Optimal Health and Retirement Policies amid Population Aging

by

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Abstract: This paper develops a simple analytical framework in which optimal health and retirement policies amid population aging can be discussed. To be efficient, these policies must recognize and exploit the dynamic complementarities between the timing of retirement, the size of lifecycle labour income and pension payments and investments in health that individuals make, for example, by purchasing medical care and that society makes by advancing medical technology. We aim to show how the traditionally separate areas of health and retirement policy can be coordinated to achieve dynamic efficiency. Under fairly general assumptions, postponing the age of retirement and greater health spending are shown to be complements in the maximization of lifecycle utility. Mandatory retirement and pension policies that change the constraints workers face can be used to induce voluntary health investments by individuals and improve society’s incentives to adopt new medical technology. Leaving a hitherto optimal mandatory retirement age unchanged as new medical technologies improve the efficacy of healthcare would be inefficient. The aggregate ability and willingness to pay for medical care and technology will be greater, the higher an economy’s per capita income, suggesting large welfare gains from postponing the average age of retirement if investments in new medical technology target the quality of life and raise the productivity of people working past a long-established mandatory retirement age.

Keywords: Medical technology, Longevity, Health policy, Retirement age

JEL classification: I12, I18, J26

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I. Introduction

The prospect of rapid population aging has made pay-as-you-go pension systems in many countries look fiscally unsustainable, unless they are fundamentally reformed. Similar claims are often made for healthcare although the demographic impact on per capita health spending is less clear. Investments in health and new medical technologies may in fact help to compress the time that people typically spend in morbidity before they die. This might mitigate the expected rise in health spending for the old. Moreover, many workers may be able to stay in their jobs well beyond the current retirement age, so that they remain contributors instead of becoming recipients of the pension system. These opportunities are often ignored and new medical technology is still widely seen as the main driver of rising per capita health spending – not as a solution, but as an additional threat amid population aging. “Can we afford to live longer in better health?” is the title of a recent generational accounting study for the EU-15 countries by Westerhout and Pellikaan (2005) in which they suggest: “To mitigate the effect of ageing on healthcare expenditures, (...) healthcare budgets may be frozen for several years, or expenditures cut, so that healthcare expenditures grow at a slower rate than GDP for several years.”

Our paper argues against such a pessimistic outlook and develops a simple optimization model to analyze the normative implications of new medical technologies, endogenous longevity and improvements in the quality of life for public pension and health policies. To be efficient, these policies must recognize and exploit the dynamic complementarity between the timing of retirement, the size of pension payments and advances in medical technology. The aggregate ability and willingness to pay for medical technology will be greater, the higher an economy’s per capita income, suggesting that there may be large welfare gains from postponing the average age of retirement if investments in new medical technology target the quality of life and raise the productivity of people working past the current retirement age. A successful coordination of health and pension policies will be essential to keep public-sector debt at sustainable levels during the 21st century, as recent estimates by the OECD (Queisser and Whitehouse, 2006) suggest that it would be politically unacceptable in many countries to make further cuts in mandatory pensions. Many poor people may not voluntarily save enough to complement their mandatory pensions so that raising the official retirement age seems to be the fairest and most reliable strategy to contain the costs of population aging.
The basic elements of our model can be summarized in terms of price and income effects on the timing of individual retirement, as illustrated by means of the utility indifference curves of a representative worker in Figure 1. In the simplest case, each worker has a fixed time budget over the lifecycle, such as $T_0T_0$, the only savings motive is consumption smoothing and there is no discounting. Assuming non-satiation in leisure and monetary income, an optimum is reached at $A$. Next, we assume retirement stops the health decline associated with working so that earlier retirement generates additional lifetime. This implies a higher relative price of work, illustrated by the new budget constraint $T_0T_1$. The new optimum, at $B$, may imply more time at work in absolute terms, but the substitution effect lets the lifetime share of work decline, depending on preferences and the relative size of income and substitution effects. In the graph, $B$ lies to the left of the ray, not actually drawn, through the origin and $A$. Finally, other health investments, such as across-the-board improvements in medical technology, move the original budget constraint from $T_0T_0$ to $T_2T_2$, equivalent to a pure income effect that leads to a new preferred optimum at $C$, where both leisure and working time are expanded in similar proportion. If an immovable mandatory retirement age had fixed working time at the level of point $A$, the welfare gains from better medical technology would obviously be smaller than those in $C$.

Our main interest is normative: How should improvements in health affect the rules about retirement age? How much investment in health would be justified with and without the opportunity to expand people’s working lives? To this end, our approach is based on a representative agent with a finite life, but largely ignores trades between generations, such as bequests, and the detailed social security design issues that are often studied within overlapping generation models in the spirit of Samuelson (1958) and Diamond (1965). We introduce population aging simply by extending individual lives. Exogenous changes in health that reduce mortality and increase life expectancy are associated with income and substitution effects that may work in opposite directions. On the one hand, a given stock of wealth has to finance consumption over a longer period of time, which is a negative income effect that tends to delay retirement. On the other hand, uncertainty about the length of life after retirement is reduced so that the relative price of retirement in terms of precautionary savings declines – a substitution effect in favor of earlier retirement. Overall, we believe that further gains in life expectancy in today’s aging societies will increase the demand for consumer goods and lifetime income, and hence the supply of labor.
In addition, one may assume that time spent working and leisure have different effects on health, so that health and life expectancy are endogenous to the timing of retirement. The health hazards of work are similar to a tax on labor income that induces early retirement. Better healthcare may lower that tax and yield income and substitution effects. For example, anticipated mortality reductions during the time in retirement may increase the demand for early retirement and thus give an additional boost to the health of the retired. By contrast, the reduction of health hazards at work may raise the supply of labor and delay retirement so that the health and remaining life expectancy of the retired rises less than it would with unchanged retirement age.

The relevant prior literature can be divided into three strands: the contribution of health to economic growth, the determinants of retirement timing, and the demand for health and longevity. Recent empirical studies in the first strand, such as Becker et al. (2005) and Nordhaus (2003), suggest we would underestimate economic growth by half if we did not include gains in health and longevity in our measure of output. Many of these gains seem to have been privately appropriated by workers spending an ever larger part of their lives in retirement. There is no consensus that the increasing divergence between retirement timing and longevity gains since the early 20th century has been efficient. However, this divergence may be one reason why microeconomic estimates of the health effect on per capita income, such as in Weil (2007), are likely to underestimate the social returns to better health substantially.

In the second strand of the literature, four major explanations with strikingly different efficiency implications have been proposed for the growing gap between retirement age and life expectancy. The first explanation is one of first-best efficiency as it sees time in retirement as a luxury good and argues that secularly rising lifetime incomes have led workers to optimally choose a larger period of leisure at life’s end (Costa, 1998). The second explanation is a case of the second-best, arguing that changes in production technology have lowered the productivity of older workers and turned them into a source of negative firm-level externalities so that employers seek to get rid of them (Sala-i-Martin, 1996). Pensions may be an efficient form of severance pay, unless older workers can be made to internalize their negative impact on younger colleagues in other ways, for example through a lower wage. The third explanation sees capital market failures, such as borrowing constraints, as preventing workers from hedging against health and mortality risks. Kalemli-Ozcan and Weil (2002) argue that declining mortality has reduced workers’ uncertainty about the date of their death.
and hence largely eliminated the uninsurable risk of dying before the fruits of saving for retirement can be enjoyed. Saving for retirement has thus become individually rational, and a falling retirement age in the population at large is consistent with a correctly anticipated increase in longevity. The fourth explanation emphasizes inertia in the political system and argues that not just mandatory retirement, but public pension programs’ high implicit rates of wage taxation have pulled older workers out of the labor force regardless of their productivity (Gruber and Wise, 1998). This may have been a relatively small problem when pensions merely served as insurance against living longer than the normal retirement age, in most countries at 65, but many of today’s elderly have substantial unused productivity potential.

Moreover, the demand for health and longevity itself may be influenced by retirement and pension policies. In this vein, Philipson and Becker (1998) argue that mortality-contingent claims, by making wealth dependent on the duration of life, can induce behavior with a positive effect on longevity. They point out that the piece-rate incentive to seek greater longevity that pension annuities create represents an inefficient moral hazard when longevity-enhancing behavior cannot be observed and contracted upon. Mandatory membership in public pension systems without risk-adjusted premiums and the implied intergenerational transfers tend to exacerbate these distortions. Moreover, Philipson and Becker (1998) argue that public healthcare systems undermine incentives for prevention and set too much incentive for treatment because people favor high levels of consumption when healthy rather than sick. Hence, public pensions and healthcare reinforce each other in distorting investments towards longevity, at the expense of quality of life.

The remainder of this paper is organized as follows. Section 2 discusses the rationale for a mandatory retirement age. Section 3 introduces a simple model of individual retirement timing, on which sections 4 and 5 build to discuss optimal investments in health and the issue of coordination between health and pension policies, respectively. Section 6 provides a general discussion and concludes.
II. Rationale

To establish a rationale for retirement, we distinguish between individual choice and legal mandates, including the regulation of retirement age. Consider the individual perspective first: Would workers want to retire in the absence of mandatory retirement and regulated pensions? And if so, does workers’ desire to retire ultimately depend on the reality of growing old and on the inevitability of death? Without mortality, people might have to work forever, unless they inherit wealth or succeed in saving sufficiently to live from a perpetual annuity.

