Imperfect Labor and Products Markets, Regulation, and the Union Wage Gap

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1 Introduction

This paper presents a theoretical model designed to capture the most important interactions between imperfect product and labor markets. This issue has attracted a great deal of attention recently (see e.g. Nickell (1999), Nicoletti, Bassanini, Ernst, Jean, Santiago and Swaim (2001), and Ebell and Haefke (2003)). The reason is, as Nicoletti et al. (2001, p. 7) put it: “... accounting for the potential cross-market effects of product and labor market policies appears to be an important element of good policy design.” The main difference of our approach to other recently published contributions (see in particular Blanchard and Giavazzi, 2003) is that we consider a dual labor market. We think that the inclusion of a secondary labor market not only increases realism, but also heals some of the logical problems of related literature. In addition, the dual labor market renders possible to calculate the ‘union wage gap’, i.e. the difference between the pay of homogeneous workers that are / are not covered by collective bargaining. Through this, we are able provide an explanation of the union wage gap, which previous literature could not explain satisfactorily.

The union wage gap derived from the model for various countries also serves as a check for the appropriateness of our model. By comparing our results to estimates from the empirical literature, we are able to verify that our approach indeed seems to account for the most important forces at work. If one accepts that the model reflects the basic interactions between imperfect markets, it is straightforward to ask how economic policy may influence the outcome in the model. We chose two parameters, one for the goods markets and one for the labor market, that may (partially) be determined by policy measures, and investigate how these parameters may be used to foster real income.

The following section introduces a simple general equilibrium model of imperfect product and labor markets. In section 3 we derive the union wage gap for some industrialized countries numerically. Section 4 identifies parameters that may be influenced by regulation / deregulation policy, and analyzes how variations of these parameters impact on the size and distribution of rents. Section 5 concludes.
2 A model of imperfect in- and output markets

Imagine a two-sector economy. One monopolistic competitive sector, producing heterogeneous goods with increasing returns to scale, and one perfectly competitive sector producing a homogeneous good with constant returns to scale. This setup has recently been supported by empirical work on scale elasticities. After an examination of trade data from 71 countries, Antweiler and Trefler (2002) conclude: “Our results point to the importance of integrating constant- and increasing-returns-to-scale industries within a single general-equilibrium framework.”

The market for labor is dichotomized as well. Some workers receive the competitive wage rate, and some workers receive the (higher) union wage rate. While firms in the homogeneous sector employ exclusively workers from the competitive labor market, production in the heterogeneous sector requires unionized labor as an input. Although in reality labor markets consist of more than only two sectors, the simplifying assumption of a dual labor market is supported by empirical evidence (see the survey in Saint-Paul (1996)).

Firms in the heterogeneous sector each employ a fixed amount of non-unionized labor. Wages paid to these workers have the character of fixed costs, because the competitive wage rate is fully determined by the technology of the homogeneous sector. In our interpretation these costs arise e.g. due to security agents, cleaner, gate keeper, and all other services, who are essential for the firm to produce goods, but whose number is yet independent of the amount actually produced. Equivalently, the fixed labor input may be seen as the corresponding amount of the homogeneous good.

2.1 Workers

There are \( N \) homogeneous workers, indexed by \( j \). Utility of a worker depends on the consumption of one homogeneous and \( n \) heterogeneous goods \( (x_{0,j}, \text{ and } x_{i,j} \text{ with } i \in \{1, 2, \ldots, n\} \text{, respectively}) \). The utility function of a representative worker is

\[
 u_j = u(x_{0,j}, x_{1,j}, \ldots, x_{n,j}) = x_{0,j}^{1-\beta} \cdot X_j^\beta
\]
with
\[ \beta \in (0, 1), \]
\[ X_j \equiv \left( n^{-(1-\rho)} \sum_{i=1}^{n} x_{i,j}^{\rho} \right)^{1/\rho}, \]
\[ \rho \in (0, 1) \]

where \( x_{0,j} \) stands for consumption of the homogeneous good, \( \beta \) symbolizes the expenditure share, and \( X_j \) is a composite index of the consumed varieties (see Dixit and Stiglitz (1977) and Blanchard and Giavazzi (2003)). \( \rho \) corresponds with the elasticity of substitution, \( \sigma \), according to the definition \( \sigma = 1/(1 - \rho) > 1 \). In comparison to the original Dixit and Stiglitz-approach, \( \rho \) is derived endogenously through the assumed relationship
\[ \rho = 1 - 1/(\zeta n), \quad \zeta n = \sigma > 1 \quad (2) \]

where the exogenous parameter \( \zeta \) determines how strong \( \rho \) and the elasticity of substitution between any two varieties depend on the number of firms.\(^1\) One possible interpretation why a higher number of firms increases the elasticity of substitution is that the varieties become closer substitutes. We see \( \zeta \) as a proxy for the degree of transparency on the products market. It is necessary that changes of the supply structure be transparent to the consumers for market entry to have an impact on consumer behavior.

Apart from \( \rho \) being endogenous the main difference to the Dixit-Stiglitz framework is the term \( n^{-(1-\rho)} \) in the definition of the composite index \( X \). The effect of this term becomes clear when we assume for the moment that consumption of each heterogeneous variety is the same, i.e. \( x_{1,j} = x_{2,j} = \ldots = x_j \). In this case we get \( X_j = n \cdot x_j \). Hence, utility depends only on the total amount of consumption. In the Dixit-Stiglitz framework, in contrast, there is a direct utility gain from an increase of the number of firms/ varieties. Here, consumers profit from an increase of the number of firms only through the reduction of mark-ups by lower market power. We follow Blanchard and Giavazzi (2003, p. 882) in considering this effect of market entry to be the most important.

\(^{1}\)For a more general formulation see Blanchard and Giavazzi (2003, p. 881).
Maximizing the utility function (1) under a budget constraint yields the demand functions

\[ X_j = \frac{\beta y_j}{P} \quad \text{and} \quad x_{0,j} = \frac{(1 - \beta) y_j}{p_0} \]  \hspace{1cm} (3)

where \( y_j \) denotes the income of worker \( j \), \( p_0 \) is the price of the homogeneous good, and \( P \) is the price index of the heterogeneous goods, defined by

\[ P = \left( \frac{1}{n} \sum_{i=1}^{n} p_i^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} \]  \hspace{1cm} (4)

(see Blanchard and Giavazzi, 2003, p. 882). Income \( y_j \) of a worker is either the union wage rate \( w_i \) or the competitive wage rate \( w_0 \). Minimizing the expenditures for a given value of \( X_j \) yields the following individual demand function for variety \( x_i \):

\[ x_{i,j} = \left( \frac{P}{p_i} \right)^{\frac{\rho}{\rho-1}} \frac{\beta y_j}{np_i} \]  \hspace{1cm} (5)

Hence, aggregate demand for this good is

\[ x_i = \left( \frac{P}{p_i} \right)^{\frac{\rho}{\rho-1}} \frac{\beta}{np_i} Y \]  \hspace{1cm} (6)

and depends linearly on the total income of workers \( Y \equiv \sum_{j=1}^{N} y_j \).

2.2 Firms

Firms in both sectors maximize profits. The homogeneous good \( x_0 \) is produced employing exclusively labor from the competitive labor market. The good serves as a numeraire. Technology is assumed to be linear (no fixed costs), and standardized without loss of information to \( x_0 = L_0 \). Market entry occurs until firms just break even. This implies together with the assumed production function that the competitive wage rate is unity: \( w_0 = p_0 \equiv 1 \). The number of firms in the perfectly competitive sector is undetermined, but must be sufficiently large to guarantee perfectly competitive behavior.

