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Firm Productivity, Wages, and Sorting

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Abstract

Increasing wage inequality is associated with changes in the degree of labor market sorting, i.e. the allocation of workers to firms. To measure sorting, we propose a new method which disentangles the respective contributions of worker and firm heterogeneity to wage inequality. Inspired by sorting theory, we infer firm productivity from estimating firm-level production functions, taking into account that worker ability and firm productivity may interact at the match level. Using German data, we find that highly productive firms display low labor shares, dominate concentrated markets, and pay lower wages than less productive firms. Sorting is positive, but lower than what wage-based measures suggest. It increases over time, driven by new matches between low-productivity firms and low-ability workers. At the top, sorting decreases, reflected in worker transitions away from high-productivity firms that pay relatively low wages. We discuss implications of our findings for the interpretation of increasing wage inequality.

Zusammenfassung

Wachsende Lohnungleichheit geht mit Veränderungen der Allokation von Arbeitnehmern zu Arbeitgebern im Arbeitsmarkt einher. Dies spiegelt sich auch in einem steigenden Sortierungsgrad des Arbeitsmarkts wider. Wir entwickeln eine neue Methode zur Messung dieses Sortierungsgrades, welche die jeweiligen Beiträge von Arbeitnehmer- und Arbeitgeberheterogenität zur Lohnungleichheit entflechtet. Inspiriert von theoretischen Modellen der Arbeitsmarktsortierung leiten wir die Produktivität der arbeitgebenden Unternehmen aus Schätzungen von Produktionsfunktionen auf der Firmenebene ab. Wir berücksichtigen dabei insbesondere, dass die Firmenproduktivität auf der Matchebene mit der Leistungsfähigkeit der einzelnen Arbeitnehmer interagieren könnte. Anhand deutscher Daten beobachten wir, dass hochproduktive Firmen niedrige Lohnquoten aufweisen, in konzentrierten Märkten operieren und geringere Löhne zahlen als weniger produktive Firmen. Der Sortierungsgrad ist positiv aber niedriger als lohnbasierte Maße nahelegen. Er steigt mit der Zeit, getrieben durch neue Matches zwischen relativ unproduktiven Firmen und weniger leistungsfähigen Arbeitnehmern. An der Spitze geht der Sortierungsgrad zurück, was sich darin widerspiegelt, dass Arbeitnehmer die produktivsten Firmen, die relativ geringe Löhne zahlen, verlassen. Wir diskutieren Implikationen unserer Ergebnisse für die Interpretation steigender Lohnungleichheit.

JEL

J24, J31, J40, J62, J64, L25

Keywords

Assortative Matching, Labor Market Sorting, Wage Inequality, Job Mobility, Unobserved Heterogeneity, Firm Productivity, Production Function Estimation

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1. Introduction

The upward trend in wage inequality in many countries, including Germany and the U.S., is a topic of high interest for both policymakers and academics.[1] The firms' role in this development has been of interest for a long time.[2] Recently, Barth et al. (2016) and Song et al. (2019) show that increasing wage inequality in the U.S. is to a large extent driven by widening wage gaps between employers.[3] Song et al. (2019) argue that increasing *wage sorting* of high-wage workers into high-wage firms is a major contributing factor. Card/Heining/Kline (2013) document the same phenomenon for Germany. Despite their important contribution to wage inequality, little is known about the nature of high-wage firms.

In this paper, we aim to fill this gap. We argue that a careful characterization of high-wage firms is necessary to understand increasing worker sorting and its contribution to rising wage inequality. The reason is that wages can be high (or low) for several reasons: first, highly productive firms may share high output with their workers through high wages. Second, highly productive firms may be able to pay lower wages by exploiting labor market imperfections, e.g. search frictions. Third, relatively unproductive (perhaps young) firms may be forced to pay high wages to retain workers or expand their workforce. These three examples show that firm productivity is a central determinant of wages. The mapping from productivity into wages, however, is not necessarily obvious.

We characterize firms based on their unobserved productivity and, to this end, develop a new way of measuring it: we estimate firm-level production functions using detailed German data, building on the latest insights in the empirical industrial organization (IO) literature (Ackerberg/Caves/Frazer, 2015; Gandhi/Navarro/Rivers, 2019). Importantly, we specifically take into account that unobserved worker ability and firm productivity may interact at the match level, which is the main driving force of labor market sorting in theory (Becker, 1973; Shimer/Smith, 2000). Building on this methodological contribution, we show that high wages are indeed not always a reflection of high firm productivity. In the data, we detect all three aforementioned examples of wage-productivity links. Moreover, worker sorting into high-productivity firms is less pronounced than sorting into high-wage firms, implying a smaller contribution of *productivity sorting* to rising wage inequality.

A common approach to quantifying the respective contributions of unobserved worker and firm heterogeneity to rising wage inequality goes back to Abowd/Kramarz/Margolis (1999)

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1 Acemoglu/Autor (2011) survey the related literature. Autor/Katz/Kearney (2006) report that the gap between the 90th and 50th percentile of the U.S. wage distribution has increased at roughly one log point per year during between the-mid 1970s and the mid-2000s. Dustmann/Ludsteck/Schönberg (2009) find that the gap between the 85th and 50th percentiles of the German wage distribution has increased at a rate of about 0.6 log points per year between 1975 and 2004. Therefore, wage dispersion in Germany has grown at roughly two-thirds of the U.S. rate since the 1970s.

2 Early work on this topic studied industry wage differentials (Dickens/Katz, 1987; Krueger/Summers, 1988; Bell/Freeman, 1991; Gibbons/Katz, 1992). Davis et al. (1991) and Groshen (1991) were among the first to study wages at the firm/establishment level. The volume by Lazear/Shaw (2009) includes a number of studies from different countries.

3 Barth et al. (2016) use LEHD data for nine U.S. states and observe workers and single establishments from 1992–2007. Song et al. (2019) use data from the U.S. Social Security Administration (SSA) in which they observe all workers and firms in the U.S. between 1978–2013.

(henceforth AKM), a seminal article that changed the way researchers use matched employer-employee data. The AKM model exploits variation of workers' individual wages across firms and variation of firms' pay across workers to identify worker and firm-fixed effects using a log-linear wage equation.[4] In the literature we review below, a common finding across countries is that estimated worker effects, reflecting differences in unobserved ability, explain the major share of wage dispersion in the data. Estimated firm effects, which can be interpreted as wage premia that firms pay to all their employees, are associated with about 20 percent of wage dispersion.

Card/Heining/Kline (2013) (henceforth CHK) for Germany and Song et al. (2019) for the U.S. are examples of papers that follow the AKM approach. They study the sources of increasing wage inequality by decomposing wage dispersion into the contributions of unobserved worker ability, firm wage premia, and wage sorting in the labor market. Wage sorting measures the extent to which workers who receive high wages are also matched with firms that pay high wages. The point that we make in this paper is that the way one measures firm heterogeneity, by the wages firms pay or by their productivity, makes a difference for understanding increasing wage inequality. We find that firms with the highest estimated productivity do not pay the highest wages.

To measure unobserved worker ability, we rely on the wage-based rank aggregation technique proposed by Hagedorn/Law/Manovskii (2017) (henceforth HLM). Using an equilibrium search model, HLM show how to identify the sign and strength of labor market sorting without imposing a log-linear wage equation.[5] HLM test their method on German data and report an estimated degree of sorting (correlation of worker and firm ranks) of 0.76, much higher than the correlation of about 0.21 which CHK report for a comparable period.[6] Our benchmark estimate of the rank correlation between wage-based worker types and productivity-based firm types, the degree of productivity sorting, is 0.15. This is relatively close to but lower than the CHK measure of wage sorting and much lower than the HLM estimate. Similar to AKM and CHK, HLM also use observed wages and worker mobility to measure both worker and firm heterogeneity. Therefore, our approach differs from HLM in the way we measure firm heterogeneity, based on estimated firm productivity rather than wages, and this yields a lower estimated degree of sorting.

We use German social security register data for our analysis, which are ideal for two reasons: first, we can directly compare our results to HLM and CHK who also work with German data. Second, the German matched employer-employee data can be linked to a variety of high-

[4] The estimation of this two-way fixed-effect model requires structural assumptions on the wage equation, in particular additive separability. Moreover, it is assumed that high-wage workers do not systematically sort into high-wage firms (exogenous mobility). See also Abowd/Creecy/Kramarz (2002) for details.

[5] Due to production complementarities (Becker, 1973), wages might be a non-monotonic function of worker and firm types. In this case, the log-linear AKM wage equation is misspecified (Gautier/Teulings, 2006; Eeckhout/Kircher, 2011; Lopes de Melo, 2018). CHK argue that log-linearity is a defensible assumption because deviations from it (as measured by the AKM residuals) appear to be small for most, but not all, combinations of worker and firm types. Using our method of measuring the firm type, we find quantitatively important deviations from wage monotonicity, see Section 6.

[6] 0.21 is the mean of two correlations that CHK measure for the time period that HLM use.

quality firm data sources, including both administrative records and surveys.[7] This allows us to estimate production functions at the establishment level[8] to infer firm productivity. We access the universe of social security registers to track all workers at all the establishments we can estimate productivity for. Thus, we are able to control for the full distribution of heterogeneous worker ability at the establishment level when estimating firm productivity to measure firm productivity *net of the effect of workforce ability*.

Moreover, using insights from the theory of labor market sorting, we take into account that worker ability and firm productivity may interact at the match level. We propose a way to disentangle the respective contributions of heterogeneous worker ability and firm productivity to output. Thus our productivity measure nets out the firm-specific effect of the average worker ability that a given firm employs at every point in time. This allows us to distinguish between firms that pay high wages due to high productivity, firms that pay high wages despite low productivity, and firms that pay low wages despite high productivity. Applying our novel firm productivity measurement technique to German data leads to three main findings that are highly relevant for understanding changing sorting patterns and increasing wage inequality.

First, we find that sorting is positive, although our measure is lower than what wage-based measures suggest (CHK, HLM). We also find that sorting is increasing over time. This is mainly driven by low-ability workers who form new matches with low-productivity firms out of unemployment. These matches are characterized by relatively low wages and low (sometimes negative) wage growth. For high-ability workers and high-productivity firms, we observe that sorting is somewhat decreasing over time. The most productive firms have reduced their hiring of the most able workers. Related to this finding, we observe that almost all worker ability types receive lower wages at the most productive firms as compared to slightly less productive firms. Similarly, low and medium-ability workers sometimes face lower wages as they move to more productive firms. Thus, we present evidence that for most workers the wage is not everywhere monotonically increasing in firm productivity.

Second, we find that firms at the top of the (estimated) productivity distribution are special. They have high revenues and labor productivity (value added per worker), but they are not large in terms of employment or their capital stock. Interestingly, their wage bills are lower than those of medium-productivity firms. They pay out relatively small shares of revenue and value added to their workers, implying low labor shares. Following sorting theory, we argue that low labor shares, relatively small wage bills, and non-monotonically evolving (falling) wages at the most productive firms might reflect a specific form of market power, option value compensation. In short, the outside option of hiring better workers may allow these firms to pay lower wages to most worker types.[9]

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[7] The data are made accessible through the research data center of the Institute for Employment Research (Institut für Arbeitsmarkt- und Berufsforschung, IAB) at the German Federal Employment Agency (Bundesagentur für Arbeit).

[8] In the German data, we do not observe firms in the legal sense but establishments, i.e. single production units. We use the terms firm, establishment, and employer interchangeably throughout the paper but always refer to establishments.

[9] This idea is closely related to another potential explanation discussed in the recent literature: monopsonic

Our finding of low firm-level labor shares at highly productive firms establishes a link to the literature on the falling aggregate labor shares, a trend observed in many developed economies. Indeed, our firm ranking appears to mirror an explanation emphasized in Autor et al. (2017) and Autor et al. (forthcoming): the emergence of so-called "superstar" firms. The idea is that, due to globalization or technological progress, highly productive firms, the superstars, become increasingly dominant.[10] Market concentration rises as a result and because superstar firms have high markups and low labor shares, the aggregate labor share falls. Our findings suggest that decreasing aggregate labor shares, rising market concentration, changing sorting patterns, and increasing wage inequality might be different reflections of the same underlying secular trend.

