ABSTRACT

A Product Market Theory of Worker Training*

We develop a product market theory that explains why firms invest in general training of their workers. We consider a model where firms first decide whether to invest in general human capital, then make wage offers for each others' trained employees and finally engage in imperfect product market competition. Equilibria with and without training, and multiple equilibria can emerge. If competition is sufficiently soft and trained workers are substitutes, firms may invest in non-specific training if others do the same, because they would otherwise suffer a competitive disadvantage or need to pay high wages in order to attract trained workers. Government intervention can be socially desirable to turn training into a focal equilibrium.

JEL Classification:   D42, L22, L43, L92

Keywords: General training, human capital, oligopoly, turnover

Hans Gersbach
Alfred Weber-Institut
Universität Heidelberg
Grabengasse 14
69117 Heidelberg
Tel.: +49-6221-54 3173
Fax: +49-6221-54 3578
Email: hans@gersbach.de

* We are grateful to Stefan Bühler, Josef Falkinger, Dennis Gärtner, Verena Liessem, Sarah Niggli, Andreas Polk, Christoph Schmidt, Josef Zweimüller and seminar audiences in Seattle (Econometric Society World Congress) and Zürich for helpful discussions. Thomas Borek provided excellent research assistance.
1 Introduction

Economists have long wondered why firms provide incentives for employees to invest in productivity-enhancing general human capital, rather than providing them only with firm-specific knowledge. After all, general training investments make the workers potentially more valuable for other employers. The famous arguments by Becker (1964) and Mincer (1974) suggest that, with competitive labor markets, firms have no incentive to bear the costs of general worker training, as the associated rents accrue fully to the employees: if training investments are contractible, workers will obtain the full increase in marginal product resulting from the investment. However, these predictions seem at odds with reality (Acemoglu and Pischke 1998, Franz and Soskice 1995, Katz and Ziderman 1990, OECD 1999). In some countries, such as e.g. Germany or Switzerland, firms voluntarily pay for apprenticeships that provide workers with general skills.\footnote{For instance, Bardeleben et al. (1995) calculate the net costs per apprentice per year in German industry as DM 20509 per year if fixed costs are included, and DM 9194 if they are not. A considerable part of this training is general - for instance, apprentices spend about 65 days per year in external schools and courses.}

In addition, firms continually pay for on-the-job training of incumbent workers.\footnote{Several recent empirical studies show that a significant part of the training is general: Baily and Gersbach (1995), Blundell et al. (1996), Goux and Maurin (1997), Loewenstein and Spletzer (1998), Regner (1995, 1997) and Vilhuber (1997, 1998).}

We provide an explanation of such general training that relies on imperfect competition on the product market. We argue in the setting of a three-stage game. In a first stage (training stage), two initially identical firms decide on how many workers they want to train. In a second stage (turnover stage), firms compete through wage offers for the trained workers; workers accept the better offer. Finally, in the product market stage, firms engage in oligopolistic product market competition. Training and turnover decisions in the first two stages determine the distribution of trained workers and thereby product market profits: Excluding the costs of training and wages, we assume that profits of a firm depend positively on their own num-
ber of trained workers and negatively on the competitor’s, as trained workers increase productivity.

When firms compete for trained workers in the second stage, equilibrium wages will correspond to the marginal contribution of a trained worker to product market profits, for a given total supply of workers. This marginal value has two components. First, an additional worker in the own firm is directly beneficial for product market profits, because it increases the own productivity. Second, the fact that the competitor does not employ this worker is beneficial because it weakens the competitor.

A first result is that no training will often be an equilibrium. In fact, the value of training for an individual firm is even negative when no competitor trains, so that the firms under consideration would be willing to pay to prevent their workers from training. Intuitively, this is true because trained workers can threaten to join the competitor, which would reduce the original firm’s payoff.

Importantly, however, this is not necessarily the only equilibrium. We develop this insight in two steps. We start by considering the second stage, assuming that training has taken place in stage 1. In a second stage equilibrium, no worker should provide a higher marginal contribution to the profits of its firm than it would to the competitor — otherwise there will be additional turnover. Building from this idea, we introduce a plausible condition for an even spread of workers across firms, ”Decreasing Returns to Attracting Workers” (DRAW). This condition will imply that industry profits are maximized if workers are spread evenly across firms. Therefore, only for an even spread of workers will the gain for any firm from poaching any number of competitors be smaller than the competitor’s loss. Thus, wage bidding guarantees that in the equilibrium of the turnover game all workers will be spread uniformly across firms, no matter how training levels were distributed before the turnover stage. We further show that, under (DRAW), wages usually decrease as additional workers are trained: Intuitively, the competition on the labor market increases.
With these results in place, we move to the second step of the argument. Clearly, with (DRAW), if one firm trains \( n \) workers, both firms will have \( n/2 \) additional workers in equilibrium. Thus - apart from the direct costs it involves - training by a firm \( i \) has four kinds of effects which result from the change in second-period equilibrium brought about by an increase in trained workers. First, training has a positive profit effect arising because firm \( i \) will employ half of the additional trained workers, thereby increasing own productivity. Second, there is a negative effect arising because half of the additional trained workers will be employed in the other firm, thereby increasing the competitor’s productivity. Third, there is a negative effect on profits arising because firm \( i \) will have to pay wages to half of the additional trained workers. Fourth, additional competition among trained workers tends to decrease wages of the inframarginal workers.

We show that the two positive effects of training often dominate over the negative effects, so that training can indeed result in equilibrium. We then proceed to strengthen the argument that softer product market competition makes the training equilibrium more likely. To this end, we use theoretical arguments as well as simulations. It turns out that a large market size or a high extent of product differentiation strengthen the training equilibria, and quantity competition makes the equilibrium more likely than price competition. Potentially, our theory should thus allow us to explain cross-sectoral differences in training expenditures. In the light of our model, the widespread perception that firms have recently become less willing to invest in general training might be the result of increased product market competition. In this respect, our paper goes beyond existing theory, which assumes perfect product market competition and thus cannot use different intensities of product market competition as an explanatory variable.

\(^3\)The above-mentioned study by Bardeleben et al. (1995) shows great differences in training intensities across different sectors. Net variable expenditures per worker and year differ between DM 1002.- (food industry) and DM 20,565 (chemical industry). In terms of gross expenditure, the differences are still large, ranging from DM 12,142.- (road construction) to DM 32,027 (chemical industry).
Finally, we discuss which institutional coordination mechanisms can move firms into the training equilibrium in cases with multiple equilibria: industry associations, wage bargaining between employer associations and unions and government intervention. This suggests that cross-country differences in the extent to which such coordination mechanisms are established might explain the differences in general worker training observed in OECD (1999), Acemoglu and Pischke (1998), Booth and Snower (1996).