Each heterogeneous good is produced by a different firm, employing respectively a fixed amount of \( \Delta \) units of labor from the competitive labor market. The amount of unionized labor input can be derived from the technology constraint

\[ L_i(x_i) = \frac{x_i}{\alpha} \]  \hspace{1cm} (7)
where the constant $\alpha$ symbolizes exogenous variable output per unionized worker. Profit $\pi$ of a representative firm reads

$$\pi_i = x_i \cdot p_i - L_i \cdot w_i - \Delta$$

After substituting $L_i$ by the technology constraint (7) and $p_i$ by the inverse demand function, maximization of $\pi_i$ yields the optimum price

$$p_i = \frac{w_i}{\alpha \rho} \quad \text{or} \quad p_i = \frac{w_i}{\alpha} \frac{\sigma}{\sigma - 1}$$

(8)

The mark-up over marginal costs is a negative function of the number of firms $n$, since $\rho$ depends positively on $n$.

Market entry is free and costless. Firms enter/exit the market until the profits of an additional firm would be negative, and profits of all incumbent firms are strictly nonnegative. In a symmetric equilibrium all firms $i \neq 0$ are equal ($x_i = x$, $p_i = p$, $L_i = L$, $w_i = w$ and $\pi_i = \pi = 0$).

2.3 Trade unions

Assuming that workers are distributed evenly across all firms in the heterogeneous sector\(^2\), the probability of a worker to receive the union wage rate is $nL/N$. The remaining workers must work for the competitive wage rate. Each trade union maximizes the expected utility of a representative worker, and bargains with a fraction $\gamma$ over the wage rate $w$ (“right-to-manage model”). The number of unions is thus $1/\gamma$.

Given our assumptions the expected utility of a representative worker is

$$U^+ = \frac{nL}{N} u_j(w, P^+) + \left(1 - \frac{nL}{N}\right) u_j(1, P^+)$$

if there is an agreement with the firms and

$$U^- = u_j(1, P^-)$$

\(^2\)If workers were distributed unevenly, some of them could increase the probability of an employment by reallocating themselves to a firm where less workers are attached.
if there is no agreement.\textsuperscript{3} $P^+$ and $P^-$ represent the price index of the heterogeneous goods, respectively in the cases of an agreement and of no agreement. The bargaining parties thus take into account that the price index differs in these two cases.

### 2.4 Timing of the model

Since our model is static, there is no chronological order of decisions, actions, and reactions. But, by assuming a specific informational status of the workers, firms and unions, we determine what may be called a logical order.

One of the main differences to Blanchard and Giavazzi (2003) is the way market entry is modelled. In their paper, firms face entry costs that play a similar role as the fixed costs do in ours. But since these costs are sunk costs, it is difficult to explain why the number of firms should shrink after a marginal deterioration of their economic situation. Blanchard and Giavazzi (2003, p. 891) argue that “firms which die are not replaced”. But it remains open why these firms should die in the model as long as profits are strictly positive. In our model all firms that enter the market actually have to bear the fixed costs. Thus, starting from a zero profit equilibrium, a deterioration of the firms’ situation leads to losses, pushing some firms out of the market. But - as the entry costs in the Blanchard and Giavazzi framework – fixed costs do not affect the wage bargain, if they arise independently of whether there is an agreement or not.

From these considerations the following logical order results: i) First market entry/exit decisions are taken. ii) Then fixed costs arise for those firms that have entered the market. iii) Wage bargains take place independently of each other. It must be assumed that unions and the corresponding firms know the resulting wages, prices and employment from all other bargaining units in the economy, e.g. through a heuristic process, which is terminated in the long-run equilibrium we look at. This common assumption allows us to abstract from the strategic interplay between different bargaining units. Once wages are determined, iv) goods

\textsuperscript{3}Since all agents fare better in the case of an agreement, this second term serves only as the ‘conflict point’ during the bargain, but is never realized.
are produced, sold and consumed. In a long-run equilibrium only those firms that can actually cover fixed costs enter the market, however.

2.5 The wage bargain

Unions and firms take into account the aggregate demand functions. They are equally aware of the responses of employment, workers’ income and prices regarding changes of the wage rate. In contrast, they take the number of firms in the heterogeneous sector as given, because it is determined “before” the bargaining.

The Nash product describing the asymmetric bargaining problem is

\[ NP = \gamma n (p_x - Lw) \cdot (U^+ - U^-)^\delta \]  

where \( \delta \) denotes the relative bargaining power of the union (Nickell, 1999, p. 3). Fig. 1 illustrates how the negotiated wage changes due to adjustments of the number of firms. \( \pi^v \equiv \gamma n (px - Lw) \) are the participating firms’ profits before the deduction of fixed costs, and \( \Theta \equiv (U^+ - U^-)^\delta \) is the additional utility the union obtains in case of an agreement, weighted by the relative bargaining power of the union. \( \pi^v \) clearly depends negatively on the wage rate \( w \). In comparison, the effect of a higher wage rate on \( \Theta \) may be positive or negative. Starting from a wage rate close to zero, the union welcomes wage increases unambiguously, since all workers remain employed in the primary sector, and utility of each worker is boosted. But if the wage rate surpasses a certain point, higher wages for employed workers have to be weighted against a loss of income for those who lose their job in the primary sector. Hence, the negotiated wage has to be between the competitive wage rate, corresponding with point \( C \), and the one the union would chose unilaterally (monopoly union, corresponds with point \( M \) in fig. 1), where the positive and the negative effects of an incremental wage rise exactly match each other. For a given curve \( P \), a point further to the right-hand side is associated with a lower wage rate.

The curves \( P^0, P^1 \) and \( P^2 \) depict which combinations of \( \pi^v \) and \( \Theta \) are feasible in principle, while the hyperbolas \( NP^0, NP^1 \) and \( NP^2 \) stand for different given values of the Nash product (9). A solution to the Nash bargaining problem corresponds with a maximum of the Nash product, depicted by points \( Q^0, Q^1 \) and \( Q^2 \).
Imagine that curve $P^0$ represents the set of combinations of $\pi^v$ and $\Theta$ that are feasible initially. The negotiated wage rate corresponds to point $Q^0$. Since firms’ revenues exceed fixed costs, newcomers are attracted, which reduces the prices of the goods, so that at any wage rate profits are lower. Feasible combinations are now represented by the curve $P^1$. The process of shrinking of the set of feasible combinations continues until the symmetric firms just break even with the wage rate that results from the bargaining (point $Q^2$). Then, a long-run equilibrium is reached.