Third, we show that using productivity-based firm types as compared to wage-based ones makes a difference for understanding the sources of increasing wage inequality. Decomposing the variance of wages into the shares explained within and between establishments reveals that, similar to the trend observed in the U.S. (Song et al., 2019), the contribution of the between-firm component to overall wage dispersion has been rising by almost 10 percent in Germany between 1998 and 2008. In 2008, between-firm inequality is almost on par in magnitude with the relatively stable within-firm component. However, this picture changes when we decompose the variance of wages using our estimated firm-productivity and worker-ability types. We find that the share of wage variance explained between firm-productivity types increases over time but only by around 4 percent. Its contribution is dwarfed by the variance shares explained within firm-productivity types and between worker-ability types, which we find to be the major sources of rising wage inequality. Thus, we find that increasing sorting of high-ability workers into high-productivity firms is quantitatively a less important source of rising wage dispersion, which is in line with the relatively low degree of productivity sorting we observe overall.

Related Literature

Only a small number of papers in the empirical literature on wage dispersion uses non-wage-based measures of firm heterogeneity. Bartolucci/Devicienti/Monzón (2018) use balance sheet data for a set of Italian firms to rank them by profits. Bagger/Lentz (2019) propose to rank firms by the share of workers they poach from other firms. Sorkin (2018) follows a revealed preference approach by applying Google's page ranking algorithm to worker flows in the U.S.. Haltiwanger/Hyatt/McEntarfer (2018) study the cyclical properties of worker flows between firms using gross output per worker as a measure of firm productivity.

Researchers have developed a variety of methods to estimate the sign and strength of labor market sorting. While the sign is usually found to be positive, reflecting positive assortative matching (PAM), there are large differences in the estimated degree of sorting across coun-

wage setting power of firms. See Berger/Herkenhoff/Mongey (2019) for a recent exploration of this hypothesis in the U.S. context.

[10] Elsby/Hobijn/Şahin (2013), Karabarbounis/Neiman (2014), Barkai (2016), De Loecker/Eeckhout/Unger (forthcoming), Dao et al. (2017), Hall (2018), and Kehrig/Vincent (2018) also study falling labor shares and different explanations for it.

tries and methods.[11]

Lise/Meghir/Robin (2016) estimate a structural model on U.S. data to get a direct measure of the elasticity of substitution between worker and firm types. They find evidence for PAM.[12]

Several papers on labor market sorting use Scandinavian data. Bonhomme/Lamadon/Manresa (2019) propose a clustering technique (finite mixture model) to identify a discrete number of firm types in an initial step before estimating a model with non-linear interactions between worker and firm types.[13] Using Swedish data, they find a benchmark correlation between worker and firm effects of 0.49. Lentz/Piyapromdee/Robin (2018) use a variant of the Bonhomme/Lamadon/Manresa (2019) method on Danish data and report a correlation coefficient of 0.28.

On the one hand, this number is higher than the correlation found using the AKM model on Danish data, which is 0.05 according to Bagger/Sørensen/Vejlin (2013). On the other hand, the Lentz/Piyapromdee/Robin (2018) correlation is lower than the correlation of 0.39 that Bagger/Lentz (2019) (henceforth BL) find using the same data. BL estimate an equilibrium search model in which sorting is driven by on-the-job search with endogenous intensity. Thus, a similarity between the Danish and the German case appears to be that structural methods (HLM and BL, respectively) deliver higher correlations than the AKM approach.[14] Overall, the degree of positive labor market sorting appears to be higher in Germany than in Denmark.[15]

Due to the increasing availability of matched employer-employee data, there is also a number of related papers in the empirical IO literature. These papers control for labor quality differences when estimating production functions. Two examples are Fox/Smeets (2011) and Irarrazabal/Moxnes/Ulltveit-Moe (2013) who study productivity dispersion.[16] Card et al. (2018) link the literature on rent-sharing, that is, the pass-through from firm productivity to wages, to the empirical literature on wage dispersion in the spirit of AKM. Bagger/Christensen/Mortensen (2014) also estimate firm-level production functions with heterogeneous labor inputs to study the sources of wage dispersion using Danish data. The main difference between our approach and Bagger/Christensen/Mortensen (2014) lies in the underlying model of labor market sort-

[11] The variation of estimated rank correlation coefficients across countries of course reflects both cross-country differences in, say, labor market institutions and methodological differences.

[12] The magnitude of their estimated substitution elasticity is not readily comparable to the rank correlation coefficients reported in most other studies.

[13] The k-means clustering algorithm essentially compares within-firm wage distributions. The number of discrete types has to be set in advance by the researcher. After clustering, firms with the same type look similar in terms of moments of the within-firm wage distribution. Note that the labeling of types is arbitrary, that is, a structural interpretation in terms of productivity requires additional assumptions.

[14] For the Danish case, the estimate of Lentz/Piyapromdee/Robin (2018) lies in between the estimates obtained with AKM (Bagger/Sørensen/Vejlin, 2013) and a structural model (Bagger/Lentz, 2019). To the best of our knowledge, the Bonhomme/Lamadon/Manresa (2019) method has not yet been applied to German data.

[15] A fascinating avenue for future research is the question to what extent this observation can be explained by different labor market institutions and industrial structures in the two countries.

[16] Fox/Smeets (2011) find that observable worker characteristics like education, gender, experience, and industry tenure have significant coefficients and explain about one fifth of the overall productivity dispersion across firms. The 90-10 percentile ratio of productivity is reduced from 3.27 to 2.68 across eight Danish manufacturing and service industries with labor quality controls. Using Norwegian data, Irarrazabal/Moxnes/Ulltveit-Moe (2013) find that 25 to 40 percent of the productivity premium of exporters is related to labor input quality differences, including unobserved worker heterogeneity. See also Syverson (2011) for an overview of the literature on productivity dispersion.

ing. While their wage equation conveniently reduces to a log-linear form that allows estimation in the spirit of AKM, the match-level complementarity that we allow for precludes log-linearity of the wage equation.

The remainder of our paper is structured as follows: Section 2 introduces the theoretical sorting model that guides our empirical approach. Section 3 describes our data. Section 4 explains our approaches to estimate worker and firm ranks and studies the properties of our rankings. Using the estimated ranks, Section 5 explores the extent of labor market sorting in Germany and documents changes over time. Section 6 relates our findings to wages and trends in wage inequality. Section 7 concludes.

2. A Model of Labor Market Sorting

The theory of labor market sorting has rich implications for the allocation of workers to firms and the determination of wages. To estimate firm productivity net of workforce ability, we use a simple model of labor market sorting to derive a structural link between firm productivity, workforce composition, output, and wages. We build on the frictional version of Gary Becker's optimal assignment model (Becker, 1973) developed by Shimer/Smith (2000): guided by a production complementarity, heterogeneous workers and firms seek to match with their optimal counterpart to maximize output and wages, but this process is hindered by search frictions. Suppose the production function at the match level is log-supermodular. As Shimer/Smith (2000) show, a search equilibrium exists under this condition and reflects positive assortative matching (PAM).[17]

We assume that worker and firm heterogeneity are one-dimensional.[18] Workers carry an identifier i and differ in terms of ability $a(i)$. Firms carry an identifier j and are characterized by productivity $\omega(j)$. Worker ability and firm productivity are distributed uniformly on the unit interval, known to all market participants, do not change over time, and cardinally measurable. Thus, their distributions imply economy-wide rankings of workers and firms, respectively. $x(i)$ denotes the rank of worker i in the ability distribution. $y(j)$ denotes the rank of firm j in the productivity distribution. Meeting rates are governed by a standard Cobb-Douglas matching function with constant returns to scale. For the illustrative model developed here, it is sufficient to assume that only unemployed workers search.[19] Match-level output is determined by the twice continuously differentiable log-supermodular match-level production function $f(a(i), \omega(j))$, which takes worker ability and firm productivity as inputs and is strictly increasing in both dimensions.[20]

[17] Shimer/Smith (2000) show that log-supermodularity (or log-submodularity) of the match-level production function is a necessary condition for existence. (Log-)supermodularity implies PAM, that is, the most productive firm's optimal partner is the most able worker, the second most productive firm's optimal partner is the second most able worker, and so forth. Conversely, (log-)submodularity implies negative assortative matching (NAM).

[18] For recent explorations of sorting with multi-dimensional characteristics, see Lindenlaub (2017), Lindenlaub/Postel-Vinay (2017), and Lise/Postel-Vinay (2018).

[19] We discuss generalizations including on-the-job-search at the end of this section.

[20] Thus, there is a hierarchy of workers and firms. Sorting is based on absolute advantage as in Shimer/Smith (2000). High-ability workers and high-productivity firms always produce more than workers and firms ranked below, regardless of the partner they are matched with. All market participants agree on the ranking of work-

The theory of labor market sorting focuses on one-to-one employment relationships between workers and firms. That is, firms hire no more than one worker. Of course, in the data we observe firms of various sizes in simultaneous matches with many workers of different types. In this paper, we do not derive conditions for sorting under many-to-one matching with search frictions. Eeckhout/Kircher (2018) make important progress in extending sorting theory to multi-worker firms. In their model, firms decide which worker type to hire and, additionally, how many workers of this particular type. The next step in the theoretical literature is to derive conditions for sorting with firms that hire multiple workers of different types, which is what we study empirically in this paper.

To understand the implications of the sorting model for firm-level output, we have to take a stance on the link between, on the one hand, complementarities at the match level and, on the other hand, production at the firm level. We assume the following: the output of every single match is determined by the interaction of worker ability and firm productivity. All matches contribute to a composite labor input that the firm uses together with its capital stock to produce goods. Firms employ multiple workers of various ability types, but there are no complementarities between worker types within the same firm.[21] The complementarity of worker ability and firm productivity, however, implies that the contribution of every single match depends on both the firm's productivity and the worker's ability.[22] Therefore, the marginal product of one unit of worker ability varies with firm productivity and, accordingly, across firms. Taking this into account is key to understanding the link between firm productivity, worker ability, and output. That is, the ability-productivity complementarity at the match level manifests itself as an identification problem at the firm level: firm productivity and the output elasticity of labor cannot be separately identified when estimating the firm-level production function as we formally show in Appendix B.

We address this identification problem by exploiting the logic of wage setting in the sorting model. Workers and firms are willing to match whenever the surplus is high enough to compensate both parties for the foregone option value of continued search. With Nash bargaining, this compensation manifests itself in surplus sharing and, thus, wages.[23] The model's value functions and the wage equation are presented in Appendix A. According to equation (A.8), the bargained wage has three components: the first component is match output, the second component captures both the worker's and the firm's option value of continued search in the labor market, and the third component is the workers income flow during unemployment, e.g. the value of home production or unemployment insurance benefits,

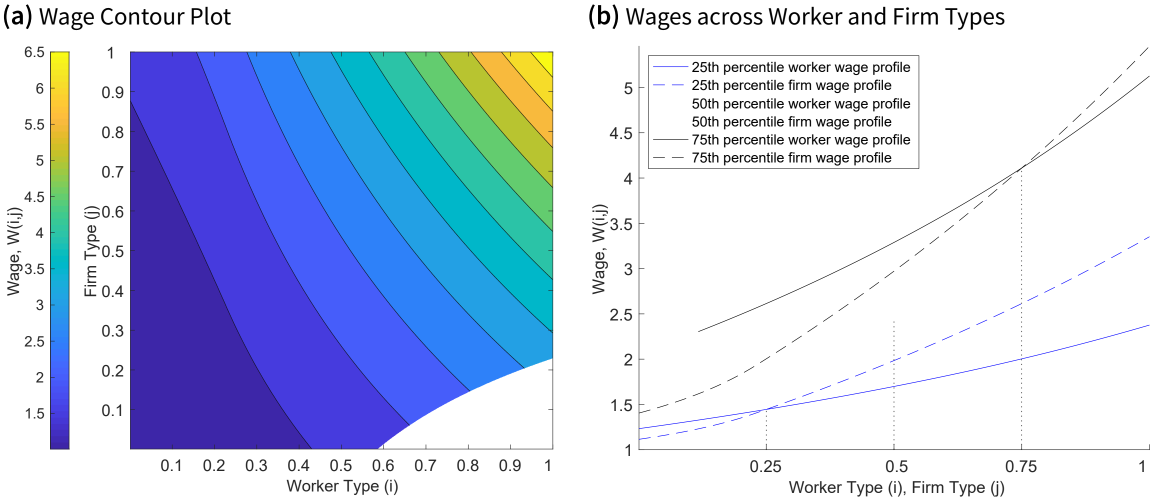
ers and firms. For comparative advantage sorting models, in which rankings may differ, see Marimon/Zilibotti (1999), Gautier/Teulings/Van Vuuren (2010), and Gautier/Teulings (2015).