Alternative explanations of why firms encourage general training investments rely on labor market imperfections caused by asymmetric information.\(^4\) Essentially, if present employers can observe ability or training investments better than potential future employers, the latter will face a lemon’s problem.\(^5\) As a result, it is difficult for the employee to sell herself on the job market. The original employer enjoys ex post informational monopsony power, the anticipation of which creates incentives to finance general training. While our theory of training focuses on product market imperfections and does not involve asymmetric information, it shares the feature of multiple equilibria with the models of asymmetric information in the labor market by Katz and Ziderman (1990), Chang and Wang (1995, 1996), Abe (1994), Prendergast (1992), Glaeser (1992), Acemoglu (1997) and Acemoglu and Pischke (1998).

Of course, as in familiar alternative theories, firms only pay for general training in our model if this pays off. However, our paper covers new ground in several respects. For instance, firms only train if others do the same. Moreover, our emphasis on the interactions between labor and product markets not only allows us to draw a connection between the intensity of competition and training. In addition, we can show how product market competition affects wages for trained workers: Essentially, with more intense product

---

\(^4\)For a survey of such explanations see Acemoglu and Pischke (1999). These authors also discuss some alternative theories that do not rely on asymmetric information (see also Stevens (1994), Kessler and Lülfesmann (2000)).

market competition, the workers’ enjoy greater bargaining power and wages increase. Thus, we see our contribution not only as another theory of general worker training, but also as a step towards an integrated analysis of product and labor markets. In the following, we spell out these arguments in more detail. In section 2, we introduce the assumptions of our model. In section 3, we present our first results, using the simplifying assumption that each firm can train at most one worker. Section 4 relaxes this assumption. In section 5, we discuss policy implications. Section 6 concludes.

2 The Model

The structure of the model is as follows. In period 1, firms $i = 1, 2$ simultaneously choose their general human capital investment levels $g^i \in \{0, 1, 2, ...\}$. Think of $g^i$ as the number of its employees receiving general training. Training a worker costs $I > 0$ for a firm. Denote firm $i$’s trained workers as $i_1, ..., i_m, ..., i_{g^i}$. At the beginning of period 2, firm $i$ can make individual wage offers $w_{i,i_m}(g^i, g^j)$ for each of their own workers and $w_{i,j_m}(g^i, g^j)$ for each of the competitor $j$’s worker ($j \neq i$). In principle, we allow wages to differ even for individuals who have the same level of human capital or belong to the same firm. We normalize wages of non-trained workers to zero. Further, we assume that the wage of the non-trained worker is also the reservation wage for the trained workers, that is, their knowledge is useless outside the industry under consideration. After having obtained the wage offers, each employee accepts the higher offer. Denote the number of trained workers

---

6As the incentive of workers to acquire general human capital is undisputed, we ignore the possibility of training investments by workers and deliberately assume that the entire training costs are borne by the firm.

7Here "wages" should be interpreted broadly, including any type of non-monetary benefits such as pleasant working environments, fringe benefits and flexible working hours which involve costs for the employer.

8We use a flexible tie-breaking rule: if $w_{i,i_m} = w_{j,i_m}$, whether the employee stays in his original firm or moves is determined by equilibrium requirements.
in firm $i$ at the end of period 2 as $n^i$. Having workers with general human capital is beneficial for the present employer; it could for instance help to reduce production costs, or to increase demand by improving product quality. This feeds into our modeling of product market competition in period 3 as follows.

**Assumption 1:** For each combination $(n^i, n^j)$ of trained workers, there exists a unique product market equilibrium with resulting gross product market profit $\pi^i(n^i, n^j) = \pi(n^i, n^j)$ for firm $i$. For firms $i = 1, 2$, $\pi^i(n^i, n^j)$ is increasing in $n^i$ and decreasing in $n^j$.

Intuitively, the higher the number of trained workers in a firm, the greater productivity and thus the higher the market profit. The higher the number of trained workers in the competitor’s firm, the higher the competitor’s productivity and thus the lower the own profit.

Assumption 1 contains several implicit statements about the training technology and product market competition. To start with, note that $\pi$ is a function of $n^i$ and $n^j$, not of $g^i$ and $g^j$. This has two immediate implications. First, if an employee leaves the firm, the original employer loses all the benefits generated by the human capital investment - the employee leaves no traces once he has left the firm. This is compatible with the assumptions in training theories that are based on labor market imperfections. Second, training a worker and hiring a trained worker are perfect substitutes. This assumption differs from the training literature which argues that one’s own workers and competitors’ workers are imperfect substitutes, because the ability of the own worker is better known. We use perfect substitutability to express the generality of human capital in its starkest form, and we shall show later on that general training can arise in equilibrium, in spite of this assumption. Finally, note that the profit function $\pi$ does not depend on $i$ directly, only on the number of trained workers. Thus, firms are treated symmetrically.

It will sometimes be convenient to assume $\pi^i(n^i, n^j)$ is defined for arbitrary positive numbers, not just for integers: $n^i \notin N$ refers to situations
such that at least one worker works part-time. In addition, we shall suppose that \( \pi \) is differentiable. We use the following notation:

- The *gross product market profit for symmetric firms*, i.e., for firms which have the same state variable: \( \pi(n) = \pi(n^i, n^j) \) with \( n = n^i = n^j \).
- *Net product market profits*: \( \Pi(n^i, n^j) = \pi(n^i, n^j) - \text{total wage payments} \).
- *Long-term payoff*: \( \Pi(n^i, n^j) - g^i \cdot I \)

Finally, we impose an assumption on product market competition that is slightly more restrictive than assumption 1: Gross profits increase if, starting from a symmetric situation, both firms increase the number of workers by the same amount.

**Assumption 2:** \( \frac{\partial \pi}{\partial n} \geq 0 \).

The game structure is summarized in the Table 1.

<table>
<thead>
<tr>
<th>Period 1:</th>
<th>Firms ( i = 1, 2 ) choose training levels ( g^i ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 2:</td>
<td>(i) Firms choose wage offers ( w_{i,m}(g^i, g^j); w_{i,m}(g^i, g^j) ).</td>
</tr>
<tr>
<td></td>
<td>(ii) Workers choose between employers, thus determining the numbers ( n^i ) of trained workers.</td>
</tr>
<tr>
<td>Period 3:</td>
<td>Product market competition results in gross profits ( \pi^i(n^i, n^j) ).</td>
</tr>
</tbody>
</table>

We can treat this game as a two period game, as all the relevant information about period 3 is contained in the reduced form profit \( \pi^i(n^i, n^j) \).