From the union’s and the corresponding $\gamma n$ firms’ point of view, the heterogeneous goods’ price index depends on the agreed wage rate because the goods prices depend on the wage rate and the number of covered workers is not negligible relative to the entire economy. If we differentiate between covered and non-covered workers and the related variables\(^4\) (in the latter case $p_i$, $L_i$ and $w_i$ carry a bar, symbolizing that these variables are regarded as being given by the bargaining parties), (4) becomes

\(^4\)As noted earlier, wages, demand, labor input etc. are the same for all firms in a symmetric equilibrium (ex post). Nevertheless, the bargaining parties consider these variables to depend on the result of the bargain ex ante if they are related to them, and as exogenous if they are related to other firms/ workers.
\[ P = \left[ \frac{1}{n} \left( \sum_{i=1}^{\gamma n} p_i^{\rho^{\gamma-1}} + \sum_{i=\gamma n+1}^{n} \bar{p}_i^{\rho^{\gamma-1}} \right) \right]^{\frac{\rho-1}{\rho}} \]  

(10)

Since workers are distributed evenly across all firms, the number of workers per firm is \( N/n \). The probability of an employment at the union wage rate is thus \( L_i/(N/n) \) for workers who are member of the considered union, and \( T_i/(N/n) \) for all other workers. Hence, the expected income of a worker equals \( (nL_i/N)w_i + [1 - (nL_i/N)] \) for members of this union and \( (n\bar{L}_i/N)\bar{w}_i + [1 - (n\bar{L}_i/N)] \) for all workers that are members of other unions (notice that the competitive wage rate is unity). Total income from the point of view of the bargaining parties is

\[ Y = \sum_{j=1}^{\gamma N} \left[ \frac{nL_i}{N}w_i + \left( 1 - \frac{nL_i}{N} \right) \right] + \sum_{j=\gamma N+1}^{N} \left[ \frac{n\bar{L}_i}{N}\bar{w}_i + \left( 1 - \frac{n\bar{L}_i}{N} \right) \right] \]

\[ = \sum_{j=1}^{\gamma N} \left[ \frac{nL_i}{N}(w_i - 1) \right] + \sum_{j=\gamma N+1}^{N} \left[ \frac{n\bar{L}_i}{N}(\bar{w}_i - 1) \right] + N \]  

(11)

The resulting wage rate is not amenable to a formal representation in general. Only in the benchmark case of decentralized bargaining (\( \gamma \to 1/n \)) a closed form can be found, which is

\[ w_i|_{\gamma \to 1/n} = \frac{\delta + \rho}{\rho(1 + \delta)} \]  

(12)

All other variables follow from the wage rate in a straightforward manner. The effects of variations of the unions’ relative bargaining power \( \delta \) and transparency \( \zeta \) are derived analytically in subsection 4.2. Except from the special case of decentralized bargaining, numerical methods are appropriate to solve for the wage rate and all other endogenous variables. This exercise is carried out in section 3 Since \( \rho \) depends on the number of firms \( n \), the latter has to be derived before we can determine the wage rate. Other variables that are related to the macro level are the price index \( P \) and total income \( Y \).

### 3 The union wage gap

In this section, we solve the model developed in the previous section numerically. Employing statistical data to distinguish a set of countries, we are able to derive a
union wage gap which is relatively close to estimates from the empirical literature. If we would restrict the analysis to decentralized bargaining ($\gamma \to 1/n$), it could also be executed analytically. Yet, this parameter turns out to be a crucial one.

### 3.1 The data

A lack of data and the objective to work out differences between countries in a stylized and focused fashion forced us to restrict the numerical analysis to seven countries: The United States, the United Kingdom, West Germany, Denmark, Canada, Italy and Japan. These countries have been chosen because the necessary data have been available at least for some years, and because they are quite different from each other with regard to the degree of centralization of the wage bargains and the union coverage rate. If the data were available, we specified the parameters for the years 1980, 1985, 1990 and 1994.

The following parameters are chosen to distinguish a country’s specific situation at different points in time: union coverage, the size of the workforce, and “the degree of centralization of the wage bargains”, which correspond with the expenditure share of the heterogeneous good ($\beta$), the number of workers ($N$), and the fraction of covered workers who are member of one union ($\gamma$). Although the latter is relatively stable over time (Kenworthy, 2000, p. 13), we account for variations of it because the considered time span is fairly long, and because $\gamma$ affects the endogenous variables strongly. It is needless to say that these three parameters cannot give a sound impression of a country’s economic situation. Yet, it turns out that they suffice to explain much of the differences in the union wage gap between the included countries.

We adjust $\beta$ until the share of workers that are covered coincides with the bargaining coverage rate, taken from Traxler (1996, p. 274), supplemented by OECD (1997, p. 71), whenever the records were comparable.\footnote{Unfortunately, the definition and measurement of bargaining coverage is not unambiguous. One difference between the reported coverage rates is that some of them adjust for the fact that in several countries not all workers have the legal right to bargain. From the role of the parameter in the model is is clear that we must take the unadjusted coverage rate. Therefore, the OECD data were only viable for those countries where all workers have the right to bargain, so that both...} That is to say, we derive
the decrease of union coverage in many countries from an assumed relative decrease of the consumers’ valuation of goods produced in the unionized sector. This means that we abstract from many causes that may have influenced the coverage rates, too, e.g. the political environment, legislative measures etc. Examples for the shift of preferences away from the unionized sector’s goods are numerous and include sectors like ‘steel’ and ‘public transports’. There are several reasons why we draw on union coverage rather than union membership (density). First, recent evidence suggests that there is no union membership non-membership wage gap among the covered employees. Measured differences between the pay of trade union members and non-members seem to be caused by various unobservable variables, which cause e.g. a concentration of members in high paying workplaces (see (Booth and Bryan, 2001)). Second, the figures for coverage account for all sort of institutional and legislative differences between the countries in the sample. For instance, in Germany all workers whose employer is member of the employers’ association are covered, regardless whether they are member of a union or not. In many countries, union wages are legally extended to cover non-union firms. In these cases union density would not reflect the number of workers who are affected by collective contracts. Therefore, union coverage is the proper concept if one wishes to measure the true impact of unionism across different countries.

The most difficult decision is regarding the appropriate measure of \( \gamma \), the degree of centralization. There is an abundance of qualitative indicators designed to describe it (for a comprehensive survey see Kenworthy, 2000). In addition, some authors claim that coordination rather than centralization would be the appropriate measurement (Soskice, 1990). For several reasons, we chose to take Iversen (1998)’s indicator of wage bargaining centralization. First, it is available annually for all countries that we included.\(^6\) Second, the Iversen indicator takes account of small changes towards a more centralized or decentralized wage setting. In comparison, other indicators, like the one published by the OECD, are much more abrasive. It must not be concealed, however, that (with the exception of Canada and the U.S.) there are considerable differences between the alternative indicators (Kenworthy, \( \cdots \))

\(^6\)The 1994 value has to be taken from 1993, which is the last one published.
Therefore, our results are not robust to the choice of indicator. We displaced the origin of the Iversen indicator such that the smallest value, corresponding with firm-level bargaining, is zero.\textsuperscript{7} \(N\) is civilian employment, taken from the US Department of Labor (2002, p. 11).\textsuperscript{8} The 1980 value is standardized to unity, respectively. It should be noted that this parametrization does not account for variations in the number of hours worked per employee.

Why have these three parameters been chosen to characterize the countries in the sample? First, it turns out that one parameter does not suffice to describe the bargaining setting adequately. For instance, bargaining takes place at the firm level in the U.S. as well as in Canada. But union coverage is significantly higher in Canada, which yields a different outcome. The same applies if one compares the situation of the U.S. with Japan. Union coverage is roughly comparable, but bargaining is more centralized in Japan, leading to a noticeable lower union wage gap. Second, this contribution sets out to derive the union wage gap from features of labor markets and product markets. Therefore, it is desirable that at least one parameter is included, which is also related to the product markets (i.e. \(N\), the ‘number of workers/ consumers’). This permits to examine if and to which extent the size of the markets has an impact on the wage rate for a given structure of the labor markets.

