$\alpha_3$	80
	.

Note: Standard Errors are in parentheses.

<sup>1</sup>Parameter restricted to lie between 0 and 80. Numerical difficulties encountered on the bound of the parameter space.

The weight of consumption in the utility function is very close to 3/4. Numerical difficulties were encountered in estimating the parameters entering the leisure-part of the instantaneous utility function. The amount of leisure available to someone entering an ALMP,  $80 - \alpha_3$ , was restricted to be strictly above 0 hours<sup>19</sup>. In practice, this was implemented by estimating a parameter  $\tilde{\alpha}_3 = \log(80 - \alpha_3)$ , and the resulting estimate of

<sup>19</sup>This requirement was partly invoked for interpretational reasons but also for pure technical reasons; i.e. in order for the derivative of the value function wrt.  $\gamma$  to be well defined.

this new transformed parameter converged to  $-\infty$ . Therefore neither standard errors on  $\tilde{\alpha}_3$  nor on  $\alpha_3$  are well defined. A similar problem was encountered with  $\alpha_2$ . For reasons of interpretation, this parameter was not allowed to be negative (it was to capture the psychological cost of entering a second period of non-work and having to meet with a case worker); see below for a possible explanation of the apparent failure of estimating this model. The value of leisure while working is estimated to be more than 300 hours a week,  $80 - \alpha_1$ , or more than three times the value of leisure while unemployed. This changes the interpretation of the parameter somewhat. Instead this estimate indicates a high non-pecuniary value attached to having a job. Remember that the model is estimated for (unskilled) prime-aged males, a group among whom having a job has a high social importance.

**TABLE A2**

PARAMETER ESTIMATES

WAGE EQUATION

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Parameter	Estimate
$\beta_0$	<b>8.243</b> (0.007)
$\beta_1$	<b>0.0125</b> ( $4.49 * 10^{-6}$ )
$\beta_2$	<b>0.0001</b> ( $2.1 * 10^{-4}$ )
$\beta_3$	<b>-0.099</b> (0.016)
$\beta_4$	<b>-0.021</b> (0.007)
$\beta_5$	<b>0.036</b> (0.010)
$\sigma^2$	<b>0.267</b> (0.004)

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Looking at the estimated coefficients in the wage equation it is seen that there is only a slight increase with experience beyond the general growth in the economy, and that the

shape of curve of the returns to ALMP indicates an initial drop in earnings capacity as a consequence of participation and that only with more than five years of ALMP-experience will there be an positive return. There will also be an estimated drop of 3.6% in earnings when re-entering the labour market.

Table A3 presents simulated choice distributions for different age cohorts. The fraction of those working is too high for all three age groups and the fraction of those choosing home production is too low. I.e. using the parameters found above and simulating paths of transitions it is found that the working alternative is chosen slightly too often. In general though, the predictive performance of the model seems quite reasonable. Turning to table 6 the limit of the basic model becomes apparent. Where the actual data displayed a high degree of persistence in choices the simulated results from the estimated model has not been able to replicate this fact. The way the instantaneous utility functions have been specified nothing apart from  $\beta_5$  and  $\alpha_2$  allows for persistency in choices, and the observed behaviour of the agents along with the specification of the utility function resulted in the counter-intuitive value of  $\alpha_2$  as discussed above.

**TABLE A3**

SIMULATED CHOICE DISTRIBUTION

Age	Choice		
	Working	Home	ALMP
24-29	94.8	4.0	1.2
	(89.7)	(8.4)	(1.9)
30-39	95.8	4.0	0.2
	(92.5)	(6.2)	(1.3)
40-49	95.4	3.6	1.0
	(92.8)	(5.9)	(1.3)

Note: Actual choices appear in parentheses.

**TABLE A4**

SIMULATED TRANSITION MATRIX

Choice ( $t - 1$ )	Choice ( $t$ )		
	Working	Home	ALMP
Working	95.6 (97.1)	3.7 (2.9)	0.6 (0.1)
Home	95.0 (39.8)	4.4 (52.9)	0.6 (7.3)
ALMP	86.9 (22.2)	11.4 (15.8)	1.7 (62.0)

Note: Actual transition probabilities appear in parentheses.