### 3 A simple example: one worker per firm

We start with a simple example. Suppose training is a \((0, 1)\)-decision, such that each firm is limited to training one worker at most. This case is obviously unrealistic, but the intuition is essentially the same as in the more general case of arbitrarily many trained workers. In sections 3.1 and 3.2, we analyze
the second stage if one or two workers, respectively, have been trained. Then we consider subgame perfect equilibria.

3.1 The turnover game for one trained worker

Consider the second stage of the game, assuming that only one of the two firms has trained its worker. To understand equilibrium wage offers, first recall that for each firm there are two reasons why having a trained worker is valuable. First, it increases the own productivity. Second, it decreases the competitor’s productivity. Accordingly, the total effect of a trained worker on gross profits is $\pi(1,0) - \pi(0,1) = [\pi(1,0) - \pi(0,0)] + [\pi(0,0) - \pi(0,1)]$.

Both firms thus have identical valuations for the worker, and wages will be bid up to $\pi(1,0) - \pi(0,1)$. Hence, being indifferent between the maximal wage offers of both firms, the worker stays with his initial employer. Thus, in equilibrium, the employer of the trained employee has a net profit $\pi(1,0) - [\pi(1,0) - \pi(0,1)] = \pi(0,1)$. Net product market profits for the competitor are also $\pi(0,1)$.

The wage offer illustrates the problem of firms when only one firm has invested in training. Essentially, the worker extracts the entire difference between the gross profits of a firm that has a trained worker and the profits of a firm that has none, and both firms are worse off than without training. This would appear to strengthen Becker’s argument (Becker 1964) that firms have no incentives to invest in general human capital. If, as assumption 1 requires, gross product market profits depend negatively on the competitor’s training level $[\pi(0,1) < \pi(0,0)]$, incentives to invest are negative rather than zero: even if $I$ were equal to zero, a firm would be better off not investing at all. A firm that invests in human capital makes itself vulnerable to the threat that its worker leaves the firm, thus hurting the firm not only by increasing its own costs, but also by decreasing the competitor’s cost. It does not matter whether the worker actually carries out the threat: if she does, the original employer will lose gross product market profits of $\pi(1,0) - \pi(0,1)$, if not, wage payments of the same amount will be necessary. As a result,
firms would even be willing to pay a positive amount of money to prevent employees from acquiring human capital even when there are no training costs. In the case of arbitrarily many workers treated in section 4 it will turn out that the effect described is always present, but countervailing effects can potentially destroy the no-training-equilibrium.

Finally, this simple example offers a first illustration of the more general principle that higher competitive intensity yields higher wages for trained workers: Wages \((π(1,0) − π(0,1))\) increase with the intensity of product market competition, because a large difference \(π(1,0) − π(0,1)\) says that there is a large payoff to being better than a competitor rather than worse, which corresponds to increasing intensity of competition in the sense of Boone (2000).

### 3.2 The turnover game for two trained workers

We next investigate the turnover game when each firm has trained one worker. Let \(G = g^1 + g^2 = n^i + n^j\). Then, to denote the value of an additional trained worker, if the own number of trained workers is \(n^i\) and the total number is \(G\), we write

\[
v(n^i, G) = \pi(n^i + 1, G - n^i - 1) - \pi(n^i, G - n^i).
\]

In our example, \(v(0, 2) = \pi(1, 1) - \pi(0, 2)\) is the value of having one of the workers rather than having none at all, given that two workers have been trained. Given the initial distribution of workers \((g^1 = g^2 = 1)\), \(v(0, 2)\) is the willingness to pay to prevent the departure of a worker to the competitor. On the other hand, \(v(1, 2) = \pi(2, 0) - \pi(1, 1)\) is the willingness to pay for a second worker. The following definition will be crucial.

**Definition 1** If \(v(n^i, G)\) is decreasing in \(n^i\), we shall say that there are Decreasing Returns to Attracting Workers (DRAW).

The equilibrium depends on whether \(v(n^i, G)\) is decreasing or increasing.
Figure 1a: The turnover game for $v(0,2) > v(1,2)$

Figure 1b: The turnover game for $v(0,2) < v(1,2)$

Figure 1: The turnover game

in $n'$. To see this, we first introduce additional notation. $w(n', G)$ is the wage payment that is necessary to obtain the services of one more worker, assuming that the number of own workers is $n'$ and the total number of workers is $G$. In our example, $w(0, 2) = v(1, 2)$ and $w(1, 2) = v(0, 2)$. To have one worker rather than none, one has to pay as much ($w(0, 2)$) as the competitor would pay for having two workers rather than one ($v(1, 2)$). To have two workers rather than one, one has to pay as much ($w(1, 2)$) as the competitor would pay for having one worker rather than none ($v(0, 2)$). Figure 1 describes the equilibrium.

Figure 1a corresponds to the case (DRAW) where

$$v(0, 2) > v(1, 2).$$  \tag{1}$$

Figure 1b corresponds to $v(0, 2) < v(1, 2)$. With Figure 1a, it is straightfor-

---

9As we will show in a specific example, both cases are possible in principle.
ward to show that, in equilibrium, \( n^i = n^j = 1 \) if \( v(0, 2) > v(1, 2) \). First, note that each firm would be prepared to pay at most \( v(1, 2) \) for a second worker. But this is less than what the firm would have to pay to attract this worker, namely \( w(1, 2) = v(0, 2) \). Thus, if both firms offer a wage of \( v(1, 2) \) to both workers, this will be an equilibrium. With these wages, both workers are indifferent between employers, and stay where they are. Clearly, there is no incentive to reduce wages unilaterally: By doing so, the firm would save the wages for one worker \((v(1, 2))\), but gross profits would fall by \( v(0, 2) > v(1, 2) \). Conversely, by increasing wages to \( w > v(1, 2) \), a firm would be able to hire the second worker. This would increase gross profits by \( v(1, 2) \) which would not make up for the increase in wage payments.\(^{10}\)

Similar arguments show that there is no pure strategy equilibrium with an even distribution of workers if \( v(0, 2) < v(1, 2) \). Intuitively, starting from a symmetric situation, for each firm poaching the second worker is more attractive than it is for the competitor to keep the worker. Further, there will be an equilibrium where one firm employs both workers, paying wages of \( v(1, 2) \) to one and \( v(0, 2) \) to the other one, which leads to a wage sum of \( \pi(2, 0) - \pi(0, 2) \). As a result, net profits are \( \pi(0, 2) \) for both firms.\(^{11}\)

Summing up, we obtain the following result.