[21] Mas/Moretti (2009) and Cornelissen/Dustmann/Schönberg (2017) estimate peer effects on worker productivity and wages in the spirit of AKM. Herkenhoff et al. (2018) and Jarosch/Oberfield/Rossi-Hansberg (2019) estimate structural models of coworker learning. The challenge that remains is the disentanglement of coworker effects on wages and output from firm-specific factors like productivity.

[22] We assume that the firm's productivity is a non-rival resource. Multiple workers do not have to "share" the firm's productivity, so we abstract from span of control issues.

[23] Nash bargaining is not a critical assumption. It is sufficient to assume that both parties' payoffs increase in match surplus. This is a feature of a broad class of bargaining games.

Figure 1: Wages in the Sorting Model



Source: The figure is based on a simulation of the sorting model presented in this section.

which, combined with the option value of search, constitutes the worker's outside option. Consequently, wages can be high for two reasons in this model: the first reason is high output (first component), for example in case both firm productivity and worker ability are high and the match-level complementarity is exploited (positive sorting).[24] There is little mismatch in this case, so outside option compensation (second component) is small. The second reason for high wages, conversely, is high outside option compensation, for example in case a low-productivity firm hires a high-ability worker and has to pay compensation for this worker's valuable outside option. As wages can be high for two different reasons, observed wages alone are not informative about the respective contributions of worker ability and firm productivity to output.[25]

We exploit the sorting model's outside option compensation mechanism to overcome the aforementioned identification problem. To illustrate our approach, Figure 1 depicts wages across worker and firm types in equilibrium. Panel (a) is a contour plot of wages. Match-specific wages increase in both the worker type i (horizontal axis) and the firm type j (vertical axis). High-ability workers employed by high-productivity firms earn the highest wages (PAM). The white area in the lower-right corner reflects negative surplus, so no matches are formed between high-ability workers and low-productivity firms in this example.[26] Note that wages do not increase uniformly in worker and firm types. This reflects the different outside options of workers[27] and firms.[28]

[24] A positive match-specific productivity shock is an alternative interpretation of this scenario: output is high for some match-specific reason, and this is reflected in a high wage.

[25] The technical reason is that the wage can be a non-monotonic function of worker ability and firm productivity in sorting models.

[26] This is qualitatively in line with what we find for the German labor market, see Section 5.

[27] Workers' income during unemployment and, thus, reservation wages are assumed to increase in ability.

[28] The firms' value of posting a vacancy is determined by free entry. Due to productivity heterogeneity, we assume that vacancy posting costs are a convex. This ensures that the distribution of vacancies in equilibrium is non-degenerate.

Panel (b) of Figure 1 zooms in and shows wage profiles of selected worker types (solid lines) and firm types (dashed lines) across the firm-productivity and worker-ability distribution, respectively. We define a wage profile as the ordered set of wages a given worker (firm) type receives (pays) in all matches with positive surplus. The wage profile of a 75th percentile worker (black solid line) displays a higher wage at all firm types as compared to workers at the 50th (red solid line) and 25th (blue solid line) percentile of the ability distribution. This reflects both higher match output and a higher outside option of the high-ability worker.

Similarly, a 75th percentile firm (black dashed line) pays higher wages in matches with all worker types as compared to firms at the 50th (red dashed line) and 25th (blue dashed line) percentile of the productivity distribution. Note that both worker and firm wage profiles diverge as the firm and worker types increase: the wage profile of a 75th percentile firm (black dashed line) has a higher slope everywhere as compared to 50th (red dashed line) and 25th (blue dashed line) percentile firms. This is a result of the log-supermodular production function we assume to induce PAM. The surplus from employing high-ability workers is higher at high-productivity firms as compared to low-productivity firms.

Central to our identification argument are the three intersections in Panel (b). The solid and dashed lines in blue, red, and black, respectively, must by construction cross in the points where worker and firm types are equal.[29] How wages change above and below these intersections is instructive. Consider the black lines, solid and dashed. If the 75th percentile worker moves to a marginally better firm, his or her wage increase (solid line) is lower than what a 75th percentile firm has to pay additionally to hire a marginally better worker (dashed line). The production function is symmetric (the output gain is equal in both cases), so the wage difference must be due to different outside options: when hiring a marginally better worker, the 75th percentile firm needs to compensate this worker for giving up both home production and the value of continued search for a better match. The firm's outside option is lower than the worker's outside option—in the model considered here it is zero due to free entry—and this explains the lower slope of the firm's wage profile. Our empirical strategy builds on this property: the worker's option value of continued search is higher than the firm's option value. In other words, the worker has to "give up more" upon matching. This property arises in a broad class of equilibrium search models.[30]

How do different outside options help us to identify firm types? As the model makes clear, observed wages can be high for two reasons: the first is high output and the second is high outside option compensation to the worker. To reliably identify firm productivity, we have to distinguish high-productivity firms that pay high wages for the former reason from low-productivity firms that pay high wages for the latter reason. Thus, we need to measure the

[29] The wage that a 75th percentile worker earns at a 75th percentile firm is equal to the wage that a 75th percentile firm pays to a 75th percentile worker.

[30] Consider a model with multi-worker firms: the firm can be matched with multiple workers at the same time, but the worker can only work for one firm at a time. Thus, the firm foregoes less option value upon matching than the worker because the firm can always hire more workers. In on-the-job-search models, the worker does not lose the option value of working for another firm upon matching. Thus, the foregone option value of search is lower. As long as the workers option value of search is higher in unemployment, e.g. if search on-the-job is less efficient than search off-the-job, on-the-job-search would not change our estimation strategy.

extent of outside option compensation that a firm pays to its workers, at least on average, to understand the relative contributions of worker ability and firm productivity to output and wages. Having such a measure, we can separate firm productivity from the productivity-dependent effect of worker ability on output and solve the identification problem.

To measure the extent of outside option compensation at the firm level, we evaluate the observed wage bill relative to a predicted benchmark wage bill that includes no outside option compensation. According to our model, differences between those two wage bills must be informative about the extent of option value compensation that a firm pays (or receives). Our benchmark model is the CHK implementation of AKM, which we replicate in Section 3.3. The log-linear AKM wage equation abstracts from option value compensation, because this kind of wage effect implies that worker and firm effects interact. Any option value effect in the data is therefore absorbed by the AKM residual, together with potential other match-specific wage effects.[31] In the following, we refer to the ratio between the observed wage bill and the AKM-predicted wage bill as the *wage bill ratio*. The wage bill ratio is convenient to work with because we can simply multiply it by the firm's observed labor input to get an adjusted labor input measure that reflects how worker ability contributes to output for a particular firm given its current productivity. For example, suppose that for a single firm the observed wage bill is higher than the AKM prediction. The wage bill ratio is bigger than one in this case, implying that the firm's average worker is of high ability relative to the firm's productivity because the average worker receives positive outside option compensation. Conversely, a wage bill ratio less than one implies outside option compensation in the other direction, from the worker to the firm. The average worker is of low ability relative to firm productivity and accepts a lower wage to compensate the firm for not waiting longer to hire a better worker. As we will see in the data, both directions of outside option compensation are quantitatively important.

3. Data

Our analysis combines three data sets provided by the Institute for Employment Research (Institut für Arbeitsmarkt- und Berufsforschung, IAB) of the German Federal Employment Agency (Bundesagentur für Arbeit, BA). Two of them contain matched employer-employee data, the "LIAB Mover Model" (LIAB) and the "Integrated Employment Biographies" (IEB). The third data set is a comprehensive establishment-level survey, the "IAB Establishment Panel" (EP). In this section, we describe the different data sets and explain how we prepare and combine them. Some descriptive information is included. Additional details on sample selection and imputation procedures are relegated to Appendix D.

[31] We use option value compensation to illustrate our estimation strategy. Match-specific productivity, another common interpretation of match-specific wage effects, would inform our estimation of firm types in exactly the same way. Our empirical strategy merely assumes that a positive AKM residual reflects a relatively high contribution of a given worker to output, conditional on the firm's productivity.

3.1. Data Sources

The LIAB is our main data set. It provides us with information about a large number of employment spells including wages, precise start and end dates, and the identities of workers and establishments, along with other characteristics. The “Mover Model” version of the LIAB is ideal for our purposes because the sampling procedure builds directly on the EP, which is our primary source of firm data. The employment histories of all movers, defined as workers who are employed by at least two EP-surveyed (and potentially more, non-surveyed) establishments over time, are drawn from the German social security registers. In addition to all (un)employment spells of these movers, employment histories of up to 500 additional workers per establishment are drawn. These workers are not movers according to the EP-based definition, although the majority of them is also employed by multiple (non-surveyed) establishments over time.[32] The fact that the LIAB sampling is based on movers between EP establishments implies that all establishments with at least one mover according to the definition above are also included in the LIAB. Therefore, when linking the two data sets using establishment identifiers, we find virtually all EP establishments in the LIAB.

The EP is a comprehensive yearly survey of establishments, that is, single production units like factories or branches.[33] We also use the terms firm or employer in the following but always refer to single establishments. The EP provides us with the necessary data to estimate production functions at the establishment level. This also implies that the number of establishments we can estimate productivity for is limited to those that participate in the EP survey. A possible concern about working with establishment-level data is that firms (in the legal sense) may consist of multiple establishments that influence one another. The German economy, however, is well-known to be characterized by a broad basis of small and medium-sized enterprises.[34] Accordingly, 80 percent of the establishments in our data (self-reportedly) belong to single-establishment firms.

In the EP data, we observe revenues, intermediate good purchases (reported as a share of revenues), value added (calculated as revenues minus intermediate good purchases), and net investments in four different categories of capital goods (buildings, production machinery, IT, and transport equipment). We supplement the EP data with some additional establishment-level covariates from the “Establishment History Panel” (henceforth BHP).[35] These include average wages, numbers of employees, and shares of full-time/part-time workers and different skill groups. As compared to the EP survey, the BHP provides reliable administrative information on firm age and a consistent industry classification.

As explained in Section 2, we construct a measure of (relative) workforce ability as a model-consistent control variable when estimating firm productivity. To disentangle the wage ef-

[32] See Alda/Bender/Gartner (2005) and Heining et al. (2012) for a detailed description of the LIAB.

[33] The EP is a representative random sample of all establishments in Germany, stratified according to size, industry, and federal state. See Kölling (2000) and Fischer et al. (2009) for a detailed description of the EP data.

[34] Also known as the “Mittelstand”, a German term that refers to firms with annual revenues up to 50 million Euro and a maximum of 499 employees according to a common definition.

[35] The BHP covers all establishments with at least one employee liable to social security on a reference date (June 30th). See Spengler (2008) for a detailed description of the BHP data.

fects of outside option compensation and firm productivity, we evaluate the observed firm level wage bill relative to the AKM-predicted wage bill. Although the LIAB contains matched employer-employee data, it is not ideal to run AKM-style wage regressions, measure workforce ability, and predict the wage bill. The LIAB does not cover the full workforce of the establishments included, only a subsample as explained above. This is problematic for two reasons: first, using the LIAB alone, we can measure worker ability only for this subsample of workers at every establishment. Extrapolating those ability measures to the establishment level is error-prone. Second, the LIAB sampling procedure limits the size of the "connected set" of workers at different firms used to separately identify worker and firm effects in AKM, rendering estimated worker and firm effects less reliable.[36]

To overcome these limitations, we bring in a third data set, the IEB, to construct our workforce ability controls. It contains all worker histories across all establishments in Germany that employ at least one employee subject to social security contributions.[37] Using the IEB, we replicate the CHK implementation of the AKM model. This enables us to more reliably estimate worker and firm effects for our period of interest (defined below) and construct the wage bill ratio, which we then merge with the EP data at the establishment level to estimate firm productivity and construct our firm ranking. Details follow in Section 4.2.