In contrast, we chose not to vary the production technology across countries and time (parameters \(\alpha\) and \(\Delta\) in the model). Even though these parameters play an important role for the level and dynamics of real wages and income, we abstract from variations in them because our focus is on the comparative-static effects of the bargaining structure and the size of the unionized sector. In addition, it is difficult to obtain reliable data on costs. The latter is equally valid for the parameters \(\delta\) (relative union bargaining power) and \(\zeta\) (determining how strong the number of varieties affects the elasticity of substitution). We chose to employ the same parameter values for \(\zeta\) and \(\delta\) for each point in time and country because a lack of data would otherwise make the outcome additionally arbitrary.

\textsuperscript{7}To avoid computational problems, a value of 0.0001 rather than literally zero is the minimum (employed for the U.S. and for Canada).

\textsuperscript{8}This source converts the national data such that they approximate U.S. concepts. The Danish values stem from .
Table 1 specifies the parameter values employed, where union coverage, symbolized by $\psi$, is given in addition to the corresponding values of $\beta$. $\alpha$ is standardized to unity for the ease of computation. Fixed labor input $\Delta$ is 0.002, which causes a ratio of fixed costs to total costs (cost disadvantage ratio, CDR) within the range 13.6%–24.5%.$^9$ Furthermore, we specify the parameters $\delta$ and $\zeta$ as 1 and 0.1, respectively, implying symmetric bargaining and a relatively weak responsiveness of the elasticity of substitution with regard to market entry.

<table>
<thead>
<tr>
<th></th>
<th>Canada</th>
<th>Denmark</th>
<th>Germany</th>
<th>Italy</th>
<th>Japan</th>
<th>U.K.</th>
<th>U.S.A.</th>
<th>Each country/year</th>
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<td>0.000</td>
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<td>0.257</td>
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<td>1.015</td>
<td>1.000</td>
<td>0.982 1.055</td>
</tr>
</tbody>
</table>

3.2 Results

Figure 2 shows the relative differences between the union wages and the competitive wage rates in the model. The highest wage gap is found in the U.S., where it equals 14.7% in 1980, increases to 16.3% in 1985 and then decreases to 15.9% in 1994. Canada features the second highest value, which is roughly 11.5%. The U.K. wage gap increases from 8.2% in 1980 to 9.7% in 1994. In contrast, the union wage gap decreases in Japan from about 10.5% in 1980 to 7.2% in 1990. In the time span,$^9$ these values are comparable with those of Elbehri and Hertel (1999).
the Danish union wage gap increases from 3.0% to 5.2%, and then decreases to 4.2% in 1994. The German and Italian values remain nearly constant from 1980 to 1990 (5.4% and 7.1%, respectively). In 1994 the Italian wage gap falls to 5.9%, whereas the German figure could not be calculated because of missing data.

Decentralized wage bargaining apparently explains the high wage gap in Canada and the U.S. Unions disregard the negative effect higher wages have on the aggregate price level, since the number of represented workers is small relative to the total workforce. The inverse accounts for Denmark, where wage bargains concern a large fraction of the workforce, and the wage gap is the lowest. Danish unions internalize the negative effect higher wages have to a great extent. This effect is well explored in the literature (for a short summary of various external effects that play a role see Boeri, Brugiavini and Calmfors, 2001), and it causes much, yet not all of the differences between the countries’ development of the wage gap.
Since Canada and the U.S. have both decentralized bargaining, and relative variations of the total workforce are similar, differences in union coverage explain why Canada’s wage gap is roughly 4 percentage points lower. But why should higher union coverage yield a lower wage? Usually, it is taken for given that coverage is a proxy for a union’s bargaining power, which is supposed to have a positive effect on the negotiated wage rate. In our framework, in contrast, a higher coverage rate is caused by a higher expenditure share of the unionized sector of the economy, so that there are more monopolistically competitive firms. This implies that the heterogeneous goods become closer substitutes ($\rho$ increases) so that the optimum prices and the firms’ ability to accrue rents diminish. Therefore, union wages decrease. The moderate increase of the wage gap in the U.K. can also be explained by changes of the expenditure share. In all other countries the expenditure shares remained relatively stable.

The decrease of the unionized sector in the U.S. in the considered time span did not lead to a relevant modification of the wage gap, however, because the relatively strong increase of the total workforce works against this effect. More workers lead to a higher number of firms, which enhances competition in the goods markets. This reduces rents and union wages. In Canada, this effect yields the moderate decrease of the wage gap from 1985 to 1990. For all other countries, the variations of the total workforce are small relative to the variations of union coverage and the degree of centralization.

What do our results imply for real income? If we deflate wages by the consumer price index (15), we find that the highest real wages in the primary sector occur in the U.S. Here, and in the U.K., we also observe the strongest increase of the real wage rate during the 1980s. The reason is, besides the modest increases of union wages in these countries, that the expenditure share $\beta$ of the goods produced by firms that face union wage bargaining has dramatically declined in the considered time span. This lowers the cost-of-living price index (15) and raises real wages. In the U.S and in the U.K. ever less workers profited ever more from the existence of trade unions. Why this happens seems a question worth being investigated.
3.3 Comparison with empirical evidence

There is no lack of empirical work on the union wage gap. The problem is rather that the existing empirical literature is partially inconsistent because of differences with regard to the employed data and methodology. Therefore, it is advisable to keep at one source to increase comparability. We chose Blanchflower (1996), supplemented by Blanchflower and Bryson (2003), since it is the only study we found that includes nearly all countries for which we parameterized the model in the considered time span (except from Denmark). The underlying data from the International Social Survey Programm Series (ISSP, 1985–1993) does not allow to control for important variables such as industry, which is likely to bias the estimations upwards. More reliable are the results in Blanchflower and Bryson (2003), using the MORG files of the CPS for the U.S. and the British Social Attitudes Surveys (BSAS) for the U.K., which we averaged for the time span 1985–1993 to make them comparable. Table 2 contrasts the estimates of Blanchflower and Bryson (2003) for the U.K and the U.S., and Blanchflower (1996) for the rest of the countries with our model’s results. For Italy, the U.K. and the U.S. the differences appear small. The estimated wage gap for Japan (47.8%) is much higher than what the model predicts (and also much higher than what we think is plausible). The estimates for Germany (3.4%) and Canada (4.8%) are lower than what our model predicts. However, Blanchflower and Freeman (1992) find a wage gap of 6% for West Germany, which again is nearby the model’s result. Blanchflower (1996, p. 23, footnote 17) cites several studies that estimate wage gaps for Canada which are similar to the outcome of our model. Altogether the model’s results seem to be relatively close to the estimated wage gaps, in particular in those cases where the latter are reliable.