**Proposition 1** : (a) Suppose two workers have been trained and condition (1) holds. There is no turnover in the period 2 equilibrium. Equilibrium wages are \( v(1, 2) = \pi(2, 0) - \pi(1, 1) \), net profits are \( 2\pi(1, 1) - \pi(2, 0) \).

(b) If condition (1) does not hold, there is no pure strategy equilibrium without

\(^{10}\)Clearly, the equilibrium wage of the turnover game is not uniquely defined: Any wage between \( v(1, 2) \) and \( v(0, 2) \) is an equilibrium. From the firm’s point of view, the equilibria are Pareto ranked: The low-wage equilibrium \( v(1, 2) \) dominates all the other equilibria.

\(^{11}\)As both firms have the same net profits, they are indifferent between having no workers or having both workers. For the firm that employs both workers, not employing the more expensive worker would lead to savings of \( v(1, 2) \), but also to gross profit losses of the same size. Similarly, attracting the cheaper worker does not increase net profits of the firm without any employee. Note, however, that the equilibrium is in weakly dominated strategies.
turnover. There is an equilibrium where one firm employs both worker, paying wages of $v(0, 2)$ to one worker and $v(1, 2)$ to the other. Net profits are then $\pi(0, 2)$ for both firms.

Understanding the equilibrium distribution of workers is thus tantamount to understanding the sign of $v(1, 2) - v(0, 2)$. First, what exactly does the value of an additional worker depend on? In short, $v(1, 2)$ and $v(0, 2)$ embody both the training technology and the nature of competition. For definiteness, suppose having a trained worker is valuable because it permits production of any output level at lower marginal costs. Thus, think of marginal costs as a decreasing function $c^i(n^i)$. Suppose that gross product market profits are a function $\pi^i(c^i, c^j)$ that is decreasing in own marginal costs and increasing in the competitor’s marginal costs. Then $\pi^i(n^i, n^j) = \pi^i(c^i(n^i), c^j(n^j))$. Poaching an additional worker means that the own costs will decrease and the competitor’s costs will increase. The strength of both effects depends on the training technology, more precisely, on the marginal cost effect of training $\left| \frac{\partial c^i}{\partial n^i} \right|$. Further, it is necessary to understand how the own cost reduction and the competitor’s cost increase translate into profit increases. Thus, the marginal profit effect of own costs $\left( \frac{\partial \pi^i}{\partial c^i} \right)$ as well as the marginal profit effect of competitor costs $\left( \frac{\partial \pi^i}{\partial c^j} \right)$ influence the value of a trained worker. Loosely speaking, the value of poaching a worker is higher the higher the marginal cost effect of training, the marginal profit effect of own costs and the marginal profit effects of competitor costs. Whether (DRAW) holds, that is, whether the value of poaching a worker is increasing or decreasing in $n^i$, thus depends on how these effects vary with $n^i$. We shall illustrate this with an example.

Suppose the two firms are Cournot competitors, producing homogeneous goods, with market demand $x = a - p$, where $x$ is output, $p$ is price and $a$ is a positive constant. Suppose marginal costs are a function of the number of trained workers with $c(0) = c$, $c(1) = c - \Delta$ and $c(2) = c - (1 + \beta)\Delta$, $\beta > 0$, $\Delta > 0$.

---

12 This is true for almost every duopoly model.
Using Proposition 1 and the standard result that profits in a Cournot duopoly with linear demand and marginal costs \((c_i, c_j)\) are \(\frac{(a-2c_i+c_j)^2}{9}\), we obtain

**Corollary 1** Suppose that in the linear Cournot example two workers have been trained. Then, after the turnover game, workers are evenly distributed if and only if

\[
(a - c) (2 - 2\beta) + \Delta (2 - 5 (1 + \beta)^2) \geq 0.
\]

Figure 2, which corresponds to \(\Delta = 0.2\), plots the combinations of the market size parameter \(\alpha \equiv a - c\) and the technology parameter \(\beta\) for which workers are evenly distributed in equilibrium. The upper line describes the upper boundary of this regime.\(^{13}\) The parameter \(\beta\) has to be sufficiently small, so that the second worker has small effects on marginal costs, and market size \((a - c)\) has to be large, that is, competition is soft.

Finally, the role of product market competition for wages is similar as in the case where only one firm has trained a worker: Again, the wage level is high when there is a high premium \((\pi(2, 0) - \pi(1, 1))\) for being ahead of the competitor rather than being equally good. Thus, intense competition and high wages correspond. This can also be seen for the Cournot example: There, wages are \((\pi(2, 0) - \pi(1, 1)) = \frac{(2\beta - 2a)\Delta + (\beta^2 + 1)\Delta^2}{9}\). Thus, these wages are decreasing in the market size parameter \(\alpha\) - but great market size is typically considered as less intense competition.

### 3.3 Subgame Perfect Equilibrium

Section 3.1 has shown that no equilibrium can exist such that only one firm trains its worker. We now show that, if (DRAW) holds, training by both firms may arise in subgame perfect equilibrium.

\(^{13}\)The lower line will be explained in the next section.
Proposition 2 Suppose each firm can train at most one worker.
(a) A subgame perfect equilibrium with training exists if and only if
\[ 2\pi(1,1) - \pi(2,0) - \pi(0,1) \geq I. \] (2)

(b) In addition, for arbitrary parameter values, there is an equilibrium without training.

Proof. We first show that (2) implies existence of a training equilibrium. First note that (2) implies (1), because 
\[ 2\pi(1,1) - \pi(2,0) - \pi(0,1) < 2\pi(1,1) - \pi(2,0) - \pi(0,2) = v(0,2) - v(1,2). \]
Therefore, if both firms train, wages are \( \pi(2,0) - \pi(1,1) \), and long-term payoffs are \( 2\pi(1,1) - \pi(2,0) - I \).
By the arguments in section 3.1, deviation to no training would result in net product market profits of \( \pi(0,1) \), from which the result follows.
To show necessity of (2) for training, first suppose (2) does not hold, but (1) does. Firms would still receive long-term payoffs of \( 2\pi(1,1) - \pi(2,0) - I \).
with training, but deviation to no training would be profitable. If (1) does not hold, proposition 1 implies that net profits are $\pi(0, 2)$ and thus long-term payoffs are $\pi(0, 2) - I$ if both firms train. By deviating to no training a firm could guarantee itself $\pi(0, 1)$.