Mortality is not expected to be eliminated in the foreseeable future, but modern medicine will continue to push the rate of mortality down, especially in older age groups. For many people in the developed world, scarcity of material resources, such as food, is no longer a binding constraint; yet biological constraints on the human lifespan seem to persist. Biological constraints are often thought to be at least partly genetically determined. To the extent that genes are responsible for cellular maintenance systems, such as DNA repair and antioxidant defense, and thus indirectly for the gradual accumulation of molecular faults in cells and organs that determines the speed of human aging and the apparent maximum lifespan of approximately a century, they may be interpreted as part of an evolutionary adaptation to man’s natural environment during the many millennia before the advent of civilization.

Natural aging and death are realities that man appears to share with all species whose mode of reproduction is sexual. To be sure, species that do not reproduce sexually also tend to die, say, of an accident, a fight, a disease, or as victim of a predator. Yet, sexual reproduction is more costly and tends to carry a higher premium on doing it early in life. One is tempted to see pre-programmed mortality as the flipside of sex. However, it would be too simple to argue that aging and death are an inevitable consequence of natural selection because the gene’s interest is best served by spending scarce food on raising the young, not on maintaining the old whose survivorship would compete with potentially more promising genetic combinations among the young, such as re-combinations through which sexual reproduction is thought to prevent the irreversible accumulation of deleterious mutations. As Dawkins (1976) points out, this sort of explanation would imply an implausible altruism of individual genes, willing to sacrifice themselves for the benefit of other genes of the same species. It would also be difficult to reconcile with the observation of negligible and even
negative senescence in many plants and some animals and with life expectancy that is often much longer than the maximum reproductive age.

A more plausible explanation, in which mortality is endogenous and consistent with a human lifespan well beyond menopause, can be derived from the so-called disposable soma theory of aging. In all organisms where the somatic line of cells, essentially the body, has segregated from the germ line, the sex cells, an increasing rate of mortality with age may be a dynamically efficient way of containing the costs of having somatic cells (Kirkwood, 1990). The body merely provides nurture and protection for the sex cells so that they can reproduce successfully. A large body at the time of maturity may be helpful in terms of fertility, in attracting mates and in raising the offspring, but as Robson and Kaplan (2007) argue, maintaining a large body over time comes at a cost in terms of quality, especially in species characterized by determinate growth. Assuming each cell incurs its own constant per-period maintenance costs, the total cost of investment in the organism’s quality will be increasing in the number of cells. Maintaining the relatively small number of germ cells is cheap, but reproductive behavior and fertility itself are costly. With convex cost functions for quality and fertility, the speed of aging and the rate of mortality can be derived as unique solutions to an optimization model, as shown by Robson and Kaplan (2007).

On this basis, forecasts of future human mortality must take into account the interplay of many factors, including work, income, education, nutrition, sanitation and access to medical care, whose relative weight may vary with the organism’s age, cohort and place and time of living. Before 1950, most of the gain in life expectancy was obtained through mortality reductions at younger ages, whereas – at least in the developed countries – improvements in survival after age 65 are now the dominant cause. Oeppen and Vaupel (2002) argue past predictions were often misled by the assumption that biological barriers impose an immovable ceiling on average longevity. Temporary slow-downs in longevity growth may be observed in laggard countries catching up, such as Japan after World War II, but best-performance life expectancy across all countries with reliable population statistics reveals a stable long-term trend that has added a quarter of a year annually for more than 150 years.

The continuation of this trend may eventually reach the point where mortality among the survivors of age intervals with major aging-related health risks, such as cancer between 50 and 70 years, no longer rises with advanced age. Among the oldest old, mortality may even decline. Negative senescence already appears to be widespread in plants and some animals characterized by indeterminate growth (Vaupel et al., 2004). In these cases, fertility also
often increases with age, while the average age of somatic cells trails the organism’s chronological age. With sufficient repair, replacement and rejuvenation, slow increases or even decreases in the average age of somatic cells are conceivable. An example is provided by the leaves of deciduous trees that are renewed every year.

In the modern knowledge economy, growth of income is potentially unbounded and the effectiveness of medical technology in repairing shocks to health, eliminating diseases as a cause of death and slowing the aging process has largely replaced nutrition, sanitation and working conditions as the ultimate constraint on best-performance life expectancy (Cutler et al., 2006a). Human organ transplantation is an example of medical technology with rejuvenating effect. With the exception of kidneys, transplantation is still relatively rare. But in the future, science may pave the way for the manipulation of stem cells to grow any kind of organ or tissue in need of replacement, and systematic rejuvenation may become commonplace. In the meantime, xenotransplants may offer another route to rejuvenation, provided current problems with immunosuppressant drugs and the risk of cross-species transfers of infectious disease agents can be overcome. As medical technology’s ability to slow human aging improves, retirement policies will have to be fundamentally reconsidered.

Taking aging and death as givens, for now, we still need to ask why people do not choose to allocate work and leisure equally across all periods of their finite lifetime. There would be no retirement in the sense of a discrete choice to leave the labor force, but only equal adjustments on the intensive margin in all periods. This would be consistent with perfect consumption smoothing even in the absence of capital markets, provided productivity is constant. The rub is that productivity cannot be constant when finite lifetimes are associated with the depreciation of human capital, in the form of health, skills and acquired knowledge. The depreciation of health, the essence of aging and often accelerated by effort at work, is an obvious reason for declining productivity.

A second reason is that skills and vintages of acquired knowledge may become obsolete as the economy’s general stock of knowledge grows. While this establishes a case for continuing education and training, distributing the acquisition of skills and knowledge equally across all periods of a finite life would not be optimal. To the extent that efficient forms of learning, such as formal schooling, require time away from work and may involve additional fixed opportunity costs, the bulk of it must be concentrated before the onset of working age, in which the fruits of learning are to be exploited. Even if health were constant, the payoff from acquiring new knowledge would decline with age, simply because the individual’s remaining
lifetime declines. The elderly thus rationally tend to have less marketable human capital and lower productivity. And the observed concentration of a large part of total lifetime leisure at the end of life is an efficient choice.

To establish an additional rationale for mandatory retirement and for a mandatory retirement age presupposes that the individual rationale is somehow incomplete or inefficient, for example due to social interactions. In this vein, Sala-i-Martin (1996) hypothesizes that public pensions around the world are linked to retirement from the workforce because positive externalities in the average stock of human capital imply a negative effect of the elderly’s lower-than-average skills on the productivity of younger workers. Pensions, by inducing retirement, may hence help to raise aggregate output, especially where rigid seniority in wage structures and job protection rule out other adjustments and where organizational inertia preclude younger workers from protecting themselves individually against the negative externalities of the elderly. Externalities in the use of human capital are considered a key aspect of the modern knowledge economy and institutions addressing the associated incentive problems are important element of governance structures.

In addition to Sala-i-Martin’s (1996) theory, which is based on a true technological externality, we see a further rationale for a mandatory retirement age in countries with equal access to universal healthcare. It is based on the pecuniary externality generally associated with third-party payer systems. Without a mandatory retirement age, workers might have an incentive to overwork their bodies by staying in the workforce too long – anticipating to bear only the burden of irreparable health shocks, not the direct pecuniary cost of care.
III. Individual Retirement Timing

To prepare the ground for our subsequent discussion of optimal choices and policies, this section discusses the implications of exogenous price and income changes for the timing of individual retirement without allowing for endogenous investments in health or adjustments in government policies. We begin by introducing the retirement choice in a simple model of utility maximization in which workers value both consumption goods and leisure and face a binary choice of working full time or not at all. Workers adjust only the fraction of total lifetime spent working, not the intensive margin. Optimality requires that the marginal cost of delaying retirement, the instantaneous utility gain from retirement, is equal to the marginal benefit, the utility of additional earnings from continued work.

A simple way to motivate a graphical illustration, such as Figure 1, is to incorporate a retirement choice in the two-period labor-supply-model suggested by Becker (2007), where the per-period utility $u_i = u_i(c_i, l_i)$ is a function of consumption goods $c_i$ and leisure time $l_i$. Our framework is characterized by time-additive separability and the amounts of leisure time in the two periods may differ, but they are determined simultaneously by an irreversible choice of the retirement age $R$ that falls either into the first or into the second period. In either case, survival of the initial period is certain, so that $S_0 = 1$, whereas the survival probability, or survivorship, in the next period $S_1(R, H)$ is a function of the retirement age $R$ and of the health state $H$ at the end of the first period. Workers’ time preference is introduced by a discount factor, $B$, so that $U = u_0(c_0, R) + BS_1(R, H) u_i(c_i, R)$ with the derivatives $S'_{iR} < 0$, $S'_{iH} < 0$ and $S''_{iH} > 0$, $S''_{iR} \leq 0$. Delaying retirement lowers survivorship, and more so the higher a given age of retirement because the risk of dying before the planned age of retirement rises with that age. Better health raises survivorship, but the marginal benefit is declining. Moreover, we assume better health mitigates the negative impact of delaying retirement so that $\partial^2 S_1(\cdot) / \partial R \partial H > 0$.