Having access to the university of German social security records, a straightforward way to rank workers and analyze labor market sorting would be to use the estimated CHK-AKM worker effects. Unfortunately, it is legally prohibited to run such a detailed analysis on the full IEB data set. For this reason, we have to rely on the LIAB to construct a worker ranking. Due to the aforementioned problems with running AKM on the LIAB, we use the worker ranking procedure developed by HLM which has one key advantage: it is based on an algorithm that compares the wages of pairs of coworkers who are employed at the same establishment at some point in time.[38] A firm wage premium, paid to both workers, increases both wages by exactly the same amount. Therefore, the ranking of two coworkers at the same establishment, which the algorithm builds on, is not affected by a firm wage premium. Thus, we circumvent the need to estimate firm effects on the LIAB.

Details about our implementation of the HLM algorithm follow in Section 4.1. It should be mentioned, however, that this ranking procedure imposes a considerable computational burden due to the large number of coworker pairs in the data. The LIAB is available for 1993–2008, but computational constraints force us to exclude the first five years from the worker ranking procedure. Thus, the time period for our analysis is 1998–2008.[39] This period is roughly split in half by the German labor market reforms implemented between 2003 and 2005.[40]

[36] This problem is due to the so-called "limited mobility bias" as emphasized by Andrews et al. (2008, 2012) and recently revisited in Borovičková/Shimer (2017) and Kline/Saggio/Sølvsten (2019).

[37] This is the universe of German social security records, CHK use the same data.

[38] The employment spells do not have to overlap.

[39] We do not discard pre-1998 observations of workers and firms though, see Section 3.4.

[40] The so-called Hartz reform package consisted of four reforms that were designed to increase labor demand (Hartz I and II), matching efficiency (Hartz III), and labor supply (Hartz II and IV).

3.2. Imputations

German social security registers are characterized by a high quality of wage data due to various plausibility checks carried out by the social security institutions. Misreporting leads to sanctions. We observe nominal gross daily wages, which we deflate using the consumer price index from German national accounts. Every wage observation corresponds to one employment spell. It can last from one day up to one year according to the reporting rules of the German social security system.[41]

A limitation of the wage data is that earnings are tracked only up to a threshold, the contribution assessment ceiling ("Beitragsbemessungsgrenze").[42] To impute the upper tail of the wage distribution in the LIAB and the IEB, we follow the procedure suggested by Dustmann/Ludsteck/Schönberg (2009) and run a series of Tobit regressions, allowing for a maximum degree of heterogeneity by fitting the regression model separately for years, education levels, and eight five-year age groups.[43] Additional details about the wage imputation can be found in Appendix D.3.

The education variable in German social security data suffers from missing values and inconsistencies. Here, misreporting has no negative consequences for employers and employees. We impute missing and inconsistent observations in the LIAB and the IEB using the methodology proposed in Fitzenberger/Osikominu/Völter (2006). Missing values cannot be imputed for about 2 percent of the data, so we drop these employment spells. Additional details about the education imputation can be found in Appendix D.2.

In the EP data, the capital stock on which an establishment operates is not reported. To estimate the capital input used for production function estimation, we use a perpetual inventory method following Müller (2008). This method approximates the establishment-level capital stock by combining information on net investments (directly available in the EP) with average economic lives (depreciation rates, available from national accounts) of the different types of capital goods we observe investment for.

3.3. Wage Regressions

First, we estimate an AKM model on the IEB data for our period of analysis, 1998-2008, including both men and women in reunited Germany. We aggregate the data to the person-year level and identify the largest connected set.[44] It contains more than 233 million person-years (roughly 22 million workers per year), corresponding to 35 million individual workers at 3.3

[41] Employers are required to file a report whenever an employee joins or leaves the establishment or, in the event of no change in an ongoing employment relationship, on December 31 each year.

[42] The average yearly censoring rate in the LIAB is 13.6 percent of wage observations. We define a wage observation as censored whenever the reported wage is higher than 99 percent of the censoring threshold.

[43] An alternative to imputing the censored part of the wage distribution would be to simply drop top-coded wages. Table D.2a in the Appendix shows that a wage variance decomposition delivers virtually identical results with and without the imputed part of the wage distribution, even though the wage variance without top-coded wages is roughly 39 percent lower.

[44] Following CHK, we focus on full-time workers between 20 and 60 years of age. To aggregate, we calculate wage sums (daily wage multiplied by the spell length in days) for all employment spells. If workers have multiple employment spells in one calendar year, we keep the employment spell that generated the highest earnings.

million establishments. 55 percent of the observations are movers between establishments. We use the CHK specification, that is, we estimate a log-linear wage equation for worker i who works at firm $j(i,t)$ in year t:

$$w_{it} = \alpha_i + \psi_{j(i,t)} + x'_{it}\beta + \varepsilon_{it}, \tag{1}$$

where w_{it} are log real daily wages, α_i is a worker-fixed effect, $\psi_{j(i,t)}$ is an establishment-fixed effect, and x'_{it} contains time-varying controls: an unrestricted set of year dummies and quadratic and cubic terms in age, fully interacted with educational attainment. ε_{it} is an error term. The adjusted R^2 of this regression is 0.92, broadly in line with CHK. The correlation of estimated worker and firm-fixed effects, sometimes interpreted as a measure of labor market sorting, is 0.27 in our time interval. This is slightly higher than what CHK report, likely due to the longer time period and broader sample we consider.[45]

Second, we run a simplified wage regression on the LIAB data for the same time period. We include time-varying observables and worker-fixed effects but omit establishment-fixed effects due to the aforementioned problem of not observing the full workforce of the establishments in the LIAB.[46] The regression equation is

$$w_{it} = \alpha_i + x'_{it}\beta + \varepsilon_{it}, \tag{2}$$

where, again, w_{it} are log real daily wages, α_i is the worker-fixed effect, x'_{it} contains year dummies and quadratic/cubic terms in age fully interacted with educational attainment. ε_{it} is the error term. Unsurprisingly, the explanatory power of regression (2) is lower compared to regression (1). The adjusted R^2 falls to 0.81.

After running the two regressions, we first use the estimated worker effects ($\hat{\alpha}_i$), firm effects ($\hat{\psi}_{k(i,t)}$, only for (1)), and coefficients on workers' observable characteristics ($\hat{\beta}$) to decompose the variance of log wages (Section 3.3.1). This allows us to compare the sources of wage dispersion in the LIAB and different IEB samples. Second, we use the results from model (2) to compute residual wages as input to the worker ranking procedure (Section 3.3.2). Third, we use the results from model (1) to compute the wage bill ratio for the estimation of firm productivity (Section 3.3.3).

3.3.1. Wage Variance Decompositions

Decomposition results are documented in Table 1. Column (a) shows a decomposition for all person-years, (b) for all women, (c) for all men, and (d) for all men in West Germany using the IEB data. The IEB subsample in column (d) is comparable to the LIAB sample (only men, West Germany) for which the decomposition is shown in column (e). In columns (a)–(d), the major share of wage variance is explained by unobserved worker heterogeneity. This finding is well-known in the literature. The worker-fixed effect explains almost half of the observed variation

45 CHK report correlations for shorter time intervals: 0.17 (1996-2002) and 0.25 (2002-2009). CHK include only men in former West Germany. We include both men and women in reunited Germany from 1998 to 2008.
46 As we argue above, this is inconsequential for the purpose of ranking workers using the HLM method.

in wages, slightly more for women and less for men. The second most important source of variation are firm-fixed effects. They explain roughly a quarter of the wage variance across the four IEB samples. The third most important source is the covariance of worker and firm effects, which explains between 12 and 19 percent of wage variance.[47] With only 2 percent, the share of wage variance explained by observable characteristics is almost negligible. The same is true for the covariances of observable characteristics with worker and firm effects. Note, however, that the main effect of time-invariant education is absorbed by the worker effect.

The residual absorbs potential match-specific wage effects like outside option compensation highlighted in our model. It explains variance shares between 7 and 9 percent across the four IEB samples. To separately assess the quantitative importance of match-specific effects, we re-estimate a fully-saturated version of regression (1) with a separate dummy for each worker-establishment pair on the full IEB sample.[48] The adjusted R^2 increases to 0.95 in the fully-saturated model. A decomposition reveals that match effects alone explain 5.8 percent of wage variation, so it accounts for almost 75 percent of the residual in specification (a). The contribution of the remaining error term is 3.5 percent. The quantitative contribution of match-specific effects, which we highlighted in our model, may appear small compared to worker effects, firm effects, and their covariance. Note, however, that we only measure their contribution to wage dispersion. Suppose workers have low bargaining power and a relatively small share of match output is reflected in wages. The quantitative importance of match-specific effects for output and productivity could still be large, as they are only partly reflected in wages.

Column (e) contains the decomposition of the variance of log wages in our LIAB sample. Reassuringly, the wage variance in this sample, 0.207, is relatively close to the variance of 0.226 in the comparable IEB sample (men in West Germany, column (d)). The remaining difference might be related to the EP-based LIAB sampling procedure. The mean wage, however, is almost identical in both samples.[49] The wage variance in the LIAB sample is also very close to the variances reported by CHK.[50] The dispersion of wages in the LIAB sample, which we run the worker ranking procedure on, is thus comparable to the IEB based samples and to what CHK find. Without firm effects, however, the decomposition assigns a higher share of the wage variance, 73 percent, to the worker-fixed effect in the LIAB sample. 17 percent is absorbed by the residual. The contribution of observable characteristics, however, is with 4 percent comparable in magnitude to the IEB samples. Residual wages, which we compute

[47] Interestingly, women are less positively sorted in terms of wages than men. This is in line with what Card/Cardoso/Kline (2016) find using Portuguese data. They argue that higher sorting of men is an important component of the gender wage gap. It reflects that men are more likely to work at high wage firms. Bruns (2019) confirms this finding for Germany.

[48] Note that the residual in regression (1) can be written as the sum of different random effects: $\varepsilon_{it} = \eta_{i,k(i,t)} + r_{it}$, where η_{ik} is a match-specific effect on the wage that worker i earns at firm k. The remaining error, r_{it}, may include additional transitory and non-transitory components.

[49] The mean wage is 4.621 in the column (d) IEB sample and 4.617 in the LIAB sample.

[50] CHK report a variance of 0.187 (standard deviation 0.432) for 1996-2002 and 0.249 (standard deviation 0.499) for 2002-2009.

Table 1: Wage Variance Decompositions

	(a) Regression (1) IEB, full		(b) Regression (1) IEB, women		(c) Regression (1) IEB, men		(d) Regression (1) IEB, men, west		(e) Regression (2) LIAB, men, west	
Var(w_{it})	0.276	(100%)	0.277	(100%)	0.245	(100%)	0.226	(100%)	0.207	(100%)
Var($\hat{\alpha}_i$)	0.126	(46%)	0.138	(50%)	0.105	(43%)	0.106	(47%)	0.152	(73%)
Var($\hat{\psi}_{j(i,t)}$)	0.068	(25%)	0.076	(27%)	0.061	(25%)	0.049	(22%)	–	
Var($x'_{it}\hat{\beta}$)	0.005	(2%)	0.006	(2%)	0.005	(2%)	0.005	(2%)	0.008	(4%)
$2 \times$ Cov($\hat{\alpha}_i, \hat{\psi}_{(i,t)}$)	0.049	(18%)	0.032	(12%)	0.047	(19%)	0.037	(16%)	–	
$2 \times$ Cov($\hat{\alpha}_i, x'_{it}\hat{\beta}$)	0.004	(0%)	0.001	(0%)	0.005	(2%)	0.006	(3%)	0.006	(3%)
$2 \times$ Cov($\hat{\psi}_{j(i,t)}, x'_{it}\hat{\beta}$)	0.003	(0%)	0.001	(0%)	0.004	(2%)	0.004	(2%)	–	
Var($\hat{\varepsilon}_{it}$)	0.021	(8%)	0.025	(9%)	0.018	(7%)	0.019	(8%)	0.035	(17%)
Sample mean wage	4.450		4.261		4.553		4.621		4.617	
R^2	0.92		0.93		0.91		0.92		0.81	
#Observations	233,117,492		82,267,794		150,849,698		123,087,610		16,361,068	

Notes: Variance decompositions of log real daily wages according to regression models (1) and (2) for various IEB and LIAB samples. Mean wages, variances, and covariances rounded to three decimal places. Source: IEB, LIAB.

next, should thus also be comparable across samples.