Part (b) relies on the arguments laid out in section 3.1: Without training, firms both obtain long-term payoffs $\pi(0, 0)$. Unilateral training leads to net product market profits of $\pi(0, 1)$ and long-term payoffs $\pi(0, 1) - I < \pi(0, 1) < \pi(0, 0)$.

Before giving a general interpretation of the result, it is helpful to analyze the equilibrium condition for the Cournot example. Training by both firms is an equilibrium if and only if

$$2\alpha (1 - 2\beta) \geq \Delta (3 + 8\beta + 4\beta^2) + \frac{9I}{\Delta}.$$ 

The lower line in Figure 2 displays the upper boundary of this parameter regime, assuming that $I = 0$. Obviously, choosing $I > 0$ would result in a downward shift of this curve. Nevertheless, for sufficiently low values of $I$, there is training in equilibrium, provided market size $\alpha$ is high, the complementarity parameter $\beta$ is low and the marginal cost effect of training the first worker ($\Delta$) is sufficiently small. The relation between $\alpha$ and the training equilibrium is particularly important: With greater market size corresponding to less intense competition, there is a negative relation between the intensity of competition and training. There are two complementary economic explanations. First, there is a scale effect: increasing market size increases output size and thus the benefits to cost reduction. Second, there is a wage effect. As noticed earlier, decreasing competitive intensity reduces workers’ bargaining power, which implies that low wages are sufficient to prevent the departure of trained workers. Clearly, lower competitive intensity therefore favors training.

What is the intuition behind the training equilibrium? It is well known (Acemoglu and Pischke 1999) that general training requires a compressed
wage structure: training increases productivity by more than it increases wages. In which sense is the wage structure compressed in our setting? In our framework, "productivity" effects are best defined as gross profit effects. To define them, suppose competitor $j$ has trained one worker.

Consider as a first reference point the situation where $i$ does not train a worker and does not poach the competitor’s worker. In this reference situation, firm $i$ thus has gross product market profits $\pi(0, 1)$ and wages $0$. Under condition (1) training would result in gross product market profits $\pi(1, 1)$ and wages $\pi(2, 0) - \pi(1, 1)$. In this interpretation, the productivity effect of training is thus $\pi(1, 1) - \pi(0, 1)$, the wage effect is $\pi(2, 0) - \pi(1, 1)$. Condition 2 is thus equivalent to the requirement that the productivity effect is greater than the wage effect.

An alternative interpretation of productivity and wage effects will be helpful in the following. Consider as a reference point the situation where firm $i$ does not train, but poaches the trained worker instead. As discussed earlier, this also results in net product market profits of $\pi(0, 1)$. However, gross product market profits in the reference point are now $\pi(1, 0)$ and wages are $\pi(1, 0) - \pi(0, 1)$. Training would result in gross profits $\pi(1, 1)$ and wages $\pi(2, 0) - \pi(1, 1)$. Clearly $\pi(1, 1) < \pi(1, 0)$. By (DRAW), $\pi(2, 0) - \pi(1, 1) < \pi(1, 1) - \pi(0, 2) \leq \pi(1, 0) - \pi(0, 1)$. Thus, in this interpretation, training implies that both gross product market profits and wages fall. Gross profits fall because the trained worker will end up with the competitor. Wages fall because the competition between trained workers reduces firms’ willingness to pay for them. Obviously, by condition (1), the wage effect is stronger than the gross profit effect, resulting in positive training effects on net profits. Again, this is consistent with the notion of a compressed wage structure.

The idea that, as a result of training, increased competition between workers may drive down wages will be central in our following generalizations. Before, we discuss the case of Bertrand competition to show that with tough product market competition, training cannot arise in equilibrium.


3.4 Bertrand competition

Under Bertrand competition, firms only earn positive gross profits if they have strictly lower marginal costs than the competitor. Thus, if higher $n^i$ corresponds to lower costs, condition (DRAW) is violated since $v(0, 2) = 0$ and $v(1, 2) = \pi(2, 0) > 0$. Thus, by proposition 1 (b), both firms obtain net profits of $\pi(0, 2) = 0$ and by proposition 2 the following result is immediate.

**Proposition 3** Suppose each firm is allowed to train at most one worker. In the Bertrand case, training never arises in equilibrium.

As Bertrand competition is essentially the same as perfect competition, this reinforces the idea that general training requires sufficiently soft product market competition, which also showed up in the Cournot example in section 3.2.

4 Arbitrary numbers of workers

So far, we have confined ourselves to the patently unrealistic case that each firm can train at most one worker. As we now show, allowing for the possibility of training additional workers does not change the basic result that firms may have incentives to invest in general training. In a sense, it even reinforces the result: Not only do training equilibria also exist for suitable parameter values, but, in addition, the no-training equilibrium might disappear.

We first show that under assumption (DRAW) - which applies verbatim to this more general case - each firm will end up with the same number of workers in the turnover game, up to integer constraints. We shall then use this result to state conditions under which equilibria with and without training exist. Finally, we shall give a detailed interpretation of these conditions.

4.1 The Turnover Game

In this section, we first state conditions under which the turnover game will lead to an even distribution of educated workers across firms, given that $G$
Proposition 4 Suppose that assumptions 1- 2 and (DRAW) hold.

(A) Then there exists an equilibrium \((n^i, n^j)\) of the turnover game such that:
   (i) If \(G\) is an even number \(2N\), then \(n^i = n^j = N\).
   (ii) If \(G\) is an odd number \(2N + 1\), then \(n^i = n^j - 1 = N\) for some \(i \in \{1, 2\}, j \neq i\).

(B) In both cases, each educated worker obtains wage offers
\[w^* (N, G) = \pi (N + 1, G - N - 1) - \pi (N, G - N)\] by both firms.

(C) There is no equilibrium with \(|n^j - n^i| > 1\).

Proof. see Appendix.\(^{15}\) ■

The intuition for this result generalizes the ideas from section 3: If (DRAW) holds and workers are distributed evenly, each firm values an additional worker less than the competitor values keeping this worker. With an uneven distribution, the firm with the smaller number of workers is willing to pay more for at least one of the competitor’s worker than he is prepared to pay for keeping him. Thus, equilibria must result in an even distribution of workers. Wages correspond to the value of an additional worker, \(\pi (N + 1, G - N - 1) - \pi (N, G - N)\).\(^{16,17}\)

\(^{15}\)The result in the appendix uses a slightly weaker requirement than (DRAW) which is useful for the example in section 4.3.

\(^{16}\)The same equilibrium obtains in a competitive labor market framework when firms take wages and demand of other firms as given. Therefore, equilibria with training can occur in a competitive labor market with demand externalities. Details are available from the authors upon request.