To keep the budget constraint simple, we follow Becker (2007) in assuming the existence of a fair annuity market that fully protects each individual against the risk of running out of resources in case of living longer than expected and against the risk of leaving unspent resources in case of an early death. This assumption rules out that the demand for retirement might rise in response to declining uncertainty about the time of death, as suggested by
Kalemli-Ozcan and Weil (2002). Let $w_i$ be the sum of wage and pension income in period $i$ in the absence of individual retirement timing and $z$ discounted additional earnings in case of later retirement, relative to what would be earned with retirement at the normal age, and assume $z$ to be an increasing function in both $R$ and $H$ with $\partial^2 z(R)/\partial R^2 < 0$ and $\partial^2 z(H)/\partial H^2 < 0$. Workers who retire before the normal age incur a negative $z$. Then, the present value of total wealth $W$ at the market rate of interest $r$ is

$$W = c_0 + S_i c_i/(1 + r) = w_0 + S_i w_i/(1 + r) + z(R, H, r) = W.$$ 

Maximizing the utility function with respect to $c$ and $R$ yields first order conditions that imply the equilibrium marginal rate of substitution between goods today and goods consumed in the future depends on the rates of time preference and interest, $u_{ic} = B(1 + r)u_{ic}$, but not on the survival rate. It is this independence of savings under full insurance and time-additive separability in the utility function that allows us to analyze continuous adjustments in the retirement age without abandoning the simple two-period set-up. Factors that influence the survival rate thus do not affect the savings rate and the choice of retirement age influences the level of savings only through the effect of the additional-earnings function, $z$, on lifetime income.

The first-order conditions also imply that the marginal benefit from earlier retirement equals the marginal cost,

$$u_{oR} + (u_{c, d} \log S_i/dR + u_{ic})S_i = u_{ic}[(c_i - w_i)(d \log S_i/dR)S_i/(1 + r) - \partial z(R, H, r)/\partial R],$$

where the marginal benefit depends on the effect of retirement on utility, the survival probability and the discount rate as well as on the level of survivorship and the level of utility in the future. The marginal benefits therefore increase with wealth. The marginal cost depends on the effect on survivorship, $\partial \log S_i/\partial R (< 0)$, on the level of survivorship, $S_i$, on the rate of interest, $r$, and on the slope of the additional-earnings function, $\partial z(R)/\partial R > 0$, which includes the retirement postponement effect on per-period wage and pension income, $w_{oR}$ and $w_{IR} (\geq 0)$.

This model suggests two reasons why people use early retirement to raise the probability of surviving in the future. First, the gain in expected lifetime raises the expected value of full wealth, net of forgone earnings, because the time gain is either used as leisure, namely in retirement, and directly enters the utility function or it is spent working to generate more income. Second, because $u$ is concave, average utility is greater than marginal utility and
additional lifetime adds average utility, while consuming more goods in a given year only adds marginal utility (Rosen, 1988). Even if some wealth had to be spread to finance consumption in more years, the undiscounted sum of annual utilities would rise because only marginal utility is lost in the years where consumption is cut to pay for the consumption in the new years, where average utility is gained. Discounting, to account for time preference, limits the utility gains from this substitution.

The specification of the utility function can make a crucial difference in this type of model. For example, if $u$ were characterized by constant returns to scale, there would be no difference between marginal and average utility and the only gain from a longer life expectancy would be the positive wealth effect of the greater endowment with time. However, when utility is concave and average utility exceeds marginal utility, retirement must happen earlier than at the wealth-maximizing age. In addition to the degree of concavity in the per-period utility function, it is mainly the degree of concavity or convexity of the additional-earnings function that determines the extent to which the optimal retirement age differs from the wealth-maximizing retirement age.

As Figure 2 illustrates, the marginal additional-earnings function, $\partial z(\cdot)/\partial R$, generally declines with declining health and thus has lower value at higher $R$. In addition, its shape varies with the way of determining workers’ pay. The function is concave if workers are paid in line with the seniority principle, with only weak reference to individual productivity. In this case, the marginal loss from retiring earlier is greatest for the oldest workers. By contrast, the additional-earnings function is convex if workers are paid strictly in line with a marginal productivity schedule that declines with work- or aging-related health deterioration. For example, elderly day laborers may find themselves in such a poor state of physical health that retiring early incurs next to no loss of wage income. Other relevant factors for the optimal retirement age include interest rates and the general level of health, which has a positive influence on the level and slope of the additional-earnings function. Improvements in health move the additional-earnings function up. And this effect is stronger for those paid in line with their marginal productivity.

Comparative statics can be used to analyze how exogenous improvements in health affect the individual timing of retirement. Better health implies a higher survival probability, $S$, and thus a gain in life expectancy – a positive endowment effect which workers will use to increase working time and time in retirement simultaneously, unless time in retirement is a
strong luxury good. In addition, better health may change the marginal effect of retirement timing on \( S_1 \). For example, better health may reduce the rate of health deterioration from work so that early retirement is less important for maintaining health over time \( \partial^2 S_1(\cdot) / \partial R \partial H > 0 \), a substitution effect in favor of work. Assuming better health raises potential income, both the level of \( z \) and its derivative with respect to \( R \) will increase with better health, \( \partial^2 z(\cdot) / \partial R \partial H > 0 \). Thus, even without specifying explicit functional forms for \( S_1 \) and \( z \), we can expect healthier workers will want to retire later. We avoid some of the ambiguity of previous analyses by ignoring any potential direct impact of health on the utility of consumption or leisure, such as greater enjoyment of leisure with better health, which would induce a substitution effect in favor of early retirement.

However, when we formally determine the effect of better health on the choice of retirement age, using the implicit function theorem,

\[
\frac{dR}{dH} = - \frac{Bu_{1R} \frac{\partial S_1}{\partial H} + \left[ Bu_1 + \frac{w_i - c_1}{1 + r} u_{oc} \right] \frac{\partial^2 S_1}{\partial R \partial H} + u_{oc} \frac{\partial^2 z}{\partial R \partial H}}{Bu_{1R} \frac{\partial S_1}{\partial R} + \left[ Bu_1 + \frac{w_i - c_1}{1 + r} u_{oc} \right] \frac{\partial^2 S_1}{\partial R^2} + u_{oc} \frac{\partial^2 z}{\partial R^2}},
\]

there is still some ambiguity. Both the numerator and the denominator may have either a positive or a negative sign. In the numerator, the first addend has a negative sign since the derivative of the survival probability \( S_1 \) with respect to \( H \) is positive and \( u_{1R} \) is negative, while the third addend has a positive sign, since better health has a positive effect on the otherwise negative impact of a higher retirement age on the additional-earnings function, \( z \). We assume the sign of the middle addend is also positive because better health mitigates the negative effect of later retirement on survival and the fact that dis-saving in old age implies \( w_i - c_1 < 0 \) is unlikely to matter because this difference is multiplied with the relatively small marginal utility of consumption in the first period. In the denominator, the first addend has a positive sign since \( u_{1R} \) and the derivative of \( S_1 \) with respect to \( R \) are both negative, while the third addend has a negative sign since the additional-earnings effect of delaying retirement declines with age. We assume the sign of the middle addend to be negative because the mortality impact of delaying retirement is greater at a higher age and the likely negative sign of \( w_i - c_1 \) does not matter.
For better health to have the effect of delaying retirement, the numerator and denominator must have different signs. This will be the case if the derivatives of the survival probability $S_1$ with respect to $H$ and $R$ are either both so large that the first addends dominate the latter two or both so small that the latter two addends alone determine the signs of the numerator and denominator, respectively. Put differently, the direct effect of changing survivorship in terms of discounted marginal utility must outweigh the sum of the implied change in the survivorship impact of delaying retirement on utility in the first and second period and of the implied change in the additional-earnings impact of delaying retirement on utility.

By contrast, better health need not lead to a postponement of retirement if – relative to the second effects – the positive marginal effect of better health on survival is very large, while the negative marginal effect of postponing retirement on survival is very small, or vice versa. A very large improvement in survival might give the negative marginal utility of delaying retirement, $u_{1,R}$, too much weight. Also a very large negative effect of delaying retirement on survival that is not accompanied by a large increase in earnings and a large effect of better health on survival establishes an obvious case in which later retirement is unattractive. In a similar vein, better health may fail to induce delayed retirement in cases of extreme divergence in the size of the second derivatives of $S_1$ and $z$ with respect to $H$ on the one hand and the derivatives of $\partial S_1(\cdot)/\partial R$ and $\partial z(\cdot)/\partial R$ with respect to $H$ on the other hand. However, as a vast empirical literature shows, healthier individuals generally retire later (see, e.g., Hostenkamp and Stolpe, 2006).
IV. Endogenous Investment in Health

The simple model developed in the previous section allows us to discuss the implications of different levels of health that are exogenously given, but not the endogenous choice of costly investments in health, such as medical care, that lower mortality and raise life expectancy. In this section, individuals are assumed to make health investments in response to non-anticipated changes in exogenous variables that determine the optimal level of health. Our focus is on the role of retirement age and we show how raising a given mandatory retirement age tends to increase workers’ incentives to invest in their health. We show this first in the context of the two-period model of section 3 and then adapt an even more stylized model with only one period, which has been introduced by Hall and Jones (2007). We adapt this model to consider feedback effects from rising income on health spending.