3.3.2. Residual Wages

We use the estimated coefficients on workers' observable characteristics from regression (2) to construct residual wages, which are the main input to the worker ranking procedure. Residual wages are defined as wages net of the effect of workers' observable characteristics. The motivation for using residual wages to rank workers is that the worker ranking should reflect unobserved worker ability differences which are independent of time-varying age and education effects. Thus, we subtract the wage share explained by observable characteristics from the observed individual wage of individual i in year t:

$$\tilde{w}_{it} = w_{it} - x'_{it}\hat{\beta}, \tag{3}$$

where $\tilde{w}_{it}$ is the residual wage (in logs). It has a standard deviation of 0.433 (variance 0.187) and is thus only slightly less dispersed than observed wages in the LIAB sample (standard deviation 0.455). Observed and residual wages are highly correlated (0.98).[51]

A concern related to the documented modest contribution of observable characteristics to wage dispersion might be that we ignore occupational effects. As a robustness check, we add 32 occupational dummies to regression (2), interacted with education and time effects.[52] Controlling for occupation does not change the results of wage regression (2). The adjusted

[51] That is, worker rankings based on observed wages and residual wages are likely very similar. We stick to the residual wage ranking to stay close to HLM on the worker side.

[52] The occupational classification in our data (Bundesagentur für Arbeit, 1988) consists of about 330 occupational codes at the 3-digit level. We use the 32 codes at the 2-digit level ("Berufsabschnitte").

R^2 stays virtually the same (0.81). The share of explained variance increases only slightly from 3.9 percent to 6.7 percent with occupational controls.[53] The correlation between baseline residual wages and residual wages net of occupational effects is very high, above 0.99. Controlling for occupations will thus not change the implied worker ranking.[54]

3.3.3. The Wage Bill Ratio

To construct the wage bill ratio introduced in Section 2, we use estimated AKM effects for both men and women from the full IEB sample (column (a) in Table 1) and predict the AKM-implied wage bill, which is the sum of estimated worker effects, the effects of workers' observable characteristics, and the firm wage premium for all workers employed at firm j in year t. We set this in relation to the sum of all observed wages in the firm, so we construct the wage bill ratio of firm j in year t as follows:

$$\frac{\sum_{i=1}^{L_{jt}} w_{ijt}}{\sum_{i=1}^{L_{jt}} (\hat{\alpha}_i + x'_{it}\hat{\beta} + \hat{\psi}_{j(i,t)})}, \tag{4}$$

where L_{jt} is the size of the firm's workforce (in heads). According to our sorting model, the observed wage bill in the numerator contains contributions of worker-ability, firm-productivity, and option value compensation. The AKM prediction in the denominator, however, does not include an option value component (absent match effects). The ratio of the two wage bills is thus informative about the extent of option value compensation, which explains the majority of the difference between the observed and the AKM-predicted wage bill.[55] In other words, the wage bill ratio mirrors the AKM residual under the assumption that the residual contains omitted match-specific effects.

How do we interpret the wage bill ratio? Suppose it is greater than one for a given establishment. This implies that the firm's average worker is of a high ability *conditional on firm productivity* because the firm pays positive outside option compensation to its average worker. We can use this information to adjust the firm's labor input in the production function to correctly reflect the value of the ability units, conditional on current productivity, it employs. A wage bill ratio smaller than one implies outside option compensation in the other direction, from the worker to the firm. In this case, the firm's average worker is of low ability *conditional on firm productivity*, and the firm receives compensation (in form of lower wages paid) from the average worker. Thus, the average worker has low ability relative to the firm's current productivity.

We compute the wage bill ratio for about 3.3 million establishments in the IEB for which we estimated the AKM effects. Interestingly, the distribution of the wage bill ratio is highly symmetric. The median ratio is 1 with a mean of 1.002, so the distribution is only slightly right-skewed. The standard deviation is 0.04. In the tails of the wage bill ratio's distribution, gaps

[53] Table D.2b shows the decomposition of wage variance including occupational controls.
[54] In line with our finding, CHK report that occupational (and industry) controls do not significantly increase the explanatory power of AKM-type wage regressions.
[55] Recall that match effects explain almost 75 percent of the residual in specification (a), Table 1.

between observed wage bills and the AKM prediction are sizable. For firms below the 10th, 5th, and 1st percentile, observed wage bills are approximately 2, 4, and 10 percent lower than the AKM prediction, respectively. Symmetrically, firms above the 90th, 95th, and 99th percentile have observed wage bills that are approximately 2, 4, and 10 percent higher than the AKM prediction, respectively. The slight positive skew of the distribution is related to the fact that the smallest ratios we observe are just below 0.5 while the highest ones are around 6.

3.4. Final Sample Selection

The two studies of wage dispersion and sorting in Germany that we want to relate our results to are CHK and HLM. Therefore, we follow those studies and restrict the final samples to men in former West Germany who are between 20 and 60 years old. We exclude part-time and marginal employment from the IEB and LIAB samples,[56] although we do use information on part-time employment for the production function estimation. Moreover, the German social security data cover only employees liable to pay social security contributions. This implies that self-employed workers, civil servants, and student workers are excluded from the analysis.

On the establishment side, we need a consistent measure of output (value added) to estimate the production function. Thus, we drop all EP establishments with missing information on revenues and intermediate good purchases.[57] Furthermore, we drop establishments above the 99th percentile of the revenue distribution to ensure that our results are not driven by outliers. For about 10 percent of the establishment years we observe in the EP, we cannot calculate the wage bill ratio due to missing estimated AKM effects.[58] All things considered, we estimate productivity for 13,669 establishments in both East and West Germany. After the production function estimation, we merge estimated productivity for the West German establishments in our LIAB sample.

To compute a worker ranking that captures unobserved ability based on observed wages, we follow a key insight in HLM that applies to a broad class of equilibrium search models with on-the-job search. Suppose the wages we observe in the data contain a firm component and a history component. The firm component, as argued before, can be taken into account implicitly by ranking workers within the same firm first, before aggregating the within-firm rankings to an economy-wide ranking. The history component includes the rent extraction ability of workers who move between different employers.[59] The potential to extract rents at a new employer depends on the workers' outside option, which can either be unemployment or the value of the current job. This value, in turn, may depend on the value of earlier jobs (the

[56] To reliably identify spells of marginal or part-time employment, we use available indicator variables in the data and, additionally, drop spells with wages below the time-varying marginal employment threshold, which is on average 12.2 euros per day over the years in our sample ("Geringfügigkeitsgrenze").
[57] The main reason for missing information is that some firms choose not to report revenues as their output measure. These are mainly financial institutions and public sector firms.
[58] Some establishments are not included in the largest connected set of workers used to estimate the AKM model, and we also have one reference establishment for which we do not estimate a firm effect.
[59] Evidence for history dependence of wages in Germany is presented in Bauer/Lochner (2019).

history). The theoretical literature makes the point that unemployed workers cannot extract rents and are compensated exactly for their value of unemployment when moving into employment (Postel-Vinay/Robin, 2002; Cahuc/Postel-Vinay/Robin, 2006). Thus, unemployed workers receive their reservation wage, which can be shown to be monotonically increasing in worker ability.[60] Thus, the reservation wage provides a clean basis for ranking workers. It contains no history component and is free of the firm component if workers are compared within the same firm.

The final sample selection step is to identify workers for whom we observe at least one spell that reveals their reservation wage. In addition to employment spells after unemployment, we also include first jobs of workers entering the labor market. In the following, we refer to this type of employment spells as OOU (out of unemployment). We exclude recalls, so rematches with the previous employer after an unemployment spell are not used to rank workers. The remaining employment spells, which begin after a worker moves from one employer to another, will be referred to as J2J (job-to-job) spells.

Finally, we produce two main samples which we use in the subsequent analysis. We refer to the first one as *All Matches*: it includes matches starting from 1993, the first year of our data.[61] There are 1,483,595 employment spells of 225,548 workers employed at 5,143 EP establishments. 83 percent (1,231,276) of employment spells are J2J spells. The remaining 17 percent (252,319) are OOU spells. The standard deviation of log wages in this sample is 0.370 (variance 0.137). We refer to the second sample as *New Matches* that are formed between 1998–2008. This sample includes 544,907 employment spells of 112,665 workers at 4,885 establishments. 71 percent (388,895) of the employment spells are J2J spells, while 29 percent (156,012) are OOU spells. The standard deviation of log wages is 0.421 (variance 0.177).

4. Ranking Workers and Firms

4.1. The Worker Ranking

HLM show that comparing wages of coworkers at the same firm who were hired out of unemployment effectively controls for both the firm and the history component of wages. The observed (residual) wages of these workers directly reflect differences in unobserved worker ability and constitute within-firm worker rankings in terms of unobserved ability. Observing rankable worker pairs at multiple firms over time, together with their other coworkers, makes it possible to piece together a global ranking of workers based on unobserved ability. HLM propose an algorithm that implements this aggregation. It merges the within-firm rankings into a global ranking of workers by solving a Kemeny-Young rank aggregation problem.[62]

[60] See result 5 on p. 57 in HLM. One needs to make sure that the wage statistic used to rank workers actually increases monotonically in worker ability to be a valid basis for ranking workers. This property is not fulfilled for e.g. a worker's average wage, which can be lower due to unemployment spells.

[61] Recall that we use only 1998–2008 to rank workers.

[62] Aggregating potentially inconsistent within-firm rankings of workers across firms has an analogy in social choice theory: the aggregation of inconsistent preference rankings across voters. The algorithm minimizes the number of disagreements between within-firm worker rankings across firms. These within-firm rankings are linked because workers move between firms over time.

Table 2: Worker Ranking Correlations

	$\bar{w}_i$	$\hat{\alpha}_i$	age	education
Correlation with $\hat{x}(i)$	0.75	0.87	0.19	0.48

Notes: the table shows correlations of our estimated individual worker ranks with individual mean wages ($\bar{w}_i$), estimated worker-fixed effects (extracted from running wage regression (2), $\hat{\alpha}_i$), and the individual-level means of age and education in our sample. All correlations are rounded to two decimals places. Source: LIAB.

Table 3: Variance Decompositions with Worker Bins

	w_{it}	$\tilde{w}_{it}$	age	education
Overall Variance	0.137	0.125	92.296	1.668
Between bins	0.105 (77%)	0.094 (75%)	4.544 (5%)	0.589 (35%)
Within bins	0.033 (24%)	0.033 (26%)	87.846 (95%)	1.090 (65%)

Notes: The table shows decompositions of the variance of log wages (w_{it}), residual wages ($\tilde{w}_{it}$), age, and education in our final sample into the respective shares explained within and between the worker bins. The age of individual workers in our sample ranges from 20 to 60. There are 6 education categories: 1 = "no degree", 2 = "vocational training", 3 = "high school", 4 = "high school and vocational training", 5 = "technical college", 6 = "university". All variances are rounded to three decimals places. Source: LIAB.

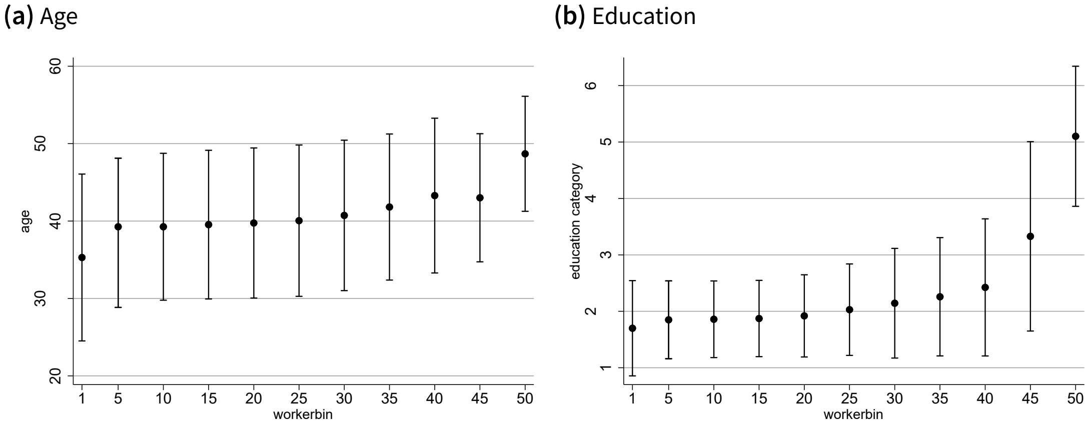
More information and technical details about our implementation of the HLM worker ranking algorithm can be found in Appendix C.