\(^{17}\)It is straightforward to show that any other wage profile where everybody is offered the same wage between \(\pi (N, G - N) - \pi (N - 1, G - N + 1)\) and \(\pi (N + 1, G - N - 1) - \pi (N, G - N)\) is also an equilibrium with an even distribution of workers. However, these other equilibria are Pareto-inefficient from the firms’ point of view, since wage costs are higher than in the equilibrium described in Proposition 4. In the following, we assume that the firms achieve an equilibrium distribution of workers at minimal wage costs, i.e. we use the Pareto criterion among firms as a selection device. Since firms make wage offers, this assumption is plausible.
As in the examples of section 3, wages are therefore highest when it pays a lot to be better than the competitor, that is, competition is intense in the sense of Boone (2000).

### 4.2 Subgame Perfect Equilibrium

We now analyze the subgame perfect equilibrium of the entire game.

Doing this for a discrete version of the game is tedious, as it requires distinguishing between even and odd numbers of workers. We use a continuous approximation instead. The main insight of the turnover game analyzed in section 4.1 are: First, if assumption (DRAW) holds, \( n^i = n^j = G/2 \), up to integer constraints. Second, the equilibrium wage equals the productivity of the marginal worker, \( M(n^i, G) \equiv \pi(n^i + 1, G-n^i - 1) - \pi(n^i, G-n^i) \). A natural extension to the continuous case would be to define \( M(n^i, G) \equiv \frac{\partial \pi}{\partial n^i} \left( \frac{G}{2}, \frac{G}{2} \right) - \frac{\partial \pi}{\partial n^j} \left( \frac{G}{2}, \frac{G}{2} \right) \) and suppose that the equilibrium wage equals this quantity: In the continuous case, this is the marginal value of poaching an employee for firm \( i \), which consists of the effect of employing more workers oneself \( \left( \frac{\partial \pi}{\partial n^i} \right) \) and of reducing the number of workers employed by the competitor \( \left( -\frac{\partial \pi}{\partial n^j} \right) \). Using these two results from the discrete game to approximate the second period equilibrium in the continuous game, we immediately obtain

**Lemma 1** Suppose (DRAW) holds. If firms choose training levels \( g^i \) and \( g^j \) and the second-period equilibrium is played, firm \( i \)'s long-term payoffs in the continuous approximation of the game are

\[
\Pi^i \left( \frac{G}{2}, \frac{G}{2} \right) - g^i \cdot I = 
\pi^i \left( \frac{G}{2}, \frac{G}{2} \right) - \left( \frac{G}{2} \right) \left[ \frac{\partial \pi^i}{\partial n^i} \left( \frac{G}{2}, \frac{G}{2} \right) - \frac{\partial \pi^i}{\partial n^j} \left( \frac{G}{2}, \frac{G}{2} \right) \right] - g^i \cdot I.
\]

Before deriving conditions for equilibria with and without training, it is helpful to see how net product market profits react to a marginal increase in

---

\(^{18}\)We did this in the working paper (Gersbach and Schmutzler, 2001)
own training. Using lemma 1

\[
\frac{\partial \Pi^i}{\partial g^i} = \frac{1}{2} \frac{\partial \pi^i}{\partial n^i} + \frac{1}{2} \frac{\partial \pi^i}{\partial n^j} - \frac{1}{2} \left( \frac{\partial \pi^i}{\partial n^j} - \frac{\partial \pi^i}{\partial n^j} - \frac{G}{4} \left( \frac{\partial^2 \pi^i}{(\partial n^i)^2} - \frac{\partial^2 \pi^i}{(\partial n^j)^2} \right) \right).
\]

Generally speaking the total effect of an additional marginal trained worker thus has the following four components.

(OPE) The own productivity effect on gross product market profits \( \left( \frac{1}{2} \frac{\partial \pi^i}{\partial n^i} > 0 \right) \)

As workers are distributed equally in equilibrium, only half of the marginal increase in the number of trained workers becomes effective in increasing product market profits for firm \( i \) under consideration.

(CPE) The competitor productivity effect on gross product market profits \( \left( \frac{1}{2} \frac{\partial \pi^i}{\partial n^i} < 0 \right) \)

The second half of the increase in trained labor will end up with the competitor, leading to a negative effect on one’s own product market profit.

(ATW) Wage payments to additional trained workers \( \left( -\frac{1}{2} \left( \frac{\partial \pi^i}{\partial n^i} - \frac{\partial \pi^i}{\partial n^j} \right) < 0 \right) \)

As half of the additional trained labor is employed by the firm under consideration, this results in additional wage payments.

(WPTW) Changes in wages per trained worker \( \left( -\frac{G}{4} \cdot \left( \frac{\partial^2 \pi^i}{(\partial n^i)^2} - \frac{\partial^2 \pi^i}{(\partial n^j)^2} \right) \right) \)

The sign of WPTW is not fully specified by our assumptions. However, the intuition from section 3 that additional competition among trained workers drives down wages if condition (DRAW) holds, carries over in many cases. Clearly, this is true if \( \pi^i \) is concave as a function of \( n^i \) and convex as a function of \( n^j \). Also, concavity of \( \pi(n, G - n) \) as a function of \( n \) implies that

\[
\frac{\partial^2 \pi^i}{(\partial n^i)^2} - \frac{\partial^2 \pi^i}{(\partial n^j)^2} = \frac{\partial^2 \pi^i}{(\partial n^i)^2} + \frac{\partial^2 \pi^i}{(\partial n^j)^2} - 2 \frac{\partial^2 \pi^i}{\partial n^i \partial n^j} + 2 \left( \frac{\partial^2 \pi^i}{\partial n^i \partial n^j} - \frac{\partial^2 \pi^i}{(\partial n^j)^2} \right) < 0
\]

and (WPTW) is therefore positive.

\[19\] This is not likely: Typically, at least \( \frac{\partial^2 \pi^i}{\partial n^j \partial n^j} < 0 \), roughly speaking, because the positive effect of trained workers on the own mark-up of a firm is higher when the other firm has less trained workers and thus faces a smaller market share.
The total effect of OPE, CPE and ATW is \( \frac{\partial \pi_i}{\partial n_i} < 0 \) for marginal changes. Increasing the number of workers in the market marginally is thus only worthwhile if the negative effect (CPE) is outweighed by the reduction in wages for inframarginal workers (WPTW).