A mandatory retirement age is a characteristic feature of pay-as-you-go pension systems that make the eligibility to receive pension payments contingent on leaving the labor force at that age and also typically impose various restrictions on early retirement. These pension systems often deviate from actuarial fairness in their financial penalties for early retirement and in their rewards for working past the retirement date if they allow it at all. In a stark simplification, we continue to assume the presence of actuarially fair annuities, so that no capital market inefficiency complicates our argument, and rule out any deviation from the mandatory retirement age. Under these assumptions, a later age of mandatory retirement and greater health spending turn out to be complements in the consumer’s utility maximization.

There are two ways of looking at this complementarity. On the one hand, a higher mandatory retirement age may make more lifetime income available to finance consumption in a given number of periods, in each of which marginal utility of consumption declines. Beyond that, some of the additional income may be used for health investments and for consumption in the additional years of life that this may generate. By contrast, if retirement was mandatory at an unchanged age, investing in health and financing additional consumption would come at the expense of consumption in earlier years. The incentives to make these investments would hence be lower.

On the other hand, one can consider the demand side for health. Better health counteracts the detrimental effect of later retirement on survivorship. Raising the mandatory retirement age without allowing workers to adjust their state of health might lead to an excessive working
life. The marginal benefit of continuing to work will then fall short of marginal costs. The optimality condition may instead hold at some point in time before the new mandatory retirement age. Investments in health address this imbalance because a higher survivorship raises expected income and causes retirement to have less of a life-saving effect at any given age so that both the level of \( z \) and its derivative with respect to \( R \) increase.

*Health production in the two-period model*

To analyze the complementarity between retirement age and health investments formally, we need to introduce a health production function. In the two-period model of section 3, this requires us to specify how these investments are allocated across periods. Any health production planned for the second period could be preempted by death since \( S_i < 1 \). We therefore assume all health investment takes place in the first period. Substituting the health production function \( m(h) \) for \( H \), where \( h \) denotes healthcare spending, we find the inter-temporal rate of substitution in consumption goods, \( u_{bc} = B(1+r)u_{tc} \), is still independent of the survival rate and its determinants, \( R \) and \( h \). The first-order conditions also imply that the marginal benefit of health spending equals marginal cost:

\[
\frac{\partial z}{\partial m} = \frac{\partial^2 z}{\partial m^2} + \frac{1}{(1+r)} \left( \frac{u_i}{u_{tc}} + c_1 - w_i \right) \frac{\partial^2 S_i}{\partial R^2} \]

The marginal benefit depends on the effect of health spending on health and of health on survivorship, on the level of survivorship, on consumption and income in the second period, and it increases with the ratio of average to marginal utility in that period. The marginal benefit also includes the positive effect of better health on the additional earnings from later retirement.

Using the implicit function theorem, we obtain the effect of raising the mandatory retirement age on \( h \):

\[
\frac{dh}{dR} = -m''(h) \frac{\partial z}{\partial m} - m'(h) \left[ \frac{\partial^2 z}{\partial m^2} + \frac{1}{(1+r)} \left( \frac{u_i}{u_{tc}} + c_1 - w_i \right) \frac{\partial^2 S_i}{\partial R^2} \right] + \left( m'(h) \frac{\partial^2 S_i}{\partial R \partial m} \right) \left( \frac{1}{u_i + u_{tc}(c_1 - w_i)} \frac{\partial^2 z}{\partial R \partial m} \right) \left( m''(h) \frac{\partial S_i}{\partial m} \right).
\]

It is a sum of three parts. The first two are unambiguously positive: first, the marginal effect of better health on additional earnings multiplied by the second derivative of health with respect to health spending, and second, the second derivative of the additional-earnings function with respect to better health plus the discounted product of the sum of average over
marginal utility and excess consumption in old age on the one hand and the second derivative of survivorship with respect to retirement age on the other hand, multiplied by the squared first derivative of health with respect to health spending. However, the third addend is likely to be negative as the numerator – with \( \partial^2 S_i/\partial m \partial R > 0 \) and \( \partial^2 z/\partial m \partial R > 0 \) – has a positive sign while the denominator has a negative sign. Only in the unlikely case of \( c_i < w_i \) could the denominator be negative. Empirical information on the relative size of the various derivatives would allow us to define more precisely the conditions under which a higher retirement age induces greater health investment. In general, the sum of the first three addends will be greater than the fourth addend and the effect of raising \( R \) on \( h \) is positive.

**Modelling the income effect on health spending**

While the preceding analysis allows health investments to have an impact on income by increasing survivorship, it does not adequately address the role of greater income as a source of greater health spending. This aspect may give rise to positive feedback and may thus facilitate sustainable population aging in the world’s developed countries. We assume a higher \( R \) always raises per-period income since it generates additional lifetime income for a shorter lifespan, given that working longer increases mortality. Nonetheless, because the objective is to maximize lifetime utility which grows with the duration of life, a negative function of \( R \), it is generally not optimal to raise \( R \) until life’s end.

For an appropriate analytical framework, we adapt Hall and Jones’ (2007) stylized model of rising health spending amid per-capita income growth. Although they do not address the retirement issue, they introduce a utility function in which average utility is always greater than marginal utility, thus motivating the pursuit of longevity that links retirement timing and health investments. To this end, they add a constant flow of utility to the time-additively separable per-period utility from consumption. In our version of the model, the representative agent whose life expectancy, \( H \), is determined by the health production function \( H = g(R, h) \) with \( \partial g(\cdot)/\partial R < 0 \), \( \partial^2 g(\cdot)/\partial R^2 < 0 \), \( \partial g(\cdot)/\partial h > 0 \), \( \partial^2 g(\cdot)/\partial h^2 < 0 \), and \( \partial^2 g(\cdot)/\partial R \partial h > 0 \), can still be considered as facing the same mortality hazard at all ages, defined as the inverse of health status, \( 1/H \).

Although an age-invariant mortality hazard may not seem realistic, especially when \( R \) affects mortality at higher ages, we think of it as a limiting case in the presence of competing risks. As \( R \) is raised, we hypothesize there will be an accelerating increase in mortality during the
additional working time. But as these changes are rationally anticipated in our model, workers make prior adjustments that lead to a new equilibrium distribution of mortality risk throughout life, as predicted by the theory of competing risks (Dow et al., 1999): complementary effects between multiple causes of death, such as hazards before and after a given age, operate to equalize the occurrence of the causes. More specifically, when later retirement raises mortality in old age, workers no longer care so much to survive to that age and let mortality rise at younger ages, too.

Preferences distinguish between the flow of per-period utility, $u$, and lifetime utility, the product of per-period utility and life expectancy. As Rosen (1988) first saw, the level of $u$ affects the optimal inter-temporal allocation in this type of specification because adding a constant raises the value placed on longevity relative to the instantaneous consumption of goods.

In the simplest version of the model, Hall and Jones (2007) assume the flow of income per period, $y$, to be constant, too. This enables them to provide insights into an essentially dynamic problem using a static model. If we were to stick to the three technically convenient assumptions that per-period income is unaffected by health spending, $R$ is fixed, and the consumer maximizes expected per-period utility $g(R, h)u(c)$, subject to the per-period budget constraint, $c + h = y$, then the optimal allocation would equate the ratio of health spending to consumption with the ratio of the elasticities of the health production function and the flow utility function. The optimal health share, $s \equiv h/y$, would satisfy $s^*/(1 - s^*) = h^*/c^* = \eta_h/\eta_c$, where $\eta_h = g'(h)h/H$ and $\eta_c = u'(c)u/c$. We can think of this solution as a benchmark.

More realistically, health spending that lets mortality decline does have an impact on per-period income. To some extent, better health will affect per-period income even if the age of retirement is held constant, because on the one hand active workers are less likely to die before reaching that age and on the other hand time spent in retirement tends to be longer as life expectancy rises. When these two opposite effects are taken into account, the first-order conditions imply $h/c = (\eta_h/\eta_c) / [1 - (\partial y(\cdot)/\partial g)(\partial g(\cdot)/\partial h)]$, which shows optimal health spending over consumption to be greater or smaller, the larger or smaller, respectively, the marginal effect of greater health spending on per-period income.

The overall effect can be decomposed into the effect of changing health spending on health, $\partial g(\cdot)/\partial h$, and the effect of changing health on income, $\partial y(\cdot)/\partial g$. While the former effect is
assumed to be always positive, it is not a priori clear how lower mortality affects per-period income. With equal distribution of lifetime income across periods and constant productivity up to retirement, the net effect depends on the relative gains in expected lifetime before and after retirement. Formally, life expectancy is $\int_0^R e^{-t/H} dt = \left(1 - e^{-R/H}\right)H$ before and $\int_R^\infty e^{-t/H} dt = He^{-R/H}$ after retirement. Depending on the given age of retirement, the gain in expected time before retirement may be larger or smaller than the gain in expected time after retirement. More specifically, when $R$ is high, the additional lifetime income due to declining mortality before retirement may be large enough to prevent a fall in per-period income even if a rising $h$ prolongs overall lifetime. However, if the gains in life expectancy are large, a parallel postponement of the retirement age may be required to generate all the additional lifetime income that is needed to maintain a given per-period income flow. If the retirement age is increased in response to declining mortality, this may ensure $\partial y/\partial h > 0$ in addition to $\partial g(\cdot)/\partial h > 0$.