The algorithm generates a global ranking of workers, that is, an estimate of the unobserved rank, $\hat{x}(i)$, for every individual worker i in our data set. To understand the properties of our estimated worker ranks, we correlate them with a number of alternative worker characteristics: individual mean wages, worker-fixed effects from regression (2), and observed age and education. Table 2 reports these correlations. Unsurprisingly, they are relatively high for individual mean wages ($\bar{w}_i$) and estimated person-fixed effects ($\hat{\alpha}_i$). However, the association is far from perfect, suggesting that the worker ranking algorithm makes substantial changes relative to other wage-based worker statistics. The correlation with estimated worker-fixed effects (0.87), which also net out workers' observable characteristics, is higher than the correlation with individual mean wages (0.75). Regarding age and education, correlations are also positive but considerably lower. Highly ranked workers are often highly educated, but there must be many deviations from this pattern to explain a correlation of only 0.48. The positive association with age is even weaker (0.19), suggesting that many young workers are ranked high, and many old workers are ranked low.

We now group all individual workers into 50 bins of equal size.[63] Let $\bar{x}(i)$ denote the bin that worker i belongs to. In the following, individual workers in the same bin are thought of as workers of the same type. To understand how the binning of workers modifies the ranking, Table 3 shows a decomposition of the respective variances of workers' observed wages, residual wages, age, and education into shares explained within and between the bins. A relatively

[63] There are about 4510 workers in every bin.

Figure 2: Age and education distributions across worker bins



Notes: Means $\pm$ one standard deviations of age and education for a selection of worker bins. The age of individual workers in our sample ranges from 20 to 60. There are 6 education categories: 1 = "no degree", 2 = "vocational training", 3 = "high school", 4 = "high school and vocational training", 5 = "technical college", 6 = "university". Source: LIAB.

homogeneous distribution of a variable within bins (low share of explained variance) indicates that workers within bins are relatively similar in the respective dimension of worker heterogeneity. The worker ranking is based on wages, implying that the share of wage variance explained within bins is relatively low: roughly one quarter for both log wages (w_{it}) and log residual wages ($\tilde{w}_{it}$).

The variation of wages between bins is thus relatively large (about three quarters). Conversely, most of the variance of age and education is within bins: 95 and 65 percent, respectively. Figure 2 illustrates how age and education vary across worker bins. For age, Panel (a) shows that the mean age across worker bins is almost flat between bins 5 and 45. Only the lowest (highest) bin has a slightly lower (higher) mean age. Standard deviations are high, however, so these differences are not statistically significant. In all bins, we find workers of almost all ages (20-60). For education, Panel (b) shows that the education gradient across bins becomes relatively steep above bin 40, but it is essentially flat below. Highly ranked workers are more likely to have tertiary degrees, while the modal worker of ranks up to bin 40 has vocational training only. Note that the dispersion of education is higher at the top of the worker ranking, so it's more common to observe high-rank workers with little education as compared to low-rank workers with tertiary degrees.

4.2. The Firm Ranking

4.2.1. The Effect of Workforce Ability on Output

We rank firms based on unobserved productivity which we infer from estimating production functions at the establishment level. This approach poses two key challenges. The first challenge is that, due to heterogeneous worker ability, the quality of labor inputs varies across firms.[64] Moreover, if firm productivity and worker ability are complements, as illustrated in Section 2, even precise measures of the within-firm distribution of worker ability are insufficient to control for the effect of the workforce's ability composition on output. The reason is that the effect of workforce ability on output, in the presence of complementarities, covaries with firm productivity. This makes it hard to separately identify the effects of firm productivity and workforce ability on output. We propose a new method to estimate firm productivity net of workforce ability that allows us to overcome the first challenge.

The second challenge for estimating production functions is a long-known endogeneity problem, the so-called "transmission bias". The industrial organization (IO) literature has emphasized at least since Marschak/Andrews (1944) that input choices, for example the demand for labor and intermediate inputs, are likely correlated with the firm's productivity.[65] To overcome the second challenge and estimate firm productivity accurately, we rely on methods from the contemporary empirical IO literature, specifically the Ackerberg/Caves/Frazer (2015) (henceforth ACF) version of the "control function" approach.[66] It is assumed that intermediate input demand is a strictly increasing function of a (scalar) unobserved productivity shock. Under this assumption, strict monotonicity allows the researcher to invert the "control function" and effectively control for unobserved firm productivity by substituting it out of the production function. ACF refine earlier approaches by allowing the intermediate input demand to depend on labor inputs. This suits our focus on worker heterogeneity well.

To overcome the first challenge and separate the effects of worker ability and firm productivity on output, we construct a novel measure of the firm level labor input that is consistent with the sorting model. This measure takes into account that the contribution of heterogeneous worker ability to output depends on the firm's productivity. It builds directly on the logic of the sorting model's wage setting mechanism that highlights the importance of option value compensation in hiring decisions. Option value compensation is informative about the relative contributions of firm productivity and worker ability to output. If, in any given match, productivity is high relative to ability, outside option compensation flows from the worker to the firm, and this lowers the wage. Conversely, if productivity is low relative to ability, outside option compensation flows from the firm to the worker, and this increases the wage. Empirically, we measure the extent of outside option compensation that a firm pays (receives) to (from) its average worker using the wage bill ratio introduced in Section 2 and computed in Section 3.3.3. It relates the observed firm-level wage bill to the wage bill predicted by AKM, a model that abstracts from outside option compensation. Differences between the observed and the AKM-predicted wage bills, thus, reflect the extent of outside option compensation at the firm level. Formally, we use the wage bill ratio to compute the adjusted labor input

64 Griliches (1957) was among the first to argue that labor inputs, which are typically measured in physical units (the number of workers or hours), are not homogeneous within and across firms if workers are heterogeneous.

65 A profit-maximizing firm optimally chooses its input demands in every period conditional on the realization of firm-level productivity. An endogeneity problem arises because the firm (or its manager) observes productivity when choosing those demands, but the econometrician does not when estimating the production function.

66 This approach was originally developed by Olley/Pakes (1996) (OP), and refined by Levinsohn/Petrin (2003) (LP) and Wooldridge (2009).

measure $L^\star_{jt}$:

$$L^\star_{jt} = L_{jt} \times \underbrace{\frac{\sum_{i=1}^{L_{jt}} w_{ijt}}{\sum_{i=1}^{L_{jt}} \left(\hat{\alpha}_i + x'_{it}\hat{\gamma} + \hat{\psi}_j\right)}}_{\text{wage bill ratio}}, \tag{5}$$

where L_{jt} is the labor input in heads (both part-time and full-time workers) of firm j in year t. The numerator of the wage bill ratio is the sum of all the observed wages of workers i who work at firm j in year t. The denominator contains the predicted wage bill according to the log-linear AKM-CHK model, again for all workers i who work at firm j in year t. It consists of the estimated worker effects, $\hat{\alpha}_i$, observable effects, $x'_{it}\hat{\gamma}$, and the firm wage premium, $\hat{\psi}_j$. Note that the denominator also varies with time because the firm's workforce composition changes from year to year.

If the observed wage bill is higher than the AKM benchmark, there is evidence for option value compensation from the firm to the worker. Thus, the firm's average worker is of high ability conditional on firm productivity. In this case, the workers' contribution to output is high relative to the effect of firm productivity. We take this into account by adjusting the labor input measure upwards. Conversely, if the observed wage bill is lower than the AKM benchmark, there is evidence for option value compensation for the worker to the firm. Thus, the firm's average worker is of low ability conditional on firm productivity. In this case, the worker's contribution to output is low relative to the effect of firm productivity. We take this into account by adjusting the labor input measure downwards. The adjusted labor input $L^\star_{jt}$ allows us to implicitly control for the firm-productivity-specific effect of workforce ability on output when estimating the productivity of firms. It serves as out labor input measure in the production function estimation to which we turn next.

4.2.2. Production Function Estimation

To estimate firm productivity, we work with the following Cobb-Douglas specification of a value-added production function in logs:[67]

$$v_{jt} = \beta_0 + \beta_l l^\star_{jt} + \beta_k k_{jt} + \omega_{jt} + z'_{jt}\gamma + \epsilon_{jt}. \tag{6}$$

v_{jt} is log value added (calculated as revenue minus expenditures for intermediate goods) of firm j in year t, β_0 is a constant, $l^\star_{jt}$ is the log of the adjusted labor input, k_{jt} is log capital input, ω_{jt} is (unobserved) productivity (or firm-level TFP), and ϵ_{jt} is a transitory shock. z'_{jt} includes additional control variables: dummies for West German establishments, three firm

[67] The ACF estimation procedure we use is designed for value-added production functions. Gandhi/Navarro/Rivers (2019) show that the ACF method, which builds on Olley/Pakes (1996) and Levinsohn/Petrin (2003), is not suitable to identify the parameters of the gross output production function without imposing further restrictions. Estimating a value-added production function implies that intermediate inputs, denoted m_{jt} below, do not enter the equation to be estimated. A common interpretation of this setting is that the gross output production function is Leontief in value added and intermediate inputs. Gandhi/Navarro/Rivers (2017) provide an in-depth analysis of the non-trivial differences between gross output and value added production functions.

age categories, and the share of part-time workers. We also include time and sector-fixed effects.[68]

To overcome the second challenge mentioned above (the endogeneity of input demands with respect to firm productivity), we follow the ACF identification strategy. It assumes a discrete time model of dynamically optimizing firms. The demand for labor and intermediate goods may change in response to realized firm productivity in the same period. In line with our sorting model and the presence of search frictions, the labor choice is allowed to have dynamic implications, for example, by affecting both current and future profits of the firm. Capital is accumulated according to $k_{jt} = \kappa(k_{j,t-1}, i_{j,t-1})$, so investment in the previous period, $i_{j,t-1}$, predetermines the capital stock. It does therefore not change with realized productivity in period t.[69] The firm's information set when making dynamic input choices includes all past and present productivity shocks $\{\omega_{j\tau}\}_{\tau=0}^{t}$, but it does not include future productivity shocks. These are assumed to evolve according to a first-order Markov process:

$$\omega_{jt} = E(\omega_{jt}|\omega_{j,t-1}) + \xi_{jt} = \rho\omega_{j,t-1} + \xi_{jt}. \tag{7}$$

Thus, firm productivity in period t is a function of the conditional expectation for ω_{jt} based on last period's realization (the Markov property) and an innovation ξ_{jt}, which is assumed to be uncorrelated with ω_{jt} and the capital stock (the firm's predetermined state variable). In the following, we assume that ω_{jt} follows and AR(1) process with parameter ρ. The control function, which is the demand for intermediate inputs, is assumed to be strictly increasing in the scalar ω_{jt}:

$$m_{jt} = f_t(l^{\star}_{jt}, k_{jt}, \omega_{jt}). \tag{8}$$

The firm's demand for intermediate inputs is thus a function of both the firm's adjusted labor input and the capital stock in addition to productivity.[70] Thus, conditional on both the adjusted labor input and capital, more productive firms use more intermediate goods in production. Due to the strict monotonicity assumption, we can invert equation (8) and write unobserved firm productivity, ω_{jt}, as a function of observables:

$$\omega_{jt} = f_t^{-1}(l^{\star}_{jt}, k_{jt}, m_{jt}), \tag{9}$$

which we then use to substitute ω_{jt} in (6), so

$$v_{jt} = \beta_0 + \beta_l l^{\star}_{jt} + \beta_k k_{jt} + f_t^{-1}(l^{\star}_{jt}, k_{jt}, m_{jt}) + z'_{jt}\gamma + \epsilon_{jt} = \Phi_t(l^{\star}_{jt}, k_{jt}, m_{jt}, z_{jt}) + \epsilon_{jt}, \tag{10}$$

[68] We use 32 sectors from the WZ93/WZ03 classification of industries available in the IAB Establishment Panel. The WZ classification of the German Federal Statistical Office is compatible to the common international classifications of industries, NACE and ISIC.