Even though we do not explicitly treat the case of more than two firms, this reasoning suggests why, for given market size, training is less likely to arise in equilibrium for large numbers of firms. First, if there are many firms in the market, (OPE) is likely to be small, if it is still true that the trained workers will be spread out equally over firms. Similarly, the effect of one’s own additional trained workers on the wage level will be small, so that it is unlikely to be a strong argument for training.

We first give conditions under which an equilibrium without training exists in the two-firm case.

**Proposition 5** Suppose that (DRAW) holds. An equilibrium without training only exists if

\[
\pi^i(g, g) - g \left[ \frac{\partial \pi^i}{\partial n^i}(g, g) - \frac{\partial \pi^i}{\partial n^j}(g, g) \right] - \pi^i(0, 0) - 2gI \leq 0 \quad \text{for all } g \geq 0. \tag{3}
\]

**Proof.** Consider firm 1. By lemma 1, deviating from \((0, 0)\) to \(g^1 > 0\) gives a profit of

\[
\pi^1 \left( \frac{g^1}{2}, \frac{g^1}{2} \right) - g^1 \left[ \frac{\partial \pi^1}{\partial n^1} \left( \frac{g^1}{2}, \frac{g^1}{2} \right) - \frac{\partial \pi^1}{\partial n^2} \left( \frac{g^1}{2}, \frac{g^1}{2} \right) \right] - g^1 \cdot I
\]

as compared to \(\pi^1(0, 0)\) in equilibrium. With \(g = \frac{g^1}{2}\), the statement follows. \(\blacksquare\)

For sufficiently low investment costs, (3) will not hold if

\[
\frac{\pi^i(g, g) - \pi^i(0, 0)}{g} > \frac{\partial \pi^i}{\partial n^i}(g, g) - \frac{\partial \pi^i}{\partial n^j}(g, g) \tag{4}
\]

for some \(g > 0\), that is, if the increase in net profit per additional worker outweighs the wage cost per worker. This is perfectly possible, as we will show in the example below.

Using similar arguments, we can derive a condition for an equilibrium with training to exist.
Proposition 6  Suppose (DRAW) holds. Suppose that there exists a $g > 0$ such that, for $k = \frac{g^i + g^j}{2}$,

$$\pi^i (k, k) - k \cdot \left[ \frac{\partial \pi^i}{\partial n^i} (k, k) - \frac{\partial \pi^i}{\partial n^j} (k, k) \right] - g^i I$$

is maximized at $g^i = g$. Then there exists a subgame perfect equilibrium with $g > 0$.

Proof. The result follows immediately from lemma 1.

As in the examples of section 3, the equal distribution of workers implied by (DRAW) is crucial. Suppose that, as an extreme counterexample, one of the firms attracts all trained workers. Then, using the logic that showed the necessity of (DRAW) for a training equilibrium in proposition 2, there can be no training: Both firms will have the same net product market profits, which amount to the gross profits of the firm without trained workers. These profits are smaller than gross profits without training. Therefore, in such cases the training equilibrium cannot exist.

4.3 Example

As a numerical example, we consider the case of price competition of two firms producing imperfect substitutes, with demand functions $D_i(p_i, p_j) = A - 10p_i + p_j$, where $0 \leq A \leq 30$. We specify the training technology as $c_i = 2 \exp (-n_i)$. Thus, marginal costs are $c_i = 2$ without training, and they decrease exponentially with training. For simplicity, we suppose each firm is restricted to training at most $g^i = 4.15$ workers. Using the logic of proposition 4, this assumption can be shown to guarantee that workers are distributed equally in the turnover game. Figure 3 plots equilibrium training levels as a function of the market size parameter. The outcome is in line with our general intuition: For low parameter values, no training takes

\footnote{Proposition 6 implies that deviations to $g^i = 0$ should not be beneficial, so that
$\pi (g, g) - g \cdot \left[ \frac{\partial \pi}{\partial n^i} (g, g) - \frac{\partial \pi}{\partial n^j} (g, g) \right] - gI - \pi \left( \frac{g}{2}, \frac{g}{2} \right) + \frac{g}{2} \left[ \frac{\partial \pi}{\partial n^i} \left( \frac{g}{2}, \frac{g}{2} \right) - \frac{\partial \pi}{\partial n^j} \left( \frac{g}{2}, \frac{g}{2} \right) \right] + \frac{g}{2} \cdot I > 0.$}
place. Around $A = 12.1$, there is a discrete jump to a training level $g^i \approx 2.45$. As $A$ increases further, training increases further.

Thus, once again, the intuition that soft competition – in this case, due to greater market size – fosters training is confirmed.

\[\text{Equilibrium Training Levels}\]

5 Welfare Results and Policy Discussion

Our analysis is partly motivated by different institutional arrangements in labor markets across the OECD. In some countries, such as Germany, firms offer apprenticeships to their workers. The knowledge acquired in such programs is mostly general in the sense of being applicable in other firms of the same industry. Nevertheless, firms bear part of the training costs. In contrast, the U.S. economy appears to generate less general training than Germany or Japan, at least at the initial stage of a worker’s life (Blinder and Kruger 1996, Acemoglu and Pischke 1998).\footnote{Training investment in later stages of a worker’s life are relatively low in Germany (OECD 1999), but the differences in the initial stage appear to be more substantial.}

In terms of our model, there are two different explanations. For simplicity, we argue in terms of the model in section 3, even though a generalization to an arbitrary number of workers is straightforward. Further, suppose that for

\[\text{Equilibrium Training Levels}\]
each industry under consideration, there is full separation between the German and the U.S. labor and product markets. Each of the two corresponds to one set of parameters of the game.

First, obviously, the relevant parameters of the game could differ for Germany and the US. Roughly speaking, Germany could be in the regime where proposition 2a applies and the U.S. in the regime where it does not. The differences might come from industry characteristics such as the intensity of competition. Alternatively, state interventions might have affected the payoff functions. Second, one could think of the game as being the same in both countries, with both countries in different equilibria. German firms have coordinated on the training equilibrium, while US firms are in the no-training equilibrium.

To understand the welfare implications of the multiple equilibria in the latter case, we restrict ourselves to the set-up in section 3 with at most one trained worker per firm. We start by looking at firm profits. It is possible that the total long-term payoffs for firms are higher in the training equilibrium.