When health spending does have an impact on per-period income, the optimal income share of health spending is determined by $s = \eta_h / \left[\eta_c (1 - (\partial y/\partial g)(\partial g/\partial h)) + \eta_h \right]$. In the limiting case of no impact that Hall and Jones (2007) study, we have $(\partial y/\partial g)(\partial g/\partial h) = 0$, and the optimal health share collapses to $s^* = \eta_h / (\eta_c + \eta_h)$. A positive impact of health spending on per-period income implies a larger health share, as long as the marginal effect of greater health spending on per-period income is smaller than one. By contrast, a negative impact implies a smaller health share than the optimal share in the limiting case.

By the envelope theorem, we know that the impact of health spending on per-period income will generally be larger if the retirement age is optimally adjusted in response to changes in health, regardless of whether the timing of retirement itself has an effect on health. If later retirement does increase mortality, this effect will be small for small changes in retirement timing as it concerns only the time after the original retirement age. Provided healthcare is sufficiently productive, there is at least some scope to boost lifetime income by delaying retirement in response to greater health spending without fully cancelling the gains in health that this spending is intended to bring about.

The main driver of health spending in Hall and Jones’ (2007) model is the demand side: rising income is associated with a falling elasticity of utility with respect to consumption, $\eta_c$, 
relative to the elasticity of health with respect to healthcare spending, $\eta_h$. Hall and Jones specify flow utility as $u(c) = b + c^{\gamma/(1-\gamma)}/(1-\gamma)$, where $\gamma$ is the constant elasticity of marginal utility. Rising income thus lets the value of a life year grow faster than the value of additional consumption so that the share of health must rise with rising per-capita income. Dormont et al. (2007, pp. 60) show that the income elasticity of health demand

$$\eta_{s/h}^* = \theta_a (b \gamma (e^{s^*})^{\gamma/(1-\gamma)} + 1/(1-\gamma)(1-s^*/s^*)\gamma [1-\gamma(1+\theta_a/(1-\gamma))(1-s^*)+\gamma]$$

depends critically on a strictly positive value of the parameter $b$. If $b$ were zero, the income elasticity would be equal to one, with $s^* = h/y = 1/(1 + \eta_c/\eta_h) = 1/(1 + (1-\gamma)/\theta_a)$.

To focus on the implications of changes in retirement age, we now assume that total lifetime income is proportional to the expected value of the duration of working life, $(1-e^{-R/H})H$. The individual budget constraint under the per-period flow of income, $y$, is now given by $c + h = y(R, H) = y(R, g(R, h)) = y\left[(1- e^{-R/g(R,h)})g(R, h)\right]$, where the budget spent in a given period is a function of $H$ and $R$. Normalizing gross wage income in each period before retirement to one, we can express the available net per-period income flow as $y = 1- e^{-R/g(R,h)}$ since $g$ is equal to life expectancy.

To motivate our assumption of equal per-period income across working life and time in retirement, we rely on the absence of time preference, the absence of any direct impact of retirement on utility and the presence of full and fair insurance against the risk of running out of resources when death comes later and of leaving unspent resources when death comes earlier than expected. For analytical convenience, we further assume that equal health spending across periods is optimal, say, because the effectiveness of health spending is age-invariant and because it is effective only in the period in which it is made. These assumptions imply some shifting of resources from the time of work to the time in retirement. We assume this takes place at the social level and do not explicitly model it here. By contrast, Hall and Jones (2007), whose model has no retirement, rule out any opportunity to shift resources from one period to another, such as physical capital or foreign trade, and thus bypass the need for actuarially fair annuities.

To determine the impact of increasing $R$ on $h$, we set up the Lagrangian $L = g(R, h)u(c) + \lambda (1- e^{-R/g(R,h)} - h - c)$ and derive the first-order condition for $h$. 
\[ \partial L / \partial h = \left( \partial g(\cdot) / \partial h \right) u(c) + g(\cdot) \partial u(c) / \partial c \left[ - e^{-Rg_{(\cdot)h}} \left( \partial g(\cdot) / \partial h \right) R' g(R, h)^2 - 1 \right] = 0, \]

where the Lagrange multiplier takes the form \( \lambda = g(R, h) \partial u(c) / \partial c \). Denoting \( \partial u(c) / \partial c \) as \( u_c \) and applying the implicit function theorem, we find

\[
\frac{dh}{dR} = - \frac{R u_c \frac{\partial g}{\partial h} \left( (R - g(\cdot)) \frac{\partial g}{\partial R} - g(\cdot) \right) + g(\cdot)^2 u_c \left( \frac{\partial g}{\partial c} + R \frac{\partial^2 g}{\partial c \partial h} \right) + e^{-Rg_{(\cdot)h} \left( \frac{\partial g}{\partial h} - u_c \frac{\partial^2 g}{\partial c \partial h} \right)} \left( u_c \frac{\partial g}{\partial c} - u(\cdot) \frac{\partial^2 g}{\partial c \partial h} \right)}{R g(\cdot) \frac{\partial^2 g}{\partial h^2} u_c \left( \frac{\partial g}{\partial h} \right)^2 + R^2 u_c \left( \frac{\partial g}{\partial h} \right)^2 + R g(\cdot)^2 \frac{\partial^2 g}{\partial h^2} + e^{-Rg_{(\cdot)h} \left( \frac{\partial g}{\partial h} - u_c \frac{\partial^2 g}{\partial c \partial h} \right)} \left( u_c \frac{\partial g}{\partial h} - u(\cdot) \frac{\partial^2 g}{\partial c \partial h} \right)}.
\]

The denominator is likely to have a positive sign since all its parts, with the exception of the third addend, are positive and the third addend is not likely to be larger in absolute value than the sum of the other three. In the numerator, the first and third parts have negative signs. Only its middle addend is positive, but while it may be larger in terms of absolute value than the first and third addend separately, it is not likely to be larger than the two other addends together. Given that numerator and denominator almost certainly have different signs, the overall effect of increasing \( R \) on \( h \) is likely to be positive.

Figure 3 illustrates the static model of Hall and Jones (2007) in the original version with only one control denoted as health spending (upper panel) and our adaptation with an additional control, the time of retirement (lower panel). When per-period income is fixed, every exogenous increase shifts the budget constraint away from the origin, such as from \( T_0 T_0 \) to \( T_2 T_2 \). The model implies that people do not increase their spending on health and other consumption proportionally, but that the share of health in income increases, as indicated by moving from point \( D \) to \( E \) on the health spending expansion curve \( V \), well above the 45°-line.

Maintaining an equal flow of income across periods, health spending in this context might mean either spending on healthcare or forgoing labor income by early retirement, but not both at the same time.

By contrast, the lower panel illustrates the implications for healthcare spending of an individual choice of retirement timing as an additional way to improve health. If the budget constraint were fixed, such as in \( T_2 T_2 \), the individual would use the retirement option to spend less on healthcare and more on other consumption goods, as indicated by moving from point \( E \) to \( F \). The new allocation in \( F \) lies on the new health spending expansion curve \( V' \) indicating a less rapidly rising health share as income rises. Increasing \( c \) would be desirable because the health gain from early retirement lowers the marginal product of additional healthcare spending. However, the lifetime budget will also be lower so that only point \( G \) on
the new budget constraint $T_RT_R$ can be reached. While health spending will clearly be lower in $G$ compared to $E$, the effect on other consumption spending is ambiguous.

The graph may also be interpreted in terms of later retirement. Per-period income rises as retirement is postponed. This makes additional health spending that increases life expectancy more attractive than additional consumption whose marginal utility declines. As a result, the marginal gain from continued work at a given age rises and keeping the retirement age constant becomes more costly in terms of forgone opportunities. This effect is reinforced when healthcare spending is added as an additional choice to increase longevity, as indicated by moving from $G$ to $E$. Later retirement and a rising health share in income are thus again shown to be complements.
V. Optimal Coordination

If the government raises a mandatory age of retirement so that per-period income rises, we predict the income share of health spending, too, will rise, calling for a further rise of the optimal retirement age. Investments in health, or exogenous shocks that improve the productivity of medical technology, can thus create a virtuous circle in which retirement age and incomes rise as mortality declines. This raises the issue of the optimal speed and coordination of the process, with normative implications for government policy.

To demonstrate the benefits of coordination, we compare the value of the indirect utility function for increases in $h$ under an exogenously fixed $R$ with its value when $h$ is accompanied by an optimal increase in $R$. The indirect utility function $\phi(\alpha)$ is defined as the maximum value of $f(x, \alpha) = H u(c)$ for given values of the set of exogenous parameters $\alpha = \{H_0, \eta, \eta_h, \eta_k\}$ and choice variables $x = \{R, h, c\}$ subject to a set of constraints $\varphi = \{R - R_0 = 0, g(R_0, h, H_0) - H = 0, y = 1 - e^{-R/g(\cdot)} = c + h\}$. The greater potential utility when retirement is flexible is an example of the well-known Le Châtelier effects, implying that long-term demand for healthcare is more elastic than short-term demand. The short term is defined as a situation in which the mandatory retirement age is held constant at the age appropriate before one or more of the exogenous parameters are changed.