[69] This accumulation mechanism is in line with the perpetual inventory method we use to approximate the capital stock in the EP data, see Section 3.2.

[70] ACF suggest to use this conditional (on labor) input demand function because this bypasses a problem of functional dependence that hinders identification of the labor input parameter in Olley/Pakes (1996) and Levinsohn/Petrin (2003).

is the final production function to be estimated. Following ACF, the estimation includes two stages. First, value added is regressed on a polynomial approximation of $\Phi_t(l^\star_{jt}, k_{jt}, m_{jt}, z_{jt})$. This does not identify any of the parameters but leads to an estimate $\widehat{\Phi}_t(l^\star_{jt}, k_{jt}, m_{jt}, z_{jt})$. In the second stage, estimated parameter values are calculated using a set of four moment conditions with GMM.[71]

Table 4 presents the results of the production function estimation. We show five different specifications in which we vary the way of controlling for workforce ability. Column (a) presents our benchmark specification in which we use the wage bill ratio to adjust the firm's labor input. Specification (b) also adjusts the labor input but uses worker ability only, measured by estimated AKM worker effects. $\widetilde{L}_{jt}$ is defined as the ratio of the sum of all the estimated worker-fixed and observable effects, $\sum_{i=1}^{L_{jt}}(\hat{\alpha}_i + x'_{it}\hat{\gamma})$ in a firm in a given year to the sample mean of $(\hat{\alpha}_i + x'_{it}\hat{\gamma})$ in that year. Again, a high amount of workforce ability has a positive effect on output, and this is reflected in $\widetilde{L}_{jt}$. As compared to the benchmark specification, however, (b) does not take into account that the effect of workforce ability on output is firm-specific. In columns (c), (d), and (e), we no longer adjust the labor input and simply use the number of workers, L_{jt}, as the labor input. Column (c) uses the mean wage of a firm's workforce to control for workforce ability. Column (d) uses the mean of estimated AKM worker effects within the firm. Column (e) uses no workforce ability controls.

The estimated output elasticity of labor is always higher than the estimated output elasticity of capital. Interestingly, the estimated parameters of the benchmark specification are most similar to column (e) with no workforce ability controls. This reflects that our labor input adjustment, the wage bill ratio, has a highly symmetrical distribution (see Section 3.3.3): for some firms the labor input is adjusted upwards, for others it is adjusted downwards. Specifications (b)–(d) control for workforce ability in ways that do not take into account that worker ability interacts with firm productivity. The estimated coefficients of the mean wage and mean AKM effect control variables are sizable and significant. Accordingly, these regressions yield higher, arguably overestimated output elasticities of labor, underestimated output elasticities of capital, and less dispersion in the estimated firm productivity, $\hat{\omega}_{jt}$.

The estimated coefficients on the additional control variables show that being a West German establishment is always positively correlated with value added. Firm age has a U-shaped effect. In column (a), as compared to firms that are less than six years old, firms between 6-15 years of age have 3.5 percent higher value added, firms between 16-25 years have 1.7 percent higher value added, and firms with more than 25 years of age have 5.8 percent higher value added. This pattern is broadly similar across specifications. Finally, a high share of part-time workers is negatively associated with value added, which seems reasonable. We always formally reject constant returns to scale of the production function due to very small (bootstrapped) standard errors. The sum of the estimated output elasticities, however, is always slightly above, most pronounced in column (b).

[71] To implement the ACF estimation procedure technically, we follow Rovigatti/Mollisi (2018).

Table 4: Production Function Estimation Results

Dependent Variable	*Value Added*				
	(a)	(b)	(c)	(d)	(e)
Labor input	0.820***	0.895***	0.863***	0.820***	0.819***
	(0.001)	(0.001)	(0.001)	(0.002)	(0.001)
Capital input	0.209***	0.190***	0.139***	0.192***	0.211***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
West German establishment	0.320***	0.303***	0.130***	0.260***	0.330***
	(0.001)	(0.001)	(0.000)	(0.001)	(0.001)
Firm age: 6–15 years	0.035***	0.046***	0.033***	0.024***	0.031***
	(0.001)	(0.001)	(0.000)	(0.001)	(0.001)
Firm age: 16–25 years	0.017***	0.027***	0.015***	-0.002	0.022***
	(0.002)	(0.001)	(0.001)	(0.002)	(0.002)
Firm age: >25 years	0.058***	0.053***	0.019***	0.028***	0.033***
	(0.001)	(0.001)	(0.000)	(0.001)	(0.002)
Part-time worker share	-0.843***	-0.817***	-0.275***	-0.850***	-0.861***
	(0.001)	(0.001)	(0.000)	(0.001)	(0.000)
Labor input variable	$L^{\star}_{jt}$	$\widetilde{L}_{jt}$	L_{jt}	L_{jt}	L_{jt}
Workforce quality control	Adjusted labor input		$\sum_{i=1}^{L_{jt}} w_{ijt}/L_{jt}$	$\sum_{i=1}^{L_{jt}} \hat{\alpha}_i/L_{jt}$	None
	–	–	0.706***	0.811***	–
	–	–	(0.001)	(0.001)	–
Year FEs	Yes	Yes	Yes	Yes	Yes
Sector FEs	Yes	Yes	Yes	Yes	Yes
Variance of $\hat{\omega}_{jt}$	0.652	0.635	0.527	0.614	0.642
Variance of mean of $\hat{\omega}_{jt}$	0.462	0.449	0.358	0.433	0.460
#Observations	40,115	40,115	37,452	37,131	38,817

Notes: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Bootstrapped standard errors (50 iterations) in parentheses. All estimated coefficients and standard errors are rounded to three decimals places. The reference category for the firm age dummies is a firm age of five years or less.
Source: BHP, EP, IEB.

Table 5: Firm Ranking Correlations

	$\bar{v}_j$	$\bar{v}_j/\bar{n}_j$	$\bar{\pi}_j/\bar{n}_j$	$\bar{n}_j$	$\bar{k}_j$	$\bar{k}_j/\bar{n}_j$	workforce education
Correlation with $\hat{y}(j)$	0.32	0.63	0.49	0.07	-0.05	-0.15	0.04

Notes: The table shows correlations of the time-invariant estimated firm ranks, $\hat{y}(j)$, with the means of the following firm statistics over time: log value added ($\bar{v}_j$), log value added per worker ($\bar{v}_j/\bar{n}_j$), profits per worker ($\bar{\pi}_j/\bar{n}_j$), the log size of the workforce ($\bar{n}_j$), the log capital stock ($\bar{k}_j$), the log capital stock per worker ($\bar{k}_j/\bar{n}_j$), and workforce education, as measured by the share of workers with tertiary education in the firm. Source: BHP, EP, IEB.

4.2.3. Firm Ranking Properties

After estimating firm productivity, we rank firms based on the mean of their estimated series of productivity realizations, $\bar{\hat{\omega}}_{jt}$. The main reason for taking the mean is that the majority of papers in the literature on labor market sorting, in relation to which we want to interpret our results, assumes permanency of worker and firm types. The worker types which we estimated using the HLM procedure are time-invariant, and so are the estimated AKM-CHK worker and firm effects that we use in parts of the analysis. Moreover, Guiso/Pistaferri/Schivardi (2005) find that firms insure workers fully against transitory productivity fluctuations but only partially against enduring productivity changes, which can have lasting effects on wages and employment. This supports our focus on permanent productivity heterogeneity to study the allocation of workers to firms and the wage distribution.[72]

We denote the estimated firm rank $\hat{y}(j)$. To ease exposition, we group all individual firms into 15 bins of equal size.[73] Let $\bar{y}(j)$ denote the bin that firm j belongs to. In the following, individual firms in the same bin are thought of as firms of the same type.

Table 5 shows correlations of our estimated firm ranks with several firm-level statistics. Since our ranking is time-invariant, we also correlate firm ranks with the within-firm means of the respective variables. Interestingly, our ranks have only a small positive correlation with firm size. The correlation with the number of workers is positive but very small (0.07), and correlations with capital (-0.05) and capital per worker (-0.15) are even slightly negative. Thus, we find that firm size is not associated with a high productivity rank of the firm. Large firms are not necessarily the most productive ones.

The correlation of the firm rank with the share of workers who hold a university degree (workforce education in Table 5) is only 0.04. This mirrors the earlier finding that worker observable characteristics can explain only a small share of wage dispersion, which is also true for firm productivity dispersion. Value added, both absolute and per worker, and profits per worker are positively correlated with estimated firm ranks.

As with the binned worker ranking, we decompose the variance of some key variables in our data into the shares explained within and between our firm bins to show in which dimension

[72] To estimate how much productivity variation we discard by using the mean, the autocorrelation of $\hat{\omega}_{jt}$, that is, the estimated ρ in equation (7), is informative. It turns out to be relatively high at 0.71, so within-firm fluctuations of $\hat{\omega}_{jt}$ over time are relatively small.

[73] There are about 343 firms in every bin. We find that 15 firm bins is a good compromise between number of firms per bin and a fine enough type space to study differences across different firm types.

Table 6: Variance Decompositions with Firm Bins

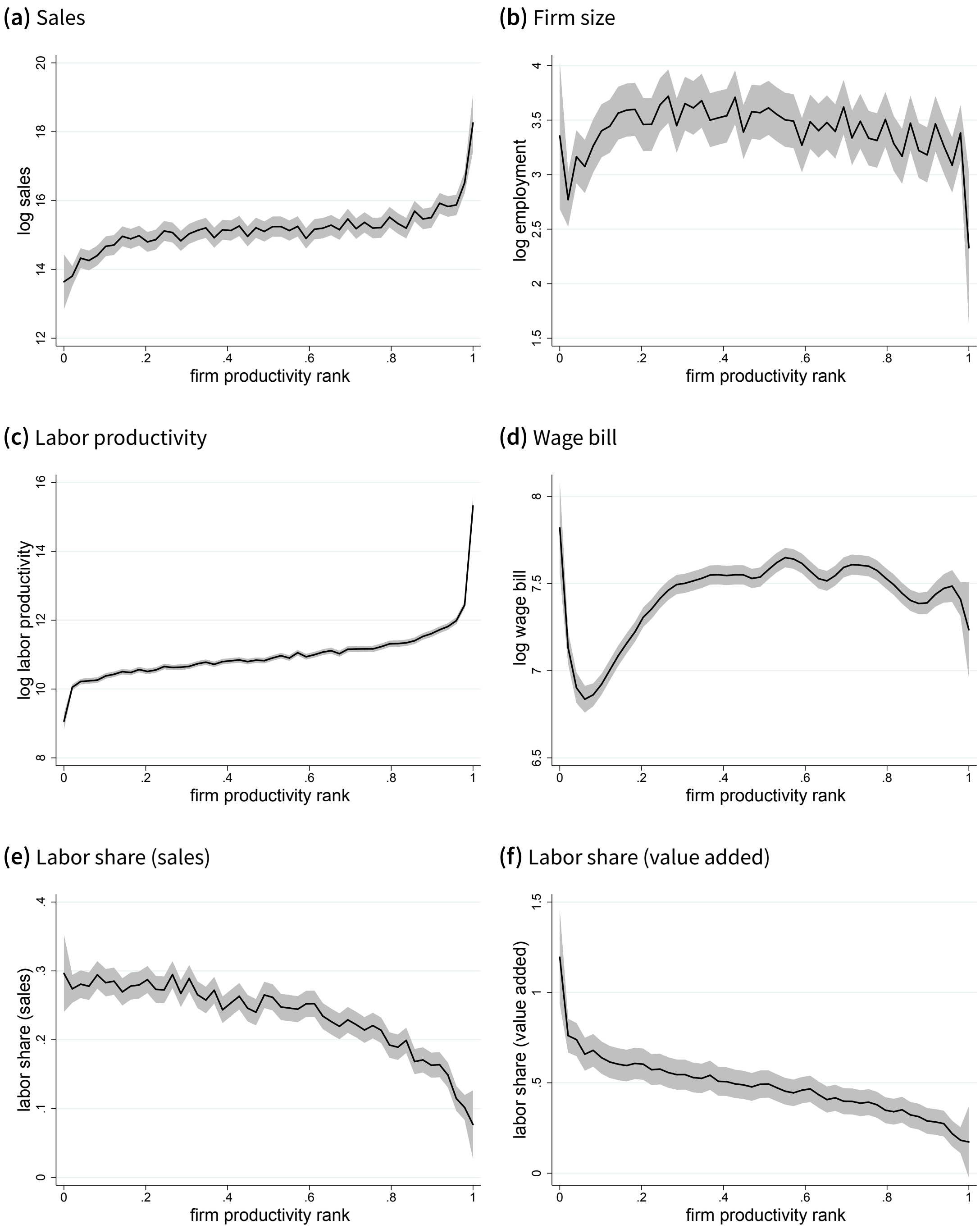
	w_{it}	$\bar{v}_j$	$\bar{v}_j/\bar{n}_j$	$\bar{\pi}_j/\bar{n}_j$	Sector
Overall Variance	0.137	1.668	0.474	1.147	51.393
Between bins	0.003 (2%)	0.589 (35%)	0.324 (68%)	0.722 (63%)	1.105 (2%)
Within bins	0.133 (98%)	1.090 (65%)	0.172 (36%)	0.479 (42%)	50.364 (98%)