One reason why estimated wage gaps may not be reliable is that empirical studies frequently use union density (membership) as a measure of union influence. This may be reasonable for the U.S., Japan and Canada, where union density and coverage are roughly equivalent. In Continental Europe substantial gaps between union density and coverage prevail, however. An extreme case is France, where union membership is about 10%, and where coverage of collective agreements is
Table 2. Union wage gap in the model and empirical evidence

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>Canada</td>
<td>11.9</td>
<td>11.0</td>
<td>11.3</td>
<td>4.8(^a)</td>
<td></td>
</tr>
<tr>
<td>Denmark</td>
<td>3.0</td>
<td>4.1</td>
<td>5.2</td>
<td>4.2</td>
<td></td>
</tr>
<tr>
<td>Germany</td>
<td>5.4</td>
<td>5.5</td>
<td>5.2</td>
<td>3.4</td>
<td></td>
</tr>
<tr>
<td>Italy</td>
<td>7.2</td>
<td>7.2</td>
<td>7.1</td>
<td>5.9</td>
<td>7.2</td>
</tr>
<tr>
<td>Japan</td>
<td>10.5</td>
<td>8.8</td>
<td>7.2</td>
<td>47.8</td>
<td></td>
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<tr>
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<td>9.6</td>
<td>9.7</td>
<td>7.3</td>
</tr>
<tr>
<td>U.S.</td>
<td>14.7</td>
<td>16.3</td>
<td>16.2</td>
<td>16.0</td>
<td>19.2</td>
</tr>
</tbody>
</table>

\(^a\) not significantly different from zero

Sources: Blanchflower (1996), Blanchflower and Bryson (2003), own calculations about 95% (see Visser (2003)). This specification biases the estimated union non-union wage gap downwards, which may partially explain the difference between the simulated and the estimated wage gap for Germany. Another important reason for differences between our results and some estimations is selection. Longitudinal evidence has shown that there is positive selection into unions among low-skilled workers, and negative selection among high-skilled workers (Hirsch, 2004), which biases the measured effect of unions on wages. In some countries, however, the results of the bargains are de facto or even legally extended to workers who are not member of a union. In these cases selection can only have a very limited impact.

How robust are our results? First, we checked the sensitivity of the results with respect to variations of the exogenous parameters \(\alpha\), \(\Delta\), \(\delta\) and \(\zeta\). None of them has a strong impact on the union wage gap. Therefore, we conclude that the parameters that describe the countries in the numerical analysis (\(\beta\), \(\gamma\) and \(N\)) are indeed decisive for the union wage gap. In contrast, employment, the elasticity of substitution, prices, and the number of firms depend more on the choice of the former parameters, which determines e.g. real income. Therefore, an analysis aiming at explaining the course of real income would have to account for their role more accurately than we do.
A second possibility to check our results is to compare computed variables (other than the wage gap) with estimates in the literature. We perform such a comparison with respect to the markup of prices on marginal costs, taken from Oliveira Martins, Scarpetta and Pilat (1996). Before we turn to the results of this comparison, some words of caution are in order, however. First, the employed method of estimation, put forward by Roeger (1995), produces a downward bias with increasing returns to scale. Thus, the estimates are likely to represent a lower bound. Second, the estimated markups are the average of sectoral markups in the period 1980-1992, weighted by 1990 production shares in manufacturing. Therefore, they can only be used to give a rough impression of the differences between countries.

Table 3. Markups on prices in the model and empirical evidence

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
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<td>20</td>
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<td>Denmark</td>
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<td>Germany</td>
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<td>21</td>
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<tr>
<td>Italy</td>
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<td>19</td>
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<tr>
<td>Japan</td>
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</tr>
<tr>
<td>U.K.</td>
<td>20</td>
<td>15</td>
</tr>
<tr>
<td>U.S.A.</td>
<td>32</td>
<td>14</td>
</tr>
</tbody>
</table>

Source: Oliveira Martins et al. (1996), own calculations

The third column of table 3 stems from Oliveira Martins et al. (1996, p. 25). The second column averages the markups we obtain in the numerical analysis for the respective countries at different points in time. Taking into account that the estimated markups are probably biased downward because of the implicitly assumed constant returns to scale, the results for Canada, Denmark, Japan and the U.K. are supported by the proving. The computed markups for Germany and Italy are quite close to the estimated values, but smaller than the latter. The ranking of the U.S. values is as supposed, but they display the largest differences. One possible explanation why the computed markup for the U.S. is too high is that here the neglected perfectly competitive sector with zero markup is the largest.
In summary, the following results seem to be fairly stable:

- The union wage gap largely depends on the degree of centralization of the bargains, and, to a somewhat lesser extent, on the expenditure share of the unionized sector’s goods and on the size of the employed labor force. At odds with a widely held view, the latter two have a negative effect on the wage gap because competition on the goods market is reinforced, which reduces the bargained wage rate.

- In contrast, the bargaining power of unions, commonly regarded as important explanatory factor, turns out to have only a limited influence on the union wage premium.

- Differences between countries with respect to real income per worker can partially be explained by the expenditure share of the unionized sector.

The fact that our model can explain some differences between the wage gaps of various countries and at various points in time leads us to the conclusion that it indeed covers some of the most important features of imperfect product and labor markets. Therefore, it is quite natural to ask how policy may influence the outcome according to the respective goal pursued, which is done in the following section.

4 Regulation of labor and goods markets

The basic theoretical model described in section 2 comprises of the equations that determine the wage rate $w$, variable employment per firm $L$, the product price $p$, demand for each variety $x$, number of firms $n$, the total income $Y$, the price index $P$ and the elasticity of substitution $\sigma$. These endogenous variables depend on the number of workers $N$, the expenditure share of the heterogeneous goods $\beta$, output per unionized worker $\alpha$, the fixed labor input $\Delta$, and on the parameters: $\delta$, $\gamma$, and $\zeta$. The latter can be thought of as being determined at least partially by policymakers:
The parameter $\zeta$ impacts on the responsiveness of the elasticity of substitution between any two varieties, $\sigma$, with respect to market entry. This parameter is referred to as ‘transparency’, because only if a market is sufficiently transparent, the number of competitors impacts on consumer behavior. Admittedly, this definition is much narrower than the vernacular meaning of ‘transparency’. But in the context of our model, we do think that transparency may actually be seen as the ability of households to perceive and compare the supply of goods by different firms. How can economic policy influence this ‘transparency’? One instance are laws that bind firms on published prices or force firms to comply with specific norms (e.g. DIN or ISO standards). Although this is only a terminological difference, it should be noted that this interpretation is rather the inverse of the one Blanchard and Giavazzi have in mind for their related parameter $\bar{\sigma}$, where increases of the parameter are named “product market deregulation”. In our interpretation transparency and substitutability are enhanced rather than worsened by regulatory measures.

This parameter symbolizes the relative bargaining power of trade unions. If its value is zero, wages are competitive. The case $\delta \to \infty$ means that the union can set wages unilaterally (monopoly union). Pencavel (2002, p. 15ff.) depicts among other things how the Thatcher administration depressed union power in Britain. Examples of policy measures include the legislation on strikes, lock-outs, union governance and the closed shop.

Although $\gamma$ is a very important parameter in the model (see section 3), which may also be influenced by policy measures, we have to exclude it from the analysis. The reason is that a positive value of $\gamma$ renders the model so complicated that it cannot be solved analytically.

We now derive how variations of $\zeta$ (product market transparency) and $\delta$ (union bargaining power) affect the equilibrium values of some selected variables. We assume decentralized bargaining ($\gamma \to 1/n$), which is the standard in the bargaining literature. We calculate the responses of the equilibrium values of selected endogenous variables to variations of the policy parameters $\zeta$ and $\delta$. In doing so,
we focus on the union wage rate, and on the variables that are determined at the macro-level, i.e. aggregate income \( Y \), the price index \( P \), the number of firms \( n \), and, derived from the latter, the degree of substitutability, represented by \( \sigma \).