The indirect utility function denoted $^2\phi(x, \alpha)$ thus features two important constraints – namely one on the generation of per-period income, using health spending as an input in the production of health, and the other on the mandatory retirement age, $R_0$. The key feature of $^2\phi(x, \alpha)$ is that it is concave around $R = R_0$, and more so than the indirect utility function $^1\phi(x, \alpha)$ with only one constraint – on per-period income. The constraint on $R$ in the short-term indirect utility function $^2\phi(x, \alpha)$ is just binding in the sense that it does not displace the solution that is obtained at the original exogenous parameter values without the mandatory-retirement-age constraint, $^1\phi(x, \alpha)$. The Le Châtelier effects are local relations that imply different comparative statics around the original solution for the choice variables. Technically speaking, the difference between the two indirect utility functions is itself a concave function of the exogenous parameters and the Le Châtelier effects are consequences of the concavity of this difference; for more details see Silberberg (1990, pp. 216–222).
To proceed, we need the unconstrained optimal choices for \(c\) and in particular for \(R\) and \(h\) as functions of the exogenous parameters. As before, the consumer maximizes \(g(R, h)u(c)\) with respect to \(c\), \(h\) and \(R\) and subject to \(c + h = y(R, g(R, h)) = 1 - e^{-R/g(R, h)}\). While we do not impose a specific functional form for \(g(R, h)\), we assume that \(g\) is monotonically increasing in \(h\) and monotonically decreasing in \(R\), with \(\partial g(\cdot)/\partial R < 0\), \(\partial^2 g(\cdot)/\partial R^2 < 0\), \(\partial g(\cdot)/\partial h > 0\), \(\partial^2 g(\cdot)/\partial h^2 < 0\), and \(\partial^2 g(\cdot)/\partial R \partial h > 0\), to ensure the maximization problem is well-defined. Perfect medical technology would raise the issue of stability because health spending might be raised until mortality drops to zero so that workers could work forever, without retiring. In this case, the maximand of our model would be unbounded, the problem ill-defined. Medical technology must hence be imperfect for health spending to run into diminishing marginal returns. This ensures mortality is always positive and retirement is optimal at some finite age, given that mortality rises monotonically and at an increasing rate with \(R\) if health spending is held constant.

Taking derivatives of the Lagrange function \(L = g(R, h)u(c) + \lambda \left(1 - e^{-R/g(\cdot)} - c - h\right)\) yields the first-order conditions

\[
\partial L/\partial c = g(\cdot)\partial u(c)/\partial c - \lambda = 0,
\]

\[
\partial L/\partial h = (\partial g(h)/\partial h)u(c) - \lambda \left(\frac{e^{-R/g(\cdot)}}{g(\cdot)}R(\partial g(\cdot)/\partial h)/g(\cdot)^2 + 1\right) = 0,
\]

\[
\partial L/\partial R = (\partial g(\cdot)/\partial R)u(c) - \lambda e^{-R/g(\cdot)}\left(R(\partial g(\cdot)/\partial R)/g(\cdot)^2 - 1/g(\cdot)\right) = 0,
\]

and \(1 - e^{-R/g(\cdot)} - c - h = 0\).

Using \(\lambda_0 = g(R_0, h_0)\partial u(c_0)/\partial c\) and \(e^{-R_0/g(\cdot)} = 1 - c_0 - h_0\), we can write the two first-order conditions for \(h\) and \(R\) as

\[
\partial L/\partial h = u(c)\partial g(\cdot)/\partial h - g(R, h)\partial u(c)/\partial c \left[1 + (1 - c - h)R(\partial g(\cdot)/\partial h)/g(R, h)^2\right] = 0
\]

\[
\partial L/\partial R = u(c)\partial g(\cdot)/\partial R - g(R, h)\partial u(c)/\partial c \left(1 - c - h\right)\left[R\partial g(\cdot)/\partial R/g(R, h)^2 - 1/g(R, h)\right] = 0
\]

Combining and using \(u'(c) = \eta_c u'/c\) where the elasticity of marginal utility, \(\gamma\), is assumed constant, \(\partial g(\cdot)/\partial h = \eta_h g/h\) where the elasticity of health with respect to health spending, \(\eta_h\), is declining, and \(\partial g(\cdot)/\partial R = \eta_R g/R\) where the negative elasticity of health with respect to
retirement age, $\eta_R$, is increasing in terms of its absolute value, we have

$$(\eta_R g / R)/(\eta_R g / h + g \eta_h u/c) = (\eta_c u/c (\eta_R - 1) g^2 )/(R \eta_h g / h),$$

which can be further simplified to $\eta_R/(\eta_R/h + \eta_c u/c) = \eta_c u/c (\eta_R - 1) g / (\eta_h / h)$. It follows that per-period health spending,

$$h = -c \eta_h (c \eta_R - (\eta_R - 1) g u \eta_c) / (g u^2 \eta_c^2 (\eta_R - 1)),$$

can be represented as a function of three important elasticities, $\eta_c$, $\eta_h$, and $\eta_R$. The optimal value of $h$ is larger the lower the initial levels of health, $g$, and utility, $u$, the lower $\eta_c$, the larger – in terms of absolute value – $\eta_R$ and the larger $\eta_h$. The ratio of $h$ to $c$ depends on the same set of exogenous parameters and rises with $c$ itself: $\partial (h/c)/\partial c > 0$.

To determine the optimal value of $c$, we plug the solution for $h$ into the first-order condition for $R$ and find two solutions for $c$ that differ in terms of the sign with which the square-root term enters the numerator. The solution for $c$ with the larger value is

$$c_1 = \{- g u \eta_c (R u \eta_c + R \eta_h - g u \eta_h - R u \eta_c \eta_R - R \eta_h \eta_R ) -$$

$$[4 g R^2 u^2 \eta_h (\eta_R - 1) g \eta_h + g^2 u^2 \eta_c^2 (R u \eta_c + R \eta_h - g u \eta_h - R u \eta_c \eta_R - R \eta_h \eta_R )^2 ]^{1/2} \}/ (2 R \eta_h \eta_R).$$

The solution with the smaller value is

$$c_2 = \{- g u \eta_c (R u \eta_c + R \eta_h - g u \eta_h - R u \eta_c \eta_R - R \eta_h \eta_R ) +$$

$$[4 g R^2 u^2 \eta_h (\eta_R - 1) g \eta_h + g^2 u^2 \eta_c^2 (R u \eta_c + R \eta_h - g u \eta_h - R u \eta_c \eta_R - R \eta_h \eta_R )^2 ]^{1/2} \}/ (2 R \eta_h \eta_R).$$

Since $\eta_h < 0$, the denominator is negative and both solutions yield a positive $c$ whose value rises with $g$ and $\eta_c$ and declines with higher values of $R$ and $\eta_h$ and with greater absolute values of $\eta_R$. Greater initial health and a higher $\eta_c$ make consumption more attractive, whereas a higher $\eta_h$ makes health spending more attractive. In a similar vein, a larger absolute value of $\eta_R$ makes later retirement less attractive and constrains increases in lifetime income that could be used to raise $c$.

For the determination of the optimal $R$, we may use the first-order condition for $h$, implying

$$R = (u \eta_h g / h - g \eta_c u/c) g^2 / [(1-c-h) \eta_h g / h] = (\eta_h - \eta_c h/c) u g / (1-c-h).$$
This expression establishes complementarity between $R$ and $h$, i.e. 
$$
\frac{\partial R}{\partial h} = -g u \eta_c/(c(1-c-h)) + (\eta_h - \eta_c h/c)g u / (1-c-h)^2 > 0, \text{ if } \eta_h \text{ is large relative to } \eta_c 	ext{ and } h \text{ is small relative to } c \text{ so that } \eta_h > -\eta_c (1-1/c). \text{ The effect of health spending on health must be sufficiently large to outweigh the opportunity cost in terms of lost utility from lower consumption. Similarly, a higher level of initial health leads to later retirement if } \frac{\partial R}{\partial g} = (\eta_h - \eta_c h/c)u / (1-c-h) > 0. \text{ Population aging is synonymous with higher levels of initial health at every age and also implies declining opportunity costs of health spending because the marginal utility of conventional consumption tends to decline relatively fast.}
$$

Deriving explicit choice functions for $R$, $h$ and $c$ in terms of the exogenous parameters only is difficult without further specifying the functional form of $g(R,h)$. Moreover, the first-order condition $e^{-R_0 / \gamma} = 1 - c_0 - h_0$ is a transcendental function that cannot be solved algebraically. In the event, we do not need explicit functional forms to sign the derivatives of the choice functions with respect to their ultimate determinants. The choice functions ultimately depend on workers’ initial endowment with health, $H_0$, the constant elasticity of marginal utility, $\gamma$, assumed to be greater than one, the consumption elasticity of utility, $\eta_c$, and the elasticity of health with respect to health spending, $\eta_h$, as well as with respect to retirement age, $\eta_R$, as follows:

$$
R^* = R(H_0, \eta_c, \eta_h, \eta_R) \text{ with } \partial R^*/\partial H_0 > 0, \partial R^*/\partial \eta_c < 0, \partial R^*/\partial \eta_h > 0, \text{ and } \partial R^*/\partial \eta_R < 0;
$$

$$
h^* = h(H_0, \eta_c, \eta_h, \eta_R) \text{ with } \partial h^*/\partial H_0 < 0, \partial h^*/\partial \eta_c < 0, \partial h^*/\partial \eta_h > 0, \text{ and } \partial h^*/\partial \eta_R > 0;
$$

$$
c^* = c(H_0, \eta_c, \eta_h, \eta_R) \text{ with } \partial c^*/\partial H_0 > 0, \partial c^*/\partial \eta_c > 0, \partial c^*/\partial \eta_h < 0, \text{ and } \partial c^*/\partial \eta_R < 0.
$$

A higher initial endowment with health, $H_0$, has a positive effect on the choice of retirement age $R$ and on per-period consumption $c$, but a negative effect on health spending $h$. A higher $\eta_c$ leads to a higher $c$ at the expense of $h$ and – due to the complementarity between $R$ and $h$ – also induces earlier retirement, a lower $R$. By contrast, a higher $\eta_h$ leads to lower $c$, higher $h$ and higher $R$. Finally, a higher absolute value of the negative elasticity of health with respect to retirement age, $\eta_R$, leads to a lower $R$ and a lower $c$, while $h$ is increased.