**Proposition 7** Suppose that training and no-training equilibria coexist, i.e. inequality (2) holds. Then, the training equilibrium is better for firms if and only if

\[ 2\pi(1, 1) - \pi(2, 0) - \pi(0, 0) \geq I. \]

Straightforward derivations show that condition (5) is never satisfied in the linear Cournot case.\(^{22}\)

Next, we compare aggregate welfare in equilibria with and without training. Aggregate welfare is defined as the sum of long-term payoffs of firms, wages and consumer surplus. Suppose that firms compete in a market with homogeneous products and demand function \(D(p)\). We denote by \(p^*(n^1, n^2)\) the equilibrium prices depending on the distribution of trained workers. In

\(^{22}\)Obviously, condition (5) may hold in other examples, for instance when products are sufficiently differentiated, so that each firm is essentially a monopolist and hence \(\pi(1, 1) \approx \pi(1, 0)\).
the training equilibrium, the sum of firms’ long-term payoffs, wages and consumer surplus is given by

\[ 2\pi(1, 1) - 2I + \int_{p^*(1,1)}^{\infty} D(p)dp. \]  

(6)

In the no-training equilibrium aggregate welfare is given by

\[ 2\pi(0, 0) + \int_{p^*(0,0)}^{\infty} D(p)dp. \]  

(7)

Therefore, we obtain:

**Proposition 8** Suppose that training and no-training equilibria coexist. Then aggregate welfare is higher in the training equilibrium if and only if

\[ 2\pi(1, 1) - 2\pi(0, 0) + \int_{p^*(0,0)}^{p^*(1,1)} D(p)dp > 2I. \]  

(8)

It is readily verified for the linear Cournot case where each firm can train at most one worker that, if it exists, the training equilibrium dominates the no-training equilibrium in terms of welfare (see section 3.3).

By combining proposition 7 and 8 for the linear Cournot case with one trained worker, we observe that firms are better off in the no-training equilibrium, but welfare is higher in the training equilibrium. If firms use the Pareto selection criterion to coordinate themselves on no-training, there could thus be a role for government policy to reach the training equilibrium. What are the mechanisms explaining why industries in a country may coordinate on the training equilibrium while others may not? Potentially, there are at least three such mechanisms.

First, wage setting institutions may promote the training equilibrium. In Germany, large employer associations and labor unions negotiate on wages and working conditions. Establishing curricula and other formal procedures
in connection with negotiations could enhance the chances of achieving the training equilibrium.

Second, government intervention could bring industries into the training equilibrium. On the one hand, the state can offer complementary investments such as schooling facilities where costless classroom education is provided. Moreover, the government can regulate the curricula and demand that apprentices take standardized exams, as in Germany. On the other hand, temporary support for general training investment may establish a social norm which will remain after direct support has been withdrawn. Apart from granting direct financial aid, governments could provide such temporary support by promoting universal acceptance of certificates from apprenticeships.

Third, in cases where the training equilibrium yields higher total payoffs for firms, industry associations themselves may facilitate coordination on a training equilibrium. For instance, by offering special courses for apprentices, paid from the associations’ budgets, the association may be able to promote participation in training programs.

Which coordination mechanisms might be at work in Germany or in other countries is beyond the scope of this paper, but the preceding considerations raise the question whether complementary activities or temporary support of the state could be useful to lead firms into the training equilibrium. As an example we have seen that government intervention can be justified in the linear Cournot case.

Perhaps the most important implication of our model is that increasing competitive intensity might destroy the training equilibrium. This suggests that the German apprenticeship might come under serious pressure should competition in manufacturing increase further. More importantly, another crucial question arises: Can the German apprenticeship survive as firms are becoming more and more exposed to competitors without such programs?
6 Conclusions and Extensions

In this paper, we provide a theory of general worker training. Our explanation of training does not rely on asymmetric information in the labor market. Instead, we require imperfect product market competition to generate equilibria with general training in a world where turnover is endogenous.

Training equilibria exist for plausible parameter values, possibly together with the no-training equilibrium. We do not claim that training is likely in all industries. The most important conditions concern the training technology and the toughness of product market competition. Competition must be sufficiently soft and returns to training must decrease sufficiently fast for turnover to be avoided and training to arise in equilibrium. The role of product market competition comes from two sources here: a standard scale effect which is familiar from the analysis of cost-reducing innovations and a wage effect that is specific to models of training with turnover.

The arguments have been cast in a duopoly framework. They appear to hold more widely in an oligopolistic framework, but it is important to investigate how the number of firms in a market affects the likelihood of a training equilibrium. As discussed in section 4.2, a plausible conjecture would be that training becomes less likely as the number of firms increases, because competition becomes more intense. One can show that this is indeed the case in the Cournot case with linear demand.\textsuperscript{23} For empirical applications we therefore hypothesize that training tends to be more likely in an industry if

- concentration is high or competitive intensity is comparatively low
- returns to training decrease sufficiently in the relevant area
- product differentiation is sufficiently strong.

\textsuperscript{23}Details about such comparative exercises where the number of firms increases with and without adjustments of the market size are available upon request from the authors.
Whether such hypotheses receive empirical support is an important step in future research for the further development of a product market theory of training.
7 Appendix: Proof of Proposition 4

Proof. For existence, we restrict ourselves to the case where \( G = 2N \) is even; the case \( G = 2N + 1 \) is similar. We first show that, given the competitor’s wage offers \( w^* (N, G) \), lowering wages is not profitable. Suppose the firm reduces its wage offer to \( k \) workers \( (k \leq N) \) so that it ends up with only \( N - k \) workers. This deviation is not profitable if

\[
\pi (N, N) - k \cdot w^* (N, G) \geq \pi (N - k, N + k).
\]

As \( w^* (N, G) = \pi (N + 1, N - 1) - \pi (N, N) \), this is equivalent to:

\[
\pi (N, N) - \pi (N - k, N + k) \geq k (\pi (N + 1, N - 1) - \pi (N, N)),
\]

which is implied by (DRAW). Thus, downward deviation is not profitable. As to upward deviations, a higher wage offer for one worker would yield an increase in gross profits of \( \pi (N + 1, N - 1) - \pi (N, N) \), which is exactly offset by the additional wage payments \( w^* (N, G) \). By (DRAW), attracting any further worker would yield additional gross profits smaller than \( \pi (N + 1, N - 1) - \pi (N, N) \) and thus smaller than the additional wage payment. Hence, there are no profitable deviations.

To show that there is no equilibrium with \( n^i < n^j - 1 \), note that the willingness of firm \( i \) to pay for an additional worker is \( \pi (n^i + 1, G - n^i - 1) - \pi (n^i, G - n^i) \), which by (DRAW) is greater than \( \pi (n^j, G - n^j) - \pi (n^j - 1, G - n^j + 1) \), which is the value of the last worker that firm \( j \) employs. \( \blacksquare \)
8 References


Chang, C. and Wang, Y. "Human Capital Investment under Asymmetric