### 4.1 Equilibrium

In a symmetric equilibrium with decentralized bargaining, many analytical expressions simplify considerably. In particular, the wage rate is given by (12), which still contains the endogenous \( \sigma \), however. In order to keep the results simple and manageable, we do not substitute all endogenous variables. Instead, we try to display all results as compact as possible. In a symmetric equilibrium the price index (4) becomes

\[
P = p_1 = p_2 = \ldots = p
\]

If \( P = p_i \), demand for one heterogeneous commodity (6) becomes

\[
x_i = \frac{\beta Y}{nP}
\]

Hence, from the technology constraint (7)

\[
L = \frac{x_i}{\alpha} = \frac{\beta Y}{\alpha nP}
\]

The cost-of-living price index \( \hat{P} \) can be derived by a weighting of the prices in both sectors with the respective expenditure shares:

\[
\hat{P} = P^\beta \cdot 1^{1-\beta} = p^\beta
\]

In the symmetric case, aggregate income (11) becomes

\[
Y = nL(w - 1) + N
\]

As in the Dixit-Stiglitz framework the number of firms/heterogeneous goods is determined through the assumption that firms’ profits are zero in equilibrium. Setting \( \pi_i = 0 \), making use of equations (14) and (8), we get

\[
(1 - \rho)px - \Delta = 0
\]

With equation (13), and from the definition of \( \rho \), we know that \( 1 - \rho = 1/(\zeta n) \). Employing this information, the zero-profits condition yields:

\[
n = \sqrt{\frac{\beta Y}{\Delta \zeta}}
\]
The three equations for the price index $P$, total income $Y$ and the number of firms $n$ determine - together with the results and definitions derived before - the simultaneous long-run equilibrium. For convenience, we now recapitulate the most important results derived in section 2:

$$
\sigma = \zeta n > 1 \quad (2)
$$

$$
p_i = \frac{w_i \sigma}{\alpha \sigma - 1} \quad (8)
$$

$$
w_i|_{\gamma \to \frac{1}{\sigma}} = \frac{\delta + \rho}{\rho(1 + \delta)} \quad (12)
$$

$$
= 1 + \frac{\delta}{(\sigma - 1)(1 + \delta)} \quad (20)
$$

It is possible to verify the correctness of the model’s outcome by the redundant equilibrium condition of the homogeneous market: $x_0^{demand} = (1 - \beta)Y = N - n(L + \Delta) = x_0^{supply}$. The following variables are derived endogenously: wage rate $w$, variable employment per firm $L$, product price $p$, demand $x$, number of firms $n$, total income $Y$, price index $P$, and elasticity of substitution $\sigma$. Other variables, like the demand for the homogeneous good may be deduced from them. The results depend on the total number of workers $N$, output per unionized worker $\alpha$, the expenditure share of the heterogeneous goods $\beta$, the unions’ relative bargaining power $\delta$, the fixed labor input $\Delta$, the degree of centralization of the bargain $\gamma$, and on the parameter $\zeta$, which indicates how strong competition on the goods market is affected by market entry.

Making use of (19) and inserting the wage rate (20), we obtain for the heterogeneous price index

$$
P = \frac{(\delta \sigma + \sigma - 1)\sigma}{\alpha(\sigma - 1)^2(1 + \delta)} \quad (21)
$$

Inserting the wage rate (20) and the number of employed workers per firm (14) into nominal income (16), we get

$$
Y = \frac{\beta Y}{\alpha P(\sigma - 1)(1 + \delta)} + N
$$

Solving this equation for $Y$, and substituting $P$ by means of equation (21) obtains

$$
Y = \frac{(\delta \sigma + \sigma - 1)}{\delta \sigma + (\sigma - \beta \delta)(\sigma - 1)/\sigma}N \quad (22)
$$
In the following subsection, we assess how variations of the policy parameters \( \delta \) and \( \zeta \) affect the equilibrium described by the above set of equations.

### 4.2 The effects of goods and labor market regulation

The policy parameters have direct and indirect effects on the endogenous variables. In abbreviated form, the functional relationships are:

\[
P = P(\delta, \sigma); \quad Y = Y(\delta, \sigma); \quad n = n(\zeta); \quad Y = \sigma(\zeta, n); \quad \text{and} \quad w = w(\delta, \sigma) \quad (23)
\]

For instance, to calculate the total effect of an increase of the union bargaining power \( \delta \) on aggregate income \( Y \), we have to take into account the direct effect of \( \delta \) on \( Y \), which is given by the partial derivative, and the effect of \( \delta \) on substitutability, \( \sigma \), which also impacts on \( Y \) indirectly. Partial and total derivatives of variable \( v_i \) with respect to variable \( v_j \) are denoted \( \partial v_i / \partial v_j \) and \( dv_i / dv_j \), respectively. The total differential of \( Y \) from equation (23) reads

\[
dY = \frac{\partial Y}{\partial \delta} d\delta + \frac{\partial Y}{\partial \sigma} d\sigma
\]

where, if we hold \( \zeta \) constant,

\[
d\sigma = \frac{\partial \sigma}{\partial n} dn
\]

and

\[
dn = \frac{\partial n}{\partial Y} dY
\]

which yields the total derivative

\[
\frac{dY}{d\delta} = \frac{\frac{\partial Y}{\partial \delta}}{1 - \frac{\partial Y}{\partial \sigma} \frac{\partial n}{\partial Y} \frac{\partial \sigma}{\partial \delta}} \quad (24)
\]

In order to derive the total effects of variations in \( \zeta \) and \( \delta \), we first evaluate the partial derivatives of \( w \), \( P \), \( Y \), \( n \) and \( \sigma \) with respect to the endogenous variables and the policy parameters. These partial derivatives are given in the appendix. Furthermore, we define:

**Definition 1**

Substitutability between goods is *high* if \( (\sigma - 1)^2 + \delta \sigma (\sigma - 2) > 0 \), and is *low*.
otherwise. This implies that, if \( \sigma \geq 2 \), substitutability is always high, and if \( \sigma \to 1 \) it is low.

It should be noted that high substitutability is the more likely case, since, for substitutability to be low, \( \sigma \) must be considerably below 2. Employing definition 1 and the partial derivatives given in the appendix, we derive the following results for the case of decentralized bargaining:

**Proposition 1**

Aggregate nominal income, the number of firms, and substitutability between goods depend positively on union bargaining power \( \delta \).

**Proof:** Using equation \((24)\) and the partial derivatives given in the appendix, the total derivative of \( Y \) with respect to \( \delta \) can be expressed as

\[
\frac{dY}{d\delta} = \frac{2 N \beta \sigma (\sigma - 1)^2 (\delta \sigma + \sigma - 1)}{\delta [\sigma^2 - \beta (\sigma - 1)] + \sigma (\sigma - 1)} + \frac{1}{2 \delta^2 \sigma^3 + \delta \sigma^2 [4 (\sigma - 1) - \beta \delta] + (\sigma - 1)^2 (2 \sigma - \beta \delta)} > 0
\]

(25)

The expression has been rearranged such that the content within every bracket has a strictly positive sign. One can then see that the total effect is unambiguously positive.