These marginal effects are local effects and are valid for both equilibria suggested by the two solutions we derived for $c$. The complementarities between $c$ and $h$ on the one hand and
between $h$ and $R$ on the other hand suggest that only one of these equilibria is efficient, presumably the equilibrium characterized by high values of $c$, $h$ and $R$, not the one characterized by low values of these choice variables.

**Short term: the indirect utility function for a fixed retirement age**

In our model, the value of the optimal adjustment of health spending in the presence of a fixed retirement age $R_0$ can be inferred from the indirect utility function $\bar{U} = \max H u(c)$ whose derivative with respect to $\eta_h$ is equal to the derivative of the Lagrangian evaluated at the optimal values of the original solution. The Lagrangian for the maximization of $F(x, \alpha) = f(x, \alpha) - \phi(\alpha)$ over the variables $x$ and $\alpha$, treated as independent variables, is $L' = f(x, \alpha) - 2\phi(\alpha) + \lambda_1 \left( g(R_0, h, H_0) - H \right) + \lambda_2 \left( 1 - e^{-R_0g(\cdot)} - c - h \right) + \lambda_3 (R - R_0)$ with first-order conditions $L'_{x} = F_{x} + \lambda_1 \phi_{\alpha x} = 0$, $L'_{\alpha} = F_{\alpha} + \lambda_1 \phi_{\alpha} - \phi_{\alpha} = 0$, and $L'_{\lambda_3} = \phi_{\lambda_3} = 0$. The first-order conditions from the differentiation with respect to $\alpha$ represent the envelope theorem.

We use the envelope theorem to understand the impact of positive exogenous shocks in healthcare productivity, such as new medical technology. At the optimal choices $x = x^*(\alpha)$, the rate of change of the indirect utility function with respect to $\eta_h$, denoted $2\phi_{\eta_h}$, in which only $c$ and $h$ are allowed to adjust to the change in $\eta_h$, is equal to the rate of change of the original Lagrangian with respect to $\eta_h$ holding $R_0$ constant. We therefore take the derivative of the Lagrangian with respect to $\eta_h$:

$$L'_{\eta_h} = F_{\eta_h} + \lambda_{\eta_h} \phi_{\eta_h} - \phi_{\eta_h} = \left[ u(\partial g/\partial h) - e^{-R_0g(\cdot)}(R_0/g)u_c(\partial g/\partial h) - g u_c \right](\partial h/\partial \eta_h).$$

The higher $h$ that is induced by the exogenous increase in $\eta_h$ affects overall utility positively through the gain in average per-period utility with better health and negatively through the marginal utility impact of the lower per-period income that a fixed $R$ implies amid rising life expectancy as well as through the expected loss of marginal utility from consumption amid rising $h$. As in previous sections, the higher average utility of additional lifetime compared with additional consumption is crucial in generating a positive effect overall.

**Long term: the indirect utility function with unconstrained retirement timing**
Using the indirect utility function $\frac{1}{\alpha} \phi(\alpha)$ without fixed retirement age yields the Lagrangian derivative

$$L_{k_h} = F_{k_h} + \lambda_{k_h} \left( \frac{1}{\alpha} \phi(\alpha) - \phi_{k_h} \right) =$$

$$\left[ u(\partial g/\partial h) - e^{-R/\gamma}(R/g)u_c(\partial g/\partial h) - g u_c \right] (\partial h/\partial \eta_h) + \left[ u(\partial g/\partial R) + e^{-R/\gamma} u_c \right] (\partial R/\partial \eta_h).$$

The first part of this expression is identical to the effect under fixed $R$, the second part adds the impact of increasing $R$. Like before, the consequences of a larger $h$ include the utility gain from living longer, the marginal utility loss from lower consumption, due to the lower per-period income when lower mortality is combined with a fixed $R$, and the lifetime marginal utility loss from lower $c$. The consequences of a higher $R$ include the loss in average utility from the greater mortality that later retirement induces and the marginal utility from additional consumption that higher lifetime income makes feasible.

**Comparison**

At the initial values of the exogenous parameters, the fixed retirement-age constraint is just binding and the Lagrange multiplier $\lambda_{k_h}$ is zero, indicating zero marginal cost of the restriction in terms of attainable values of the indirect utility function. The two indirect utility functions have the same value at that point. However, as $\eta_h$ increases, the fixed retirement-age constraint becomes strictly binding and reduces the maximum value of the indirect utility function since $R$ is no longer available as a choice variable. The constrained short-term indirect utility function is therefore more concave, or less convex, than the indirect utility function without the retirement age constraint. Looking at second-order changes, Taylor series expansions can be used to show that the Hessian matrix of second partials of $\frac{2}{\alpha} \phi(\alpha) - \frac{1}{\alpha} \phi(\alpha)$ is negative semidefinite, which implies that $\frac{2}{\alpha} \phi(\alpha) - \frac{1}{\alpha} \phi(\alpha)$, too, is a concave function of the exogenous parameters.

**Illustration**

We now illustrate the complementarity of $h$ and $R$ in the production of health by means of a few simple graphs of intertemporal health transitions. Stable fixed points in these transitions correspond to equilibria identified in our formal model. The underlying idea is that we interpret the indirect utility function as the value function of a dynamic programming
problem, which can then be expressed in terms of the familiar Bellman equation. To this end, we divide the value function

\[ V(g) = \max_{c,h,R} g(R,h) u(c) \quad \text{s.t.} \quad e^{-R/\delta} = 1 - c - h \]

by \( g(R,h) \) and obtain the Bellman equation with discounting only for mortality, not for time preference,

\[ V(g) = \max_{c,h,R} u(c) + \left[ 1 - 1/g(R,h) \right] V(g) \quad \text{s.t.} \quad e^{-R/\delta} = 1 - c - h. \]

In the budget constraint, \( e^{-R/\delta} = 1 - c - h \), the left-hand side will be smaller the larger \( R \); either \( c \) or \( h \) – or both to a smaller degree – can hence be increased as \( R \) increases. However, a higher \( R \) also lowers \( g(R,h) \) and thus raises the marginal product of \( h \) so that \( h \) must be increased as part of an optimal policy.

Figures 4 and 5 illustrate the dynamic programming approach. In Figure 4, Panel A introduces the basic idea of a health transition function in the absence of death. Health is here defined in terms of quality of life and we assume that with no medical intervention and no retirement, the present state of health \( H_t \) is the best predictor of the uncertain state of health in the next period, \( H_{t+1} \), as indicated by the diagonal \( N \). If medical care becomes available, the dotted line may be reached: any state of health below the optimum, standardized at one, can be improved by medical intervention. If retirement is an additional option, the recovery from suboptimal states of health will be even faster so that health transitions are described by the curve labeled \( R \).

In Panel B of Figure 4, we assume that workers face a health state-dependent survival probability below one. In the absence of both medical care and retirement, the expected state of health in period \( t+1 \), taking into account the irreversible event of death, will always be below the state of health realized in period \( t \), as indicated by the convex curve labeled \( N \). With medical intervention, the decline in expected health can be slowed so that health transitions are described by the convex curve labeled \( M \). If retirement is available as an additional strategy to preserve a worker’s health, the diagonal labeled \( R \) may be reached. This is only an example, the improvement of health may actually be above or below the diagonal.

In Figure 5, Panel A introduces a health transition function that would be plausible under conditions of work-related health deterioration. Because the marginal efficiency of health
spending is assumed to decline, maintaining health at its maximum value of one would not be optimal. Instead, we assume a fixed point at $A$ that is locally stable because any health shocks that do not move the worker’s health below $B$ can still be partially repaired from one period to the next. However, when a worker’s health falls below $B$, recovery of health is no longer feasible while the worker continues to work and continued work will lead to death, as indicated by the stable fixed point at zero.

Panel B of Figure 5 illustrates a new health transition function for a retiring worker who retains access to medical care. Because some of the health shocks previously suffered from work may be irreparable while working, but have no lasting effect in retirement, the upper stable fixed point moves from $A$ to $E$. Moreover, the minimum level of health from which recovery is still feasible moves from $B$ to $D$. Finally, we assume there is a second stable fixed point at $C$ that avoids death. This accounts for the observation that there are many diseases that have a permanent adverse effect on health but need not be fatal if the person receives proper cure and care in full retirement from work.