The direction of the effect of a change in the union bargaining power on the number of firms is unambiguous and yields from the signs of the total derivative of \( Y \) and the partial derivative given in the appendix. It can be expressed as

\[
\frac{dn}{d\delta} = \frac{\partial n}{\partial Y} \frac{dY}{d\delta} > 0
\]

(26)

What is the economic reason for this result? Intuition would suggest that the strength of a union impacts negatively on the profits of firms so that the resulting equilibrium comes along with fewer instead of more firms. The result can be explained by a sort of collusion between the bargaining parties. Wage increases lead to proportional increases of the firms’ prices. Since the competitive wage rate remains unchanged, the firm’s total costs rise less than proportional. Thus, covered workers and firms profit to the disadvantage of workers who are not covered by the collective agreement.
Substitutability is measured by $\sigma$ or $\rho$, which depend directly on transparency, $\zeta$, and on the number of firms, $n$. Therefore, employing eqs. (26) and the partial derivative given in the appendix, and holding $\zeta$ constant, the effect of an increase of the relative union bargaining power, $\delta$ on $\sigma$, obtains as the unambiguously positive total derivative

$$ \frac{d\sigma}{d\delta} = \frac{\partial\sigma}{\partial n} \frac{dn}{d\delta} > 0 $$

(27)

□

Proposition 2
Product market transparency $\zeta$ impacts positively on substitutability between goods. The effect on nominal income depends on whether substitutability is high or low (see definition 1). By contrast, increasing transparency unambiguously decreases the equilibrium number of firms.

Proof: The total derivative of $\sigma$ with respect to transparency, $\zeta$, and holding $\delta$ constant, is

$$ \frac{d\sigma}{d\zeta} = \frac{\partial\sigma}{\partial\zeta} + \frac{\partial\sigma}{\partial n} \frac{dn}{d\zeta} $$

Substitution of the partial derivatives given in the appendix and rearranging yields

$$ \frac{d\sigma}{d\zeta} = \frac{\{(\sigma^2 - \beta(\sigma - 1))\delta + \sigma(\sigma - 1)\}\delta\sigma + \sigma(\sigma - 1)n}{\beta\sigma^2(2\sigma - \beta) + 4\delta\sigma^3(\sigma - 1) + (2\sigma - \beta\delta)(\sigma - 1)^2} > 0 $$

Since the content of each bracket is positive, it the whole expression is positive, too. The result does not hinge on the degree of substitutability.

The total derivative of $Y$ with respect to $\zeta$ is

$$ \frac{dY}{d\zeta} = \frac{\partial Y}{\partial \sigma} \frac{d\sigma}{d\zeta} $$

(28)

The sign of the partial derivative $\partial Y/\partial \sigma$ is ambiguous in general. Economically, this is caused by the following trade-off: On the one hand an increase in $\sigma$ decreases the mark-up, which increases demand and, thereby, augments employment and total income. On the other hand, wages decrease through the reduction of rents, which impacts negatively on total income. If $\sigma$ is high, the latter effect prevails, $\partial Y/\partial \sigma$ is negative (see eq. A1 in the appendix). Hence the total derivative has a negative sign. Since the scope for 'low substitutability’ is defined quite narrowly, this is the most likely case.
The derivative of the number of firms with respect to market transparency is
\[
\frac{dn}{d\zeta} = \frac{\partial n}{\partial \zeta} + \frac{\partial n}{\partial Y} \frac{dY}{d\zeta}
\]  
(29)
After substituting the partial derivatives given in the appendix, with \(\sigma = 1/(1-\rho)\), and the total derivative of \(Y\), (28), the expression becomes
\[
\frac{dn}{d\zeta} = -\frac{\{[1 - \beta \rho(1 - \rho)]\delta^2 + \delta \rho(2 - \beta) + \rho^2 + \beta \delta \rho^2 (2 - \rho) + \beta \rho^3 (1 - \rho)\}}{2\delta^2 (1 - \beta \rho) + \beta \delta \rho^2 (1 + 2\delta) + \beta \rho^3 (1 - \rho) + \delta \rho (4 - \beta) + 2\rho^2}
\cdot n^2 (1 - \rho) < 0
\]
Again, the terms have been grouped such that the expression within every bracket is strictly positive. Accordingly, the entire derivative is negative. This result is independent of whether substitutability is high or not, hence the first term on the right-hand side of eq. (29) (the partial derivative of \(n\)), which is clearly negative, eliminates the ambiguity of the second term, which stems from the possibly positive effect on nominal income.

The result concerning variations of market transparency is to be expected, whereas it may be surprising that stronger unions increase substitutability in the model. The reason is that stronger unions increase total nominal income, which has a positive effect on the number of firms.

**Proposition 3**
The union wage and the price-index of the heterogeneous goods depend positively on the union bargaining power and negatively on product market transparency.

**Proof:** See the appendix.

Clearly, the effects on \(p_i\) and on the cost-of-living price index \(\bar{P}\) have the same direction as those on \(P\). Deriving the effects on real income, i.e. on total nominal income, deflated by the cost-of-living price index, is somewhat more demanding, however. Since both, cost-of-living price index and total nominal income depend positively on the bargaining power of the unions, and negatively on product market transparency, it is interesting to see which effect on real income prevails.

**Proposition 4**
Total real income depends negatively on the bargaining power of the unions and depends positively on product market transparency.

**Proof:** See the appendix.
The latter results are unambiguous. In particular, they do not hinge on the degree of substitutability. It should be noted that not all workers fare worse with stronger unions. Real income of a primary-sector worker increases due to an increase of the bargaining power of the corresponding union. But real income of a secondary-sector employed falls, since his nominal income remains the same, while the price-index rises. The inverse reasoning applies for the effects of an increase of transparency.

**Proposition 4** is particularly interesting, because real income is the natural target of interventions in the model. It suggests that a government that grasps at increasing real income should impair unions and foster market transparency. In the model, such a combined policy would have a number of side-effects, however. Most importantly, both measures impact negatively on the number of firms in the heterogeneous sector of the economy. Nominal income, wages, and prices would decrease. The effect on substitutability is undetermined, since the decrease of union power impacts negatively, and the increase of transparency impacts positively on it. **Proposition 4** implies that unions disregard the negative impact of wage increases on the price-level, which harms workers who are not covered by the bargain. In other words, they partially externalize the negative effects of higher wages. In the model, the extent to which unions take this effect into account can be varied by the policy parameter $\gamma$, the degree of centralization of the bargain. This parameter has been set to zero in this section, because otherwise the total effects of variations in $\delta$ and $\zeta$ could not have been derived analytically.

## 5 Summary

The theoretical model developed in this paper is designed to analyze the interplay of imperfections on product and labor markets. One sector of the products markets is characterized by monopolistic competition, while the other is perfectly competitive. The labor market is dichotomized as well: some workers receive the higher wage rate determined by collective agreements, and some receive the competitive wage rate.
Our approach proves fruitful in explaining the so-called 'union wage gap', i.e. the difference in pay between workers who are or are not covered by collective bargaining. Employing statistical data characterizing some chosen countries, we numerically calculate the union wage gaps and compare the results with recent empirical estimates. In general, these figures are quite close. This is remarkable, since we neglected factors such as differences in the technology or the tradition of worker-employer relationships in the model, which thus seem to be of minor importance for the wage gap. We conclude that the model comprehends the most important channels through which products and labor markets interact.

Since we think that the model is capable to deal with imperfections on products and labor markets and their interactions, it is straightforward to ask how economic policy may influence the outcome in order to improve the conditions for workers and/or firms. We chose two parameters which are partially determined by regulatory/deregulatory measures: ‘Product market transparency’ and ‘union bargaining power’.