How would an exogenous increase in $\eta$, such as a boost in healthcare productivity from new medical technology, change the picture? In terms of Figure 5, Panel B, a higher $\eta$ would move point $A$ to the right and point $B$ to the left, narrowing or eliminating the intervals on the $H_t$-axis from which a sick worker must resort to retiring early in order to reach the stable fixed point with good health, point $E$. 
VI. Discussion and Conclusion

The management of healthcare in ageing societies is a focal challenge for the world economy in the 21st century. Two issues that are interrelated will require particular attention: financing the rising demand for healthcare and investing in the appropriate medical technology. Solving the financial side will require anticipating the opportunities created by new medical technologies. Setting the right priorities for investments in new technologies will require anticipating the financial constraints and rewards. Hall and Jones (2007, p. 30) predict that “maximizing social welfare in the United States will require the development of institutions that are consistent with spending 30 percent or more of GDP on health by the middle of the century.”

In this paper, we have explored the idea that health and retirement policies must be coordinated to maximize welfare when population aging is mainly driven by gains in health. The social costs of early retirement are likely to increase amid population aging, representing an unprecedented investment opportunity. The marginal utility of conventional consumption is likely to diminish much faster than the social returns in the production of health. Raising the funds for the development of new medical technology will require that many workers retire later. Yet, even countries with the most rapidly aging populations have so far made only timid steps towards longer individual working lives. We have argued that rising health spending and better health will make much bolder steps possible.

The key question – how much can the health of the elderly be expected to improve – is an empirical one. It is often framed in terms of two competing hypotheses, known as morbidity expansion and morbidity compression, and a synthesis in terms of a dynamic equilibrium. The essence of these hypotheses is shown in Figure 6, Panel A. Morbidity expansion forecasts that more and more people will spend a rising share of their total lifetime in a state of chronic morbidity, often suffering from multiple degenerative diseases that defy a comprehensive diagnosis and cure. Greater consumption of drugs is predicted to keep people alive at a reduced quality of life, a development often thought to be primarily a consequence of the proliferation of non-causal therapies on the supply side, but that may also be attributable to people’s excessive demand for longevity under the hypothesis of mortality-contingent claims (Philipson and Becker, 1998).
Morbidity compression is more optimistic about medical progress and predicts that future longevity gains will be accompanied by an increasing compression of sick time in the vicinity of death. Population aging would hence not force lifecycle per capita healthcare expenditures to rise very much and may even let them fall. The dynamic equilibrium hypothesis synthesizes these two extreme predictions: Rising longevity will allow the average lifetime share that people spend in morbidity to fall, even as total sick time before death does not shrink in absolute terms.

We think that both the morbidity compression and the dynamic equilibrium hypotheses are consistent with large welfare gains from the coordinated postponement of retirement and a rising health share in income amid population aging, provided health spending helps to further reduce morbidity and mortality among the elderly. In addition, these hypotheses suggest to us that also the type of new medical technology and the direction of biomedical research may become legitimate targets for government-sponsored coordination in the context of health and retirement policies. How can such coordination be successful and why has it not happened before?

The government’s coordination problem is analytically similar to the case of policy complementarities in fighting European unemployment that has been studied by Coe and Snower (1997). They define a group of policies as complementary when the effect of each policy on the target is greater through joint implementation with the other policies than in isolation. They argue that policy complementarities arise from institutional complementarities that are rooted in a broad range of labor market rigidities. These institutional rigidities may reinforce one another so that failing to exploit the policy complementarities can create large burdens elsewhere in the economic system.

In reality, governments may fail to realize and exploit the relevant complementarities for a multitude of reasons. One proximate reason in our context is that health and retirement policies are often designed in different government departments, and decided upon at different points in times. Lack of coordination may stem from the fact that the responsible people simply have not realized the complementarities and associated opportunities. Moreover, when pension and healthcare systems are organized along separate rationales, those with the necessary expertise and influence in one system may suffer from regulatory capture or may face a prisoner’s dilemma: Postponing the mandatory retirement age unilaterally without substantial investments that improve workers’ health is neither likely to be efficient, nor
acceptable on distributional grounds. On the other hand, substantial increases in health spending without extending people’s working lives is unlikely to be fiscally sustainable.

We now briefly outline three other difficulties, and perhaps more fundamental reasons, that governments face in the coordination of health and retirement policies. More research will be required in each of these areas.

**Lack of good scientific evidence.** Dormont et al. (2007) summarize key insights from the dispersed literature about the empirical linkages between health expenditures, health status and welfare and see technological progress, instead of aging, as the main driver of growing health spending. Although spending growth appear to be mostly due to the extension effect, where more and more goods are added to the medical consumption basket without replacing older varieties, the authors believe current spending levels may still be far below the socially desirable level. However, they also argue that the problem of institutional feasibility and fiscal sustainability must be addressed, calling for a particularly careful design and for greater accountability of public institutions in the provision of healthcare.

Dormont et al. (2007) also argue that workers’ health could be a greater source of economic growth if institutional incentives for early retirement were changed and the effective retirement age increased. Many rich countries may have lost the potentially positive impact of past gains in health on economic growth by imposing a fixed and relatively low retirement age. An extension of working lives would be required to turn these productivity gains at the individual level into a substantial contribution to economic growth. Using at least part of the financial gains for health investments targeted at those most likely to benefit in terms of labor productivity may help to reconcile the objective of equity and efficiency in retirement, pension and health policies.

A number of recent empirical studies provide estimates of the impact that new medical technology has had on the timing of retirement, in spite of existing institutional rigidities. For example, Bui and Stolpe (2007) examine the evolution of early retirement due to disease and injury in the German labor force between 1988 and 2004. They show that new drug launches have substantially helped to reduce the loss of labor at the disease-level over time with each new chemical entity saving on average around 200 working years in every year of the observation period. In another study of disability retirement, Cutler et al. (2006b) explore the role of improved medical treatment for cardiovascular disease in explaining the decline in disability that has been observed for more than two decades in the US. They conclude that
specific pharmaceutical product innovations explain more than 50 percent of the observed reductions in disability and deaths over time, controlling for the utilization of other types of medical care inputs, such as hospitalizations.

**Capital market imperfections.** One of the more fundamental reasons is that full and actuarially fair insurance against the risk of running out of resources before life’s end or of leaving unspent resources at the time of death is not feasible when medical technology is imperfect, as it must be when technological innovation in medicine is endogenous. As Kalemli-Ozcan and Weil (2002) as well as Philipson and Becker (1998) point out, various capital market imperfections may have a decisive influence on individual retirement behavior and the demand for medical care. We add the observation that, when retirement removes work-related health hazards and a worker’s health is private information so that the timing of retirement is subject to moral hazard, as in Cremer et al. (2004), full insurance and actuarial fairness may no longer be available.

Furthermore, the insurance principle is challenged by the pervasive presence of competing risks in both healthcare and retirement timing. A given set of constraints in healthcare and a given age of retirement can be interpreted as risk factors for specific health problems, such as diseases that require an expensive cure and a break from work for recovery. Whichever the binding constraint, it represents the limiting factor in a Leontief-type production function of health. Individual risks may hence be no longer independent. For example, the competing risks hypothesis, as analyzed by Dow et al. (1999), implies that new medical technologies eliminating one source of risk may automatically increase the impact of other sources of risk on survivors, creating externalities among the suppliers of new technologies that may prevent them from fully appropriating the social returns to their inventive effort. These incentive problems may not only establish a rationale for coordination in the direction of biomedical research, but also suggest that designing policies on specific medical technologies for aging-related diseases and retirement schemes for specific groups of workers in context might become a fruitful prospect.

**Lack of international coordination.** Many countries now face the prospect of aging populations. The markets for medical technology, above all for pharmaceuticals, have long become global, largely a reflection of strong economies of scale in the exploitation of the fixed costs of research investments. To set efficient innovation incentives for international investors in these technologies requires a global response that aggregates the willingness-to-pay of all countries likely to benefit, notwithstanding differences in preferences they may
have. As shown in Figure 6, Panel B, citizens of countries pursuing purely national health policies may face different trade-offs between average health and financial wealth, such as indicated by the transformation curves I and II. Country I has developed and adopted medical technologies that are generally more expensive, but also produce greater levels of health than the medical technologies used in country II. These different investment priorities may partly reflect different preferences, as indicated by the tangencies of social welfare indifference curves in points $A$ and $B$. Both countries may reach the superior transformation curve III and maximize social welfare at $C$ and $D$, respectively, if they coordinate their investments in new medical technology and exploit the inherent global economies of scale in the generation, evaluation and diffusion of new medical technology. For example, a country moving from $A$ to $D$ already has a good level of population health and uses a postponement of retirement age to maximize its financial wealth. Another country may be moving from $B$ to $C$, as it is already wealthy and wishes to use the postponement of retirement age to finance greater health spending.
References


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Figure 6, Panel A: Competing Views on Population Aging

Three Scenarios for the Evolution of Healthy Life Expectancy

Today

Years in good health

Years in bad health

1. Morbidity expansion

2. Morbidity compression

3. Dynamic equilibrium

Figure 6, Panel B: Society’s Trade-off between Pensioners’ Health and Wealth