The former has a strong negative effect on the formation of rents by enhancing substitutability. This causes nominal wages, prices and the number of firms to decline, whereas real income increases. Union bargaining power impacts positively on nominal wages and income, but negatively on total real income. It has a positive effect on the number of firms and the degree of substitutability.

References


Appendix

A. Partial derivatives

The wage rate is given by (20) and reads

$$w = 1 + \frac{\delta}{(\sigma - 1)(1 + \delta)}$$

Its partial derivatives with respect to $\delta$ and $\sigma$ are:

$$\frac{\partial w}{\partial \delta} = \frac{1}{(\sigma - 1)(1 + \delta)^2} > 0 \quad \text{and} \quad \frac{\partial w}{\partial \sigma} = \frac{-\delta}{(\sigma - 1)^2(1 + \delta)} < 0$$

The price index for heterogeneous goods equals (21):

$$P = \frac{(\delta \sigma + \sigma - 1)\sigma}{\alpha(\sigma - 1)^2(1 + \delta)}$$

Its partial derivatives with respect to $\sigma$ and $\delta$ are

$$\frac{\partial P}{\partial \delta} = \frac{\sigma}{\alpha(\sigma - 1)^2(1 + \delta)^2} > 0 \quad \text{and} \quad \frac{\partial P}{\partial \sigma} = \frac{-(\sigma + 2 \delta \sigma - 1)}{\alpha(\sigma - 1)^3(1 + \delta)} < 0$$

Aggregate nominal income can be expressed as (22)

$$Y = \frac{(\delta \sigma + \sigma - 1)\sigma N}{\delta \sigma^2 + (\sigma - \beta \delta)(\sigma - 1)}$$

and its partial derivatives read

$$\frac{\partial Y}{\partial \delta} = \frac{\beta \sigma(\sigma - 1)^2 N}{[\delta \sigma^2 + (\sigma - \beta \delta)(\sigma - 1)]^2} > 0 \quad \text{(A1)}$$

$$\frac{\partial Y}{\partial \sigma} = \frac{-\beta \delta N [(\sigma - 1)^2 + \delta \sigma(\sigma - 2)]}{[\delta \sigma^2 + (\sigma - \beta \delta)(\sigma - 1)]^2}$$

The sign of the latter derivative depends on the sign of $(\sigma - 1)^2 + \delta \sigma(\sigma - 2)$, which is ambiguous in general. If substitutability is high (see definition 1), the term is positive, and the partial derivative has a negative sign.

The number of heterogeneous firms is (see equation (17))

$$n = \sqrt{\frac{\beta Y}{\Delta \zeta}}$$
which yields the partial derivatives
\[
\frac{\partial n}{\partial Y} = \frac{n}{2Y} > 0 \quad \text{and} \quad \frac{\partial n}{\partial \zeta} = -\frac{n}{2\zeta} < 0
\]

Substitutability is measured by \(\sigma\):
\[
\sigma = \zeta n
\]
(see equation (18)). Its partial derivatives are respectively
\[
\frac{\partial \sigma}{\partial n} = \zeta > 0 \quad \text{and} \quad \frac{\partial \sigma}{\partial \zeta} = n > 0
\]

B. Proof of proposition 3

The signs of the total derivatives of \(w\) and \(P\) with respect to product market transparency, \(\zeta\), follow directly from the results of proposition 2 and the partial derivatives with respect to \(\sigma\):
\[
\frac{dw}{d\zeta} = \frac{\partial w}{\partial \sigma} \frac{d\sigma}{d\zeta} < 0
\]
and
\[
\frac{dP}{d\zeta} = \frac{\partial P}{\partial \sigma} \frac{d\sigma}{d\zeta} < 0
\]

Deriving the direction of the impact of more powerful unions on the nominal union wage rate and the price-index requires a closer look because there are respectively two opposite effects:
\[
\frac{dw}{d\delta} = \frac{\partial w}{\partial \delta} + \frac{\partial w}{\partial \sigma} \frac{d\sigma}{d\delta} > 0
\]

and
\[
\frac{dP}{d\delta} = \frac{\partial P}{\partial \delta} + \frac{\partial P}{\partial \sigma} \frac{d\sigma}{d\delta} > 0
\]

Substitution of the partial derivatives and transformations (\(\sigma = 1/(1 - \rho)\)) yield
\[
\frac{dw}{d\delta} = \frac{\beta \rho^3 (1 - \rho) + 2\rho^2 + 2\beta \delta \rho^2 + 2\delta \rho (2 - \beta) + \delta^2 [2 - 3\beta \rho (1 - \rho)]}{2\delta^2 (1 - \beta \rho) + \delta \rho (4 - \beta) + 2\rho^2 + \beta \delta \rho^2 + 2\beta \delta^2 \rho^2 + \beta \rho^3 (1 - \rho)} \frac{1 - \rho}{\rho (1 + \delta)^2} > 0
\]
and
\[
\frac{dP}{d\delta} = \frac{2\delta^2 [1 - 2\beta \rho (1 - \rho)] + \delta \rho (4 - 3\beta) + \rho^2 (2 - \beta) + 2\beta \delta \rho^2 + \beta \rho^3 (2 - \rho) + \beta \delta \rho^3}{2\delta^2 (1 - \beta \rho) + \delta \rho (4 - \beta) + 2\rho^2 + \beta \delta \rho^2 + 2\beta \delta^2 \rho^2 + \beta \rho^3 (1 - \rho)} \frac{1 - \rho}{\alpha \rho^2 (1 + \delta)^2} > 0
\]

The terms have been arranged such that the content of each bracket is strictly positive (note that \(\rho (1 - \rho) \leq 1/4\), since \(0 < \rho < 1\)). □
C. Proof of proposition 4

Total real income is defined as total nominal income, $Y$, deflated by the cost-of-living price index $\hat{P}$. The total derivative of real income with respect to union bargaining power, $\delta$ is

$$\frac{dY}{d\delta} = \frac{dY}{ds} \frac{\hat{P}}{\hat{P}^2} = \frac{1}{\hat{P}} \cdot \left( \frac{dY}{d\delta} - \frac{Y \beta dP}{\hat{P} \cdot d\delta} \right)$$

Inserting the previous results in (25) and (A2), and simplifying gives

$$\frac{dY}{d\delta} = \frac{2\delta^2(1-\beta\rho) + 2\delta^2\rho(1-\beta) + \delta \rho(4-3\beta) + \rho^2(2-\beta)(1+\delta) + \beta \rho^3}{2\delta^2[1-\beta \rho(1-\rho)] + \delta \rho(4-\beta) + \beta \delta \rho^2 + 2\rho^2 + \beta \rho^3(1-\rho)} \cdot \frac{\beta Y(1-\rho)}{(1+\delta)(\rho+\delta)} \cdot \frac{1}{\hat{P}} < 0$$

An analogous proceeding yields

$$\frac{dY}{d\zeta} = \frac{2\delta^2[(1-\beta\rho(1-\rho)] + \delta \rho(1-\beta) + (3-2\rho^2) + \beta \rho^2 + \rho^2(1-\rho)}{2\delta^2[1-\beta \rho(1-\rho)] + \delta \rho(4-\beta) + \beta \delta \rho^2 + 2\rho^2 + \beta \rho^3(1-\rho)} \cdot \frac{\beta Y(1-\rho)}{\rho \zeta} \cdot \frac{1}{\hat{P}} > 0$$

Since the expressions have been arranged such that the content of each single bracket is positive, the signs of the total derivatives can easily be verified. □