Notes: The table shows decompositions of the variance of log wages (w_{it}), log value added ($\bar{v}_j$), log value added per worker ($\bar{v}_j/\bar{n}_j$), the firm's sectoral classification, and log profits per worker into the respective shares explained within and between the firm bins. For the sectoral classification, we use the WZ93/WZ03 classification of industries available in the IAB Establishment Panel, which is compatible to the common international classifications of industries, NACE and ISIC. We use 32 industries, roughly classified as follows: 1-2 = "Agriculture & Mining", 3-18 = "Manufacturing", 19-20 = "Construction", 21-23 = "Retail Trade", 24-32 = "Service Sector". Source: BHP, EP, IEB.

the bins are internally homogeneous and in which they are not. Table 6 shows this decomposition. Importantly, our firm bins are not homogeneous in terms of wages. 98 percent of the observed variance of log wages is explained within the firm bins. This reflects that our firm ranking is constructed independently of the wages that firms pay. High-paying firms can have a low rank if indicated by low estimated productivity. Two thirds of the variance of log value added are explained within the bins. Conversely, the bins are more homogeneous internally in terms of log profits per worker and log value added per worker, where the majority of variance is between the bins. This is in line with the moderate positive correlations shown in Table 5. The sector a firm operates in has a very high share of within variance and, thus, is also not a predictor of the firm rank. In every bin we can find firms from almost all sectors. We also checked whether there is a clear relation of the firm ranking with the prevalence of collective bargaining and employee representation. This is not the case: the dispersion of those attributes within firm bins is huge.

Figure 3 shows six firm performance measures across firm ranks. Log sales (Panel (a)) are increasing in firm rank, albeit not linearly. The slope is mostly flat between the 20th and 90th firm rank and steeply increases above. Thus, the most productive firms have the highest sales. Such a clear relation does not exist with firm size. In Panel (b), the relation between firm ranks and log employment (measured in heads) is very noisy. If anything, the least and most-productive firms appear to be somewhat smaller than the broad group of medium-productivity firms. The least-productive firms, however, are bigger than slightly more productive firms. In Panel (c), log labor productivity (value added per worker) mirrors the sales pattern and is increasing in the firm rank. The least productive firms have very low labor productivity. The slope is then relatively flat but shoots up again for the very productive firms. The log wage bill across firm ranks, in Panel (d), exhibits an interesting pattern: it mirrors (b) to some extent but is much less noisy. Very unproductive firms have high wage bills. Our model suggests that outside option compensation must play a role here. The graph reaches its minimum just below the 10th firm rank and increases up until the 50th rank. For the upper half of firms, the graph continues in a non-monotonic, wave-like pattern and decreases for the highest ranks. Thus, importantly, highly productive firms do not have the largest wage bills. Finally, Panels (e) and (f) show labor shares, computed with sales and value added, re-

Figure 3: Firm Performance Measures by Estimated Firm Rank



Notes: Estimated univariate kernel densities of selected firm performance measures across estimated firm ranks, normalized between zero and one. The kernel is estimated using an Epanechnikov kernel function. The bandwidth is 0.01 for (a)–(d) and 0.02 for (e)–(f). The qualitative findings are robust to the bandwidth choice. 95 percent confidence bands in gray. Source: Source: BHP, EP, IEB.

spectively. They are clearly falling in estimated firm ranks. Remarkably, we find that some of the least productive firms pay out more than 100 percent of their value added to the workers, in line with their high wage bills.[74]

Our findings seem related to a stylized fact that has generated a lot of attention in the literature in recent years: the fall of the aggregate labor shares in many developed economies. Autor et al. (forthcoming) emphasize the role that so-called "superstar" firms play for this development. The idea is that, due to globalization or technological progress, highly productive firms, the superstars, become increasingly dominant.[75] Market concentration rises as a result, and because superstar firms have high markups and low firm-level labor shares, the aggregate labor share falls.

Figure 3 suggests that our firm ranking places those superstar firms at the very top of the ranking. Those firms have high sales and high labor productivity, but they are not big in terms of employment and have wage bills that are lower than those of less productive firms. In our sorting model, those smaller wage bills could be explained by the outside option compensation that highly productive firms receive from the majority of their workers. Accordingly, only small shares of sales and value added are paid out to workers as wages and labor shares are low.

Because Autor et al. (forthcoming) highlight increasing market concentration as one driving force of the declining aggregate labor share in the U.S., we check whether the firms at the top of our ranking indeed operate in more concentrated markets. To this end, we look at all the sectors that firms in a given bin operate in.[76] We calculate Herfindahl-Hirschman Indices (HHI) of market concentration for all sectors and, additionally, the sales and employment shares of our ranked firms. Doing this for all firm bins yields Figure 4. Panel (a) shows that the concentration measure is U-shaped across firm bins for both employment and sales concentration. This implies that, first, high-productivity firms do indeed operate in more concentrated markets as compared to medium-productivity firms. But, second, this also tends to be true for low-productivity firms.

One way of explaining this pattern is that high and low-productivity firms actually compete with each other in the same sectors, whereas medium-productivity firms are somewhat isolated in less concentrated markets. Panel (b) provides some evidence for this. The actual sales and employment shares are indeed high only for high-productivity firms, so they are the dominant firms in their respective sectors. The low-productivity firms may have to compete with a superstar in their sector and thus have lower employment and sales shares.[77]

[74] Unsurprisingly, though, we observe that these firms do not survive long in our data.

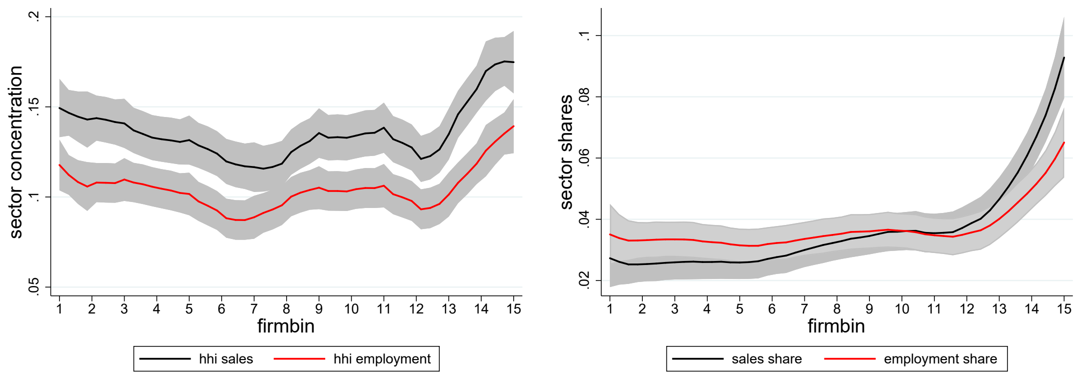
[75] Elsby/Hobijn/Şahin (2013), Karabarbounis/Neiman (2014), Barkai (2016), De Loecker/Eeckhout/Unger (forthcoming), Dao et al. (2017), Hall (2018), and Kehrig/Vincent (2018) also study falling labor shares and different explanations for it.

[76] For this exercise, we use the industry classification available in our EP data, WZ93/WZ03, which is compatible with the common international classifications of industries, NACE and ISIC. We work at the three-digit level which corresponds to 183 distinct sectors in our sample.

[77] The literature on declining aggregate labor shares focuses on the *trend* of a falling labor share over time. While we find that firms with high estimated productivity exhibit the properties of superstar firms discussed in the literature, we do not analyze in this paper to what extent this contributes to the aggregate labor share trend in Germany, which has been falling from the mid-1990s until 2007 and recovered somewhat thereafter (according to EUKLEMS data). We view this as an interesting area for further research.

Figure 4: Market Concentration and Sector Shares by Firm Bins

(a) Herfindahl-Hirschman indices (HHI) (b) Sector shares



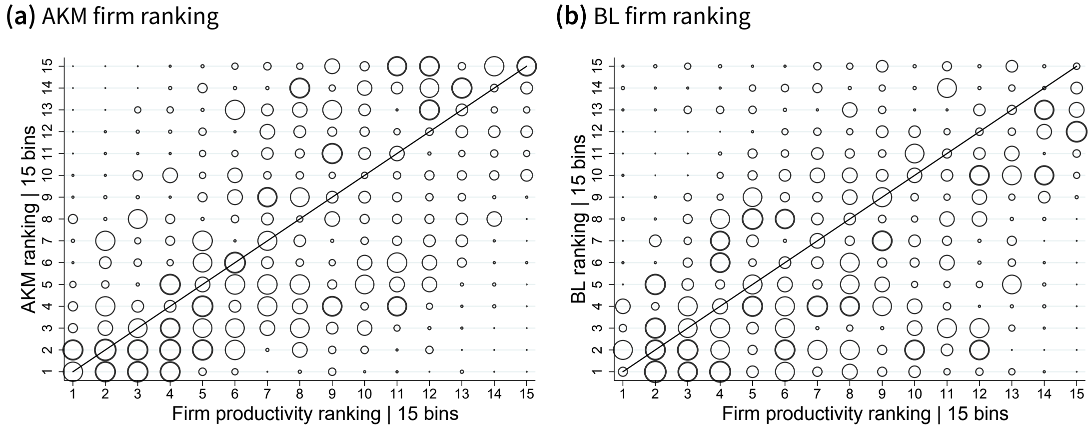
Notes: The plots show estimated univariate kernel densities of Herfindahl-Hirschman indices (HHI) and sectoral employment and sales shares across firm bins. HHIs and sector shares are computed at the three-digit level for 183 sectors of the WZ93/WZ03 industry classification, which is compatible to the common international classifications of industries, NACE and ISIC. The kernel is estimated using an Epanechnikov kernel function. The bandwidth is 1. 95 percent confidence bands in gray. Source: BHP, EP, IEB.

The final test we perform with our estimated firm ranking compares it to other firm ranking techniques used in the literature on wage dispersion and labor market sorting. We create two alternative firm rankings. One based on AKM-CHK firm-fixed effects, the other based on the concept of the "poaching rank" introduced by Bagger/Lentz (2019) (BL). Inspired by on-the-job-search models, the poaching rank is based on the idea that high-paying firms, which are also highly productive in this class of models, poach workers from other firms rather than hiring unemployed workers. Thus, the more workers a given firm hires out of unemployment, the lower is its inferred poaching rank.[78]

Overall, the correlations of our productivity ranking with the firms' AKM-CHK and BL ranks are positive: 0.171 and 0.275, respectively. To analyze the rankings' relation more deeply, we create 15 firm bins based on AKM-CHK firm effect ranks and the BL poaching ranks and compare them to our 15 productivity-based firm bins. This allows us to observe how firms with a given productivity rank are distributed across the AKM-CHK/BL bins. In Figure 5, our productivity bins on the horizontal axis are plotted against AKM-CHK bins (a) and BL bins (b). We find that the AKM-CHK firm effect has a tendency to rank firms below their productivity rank, roughly, in the lower half of the productivity distribution. Here, observations are concentrated below the 45 degree line. Conversely, AKM-CHK effects rank firms above their productivity rank in the upper half of the distribution, where we see more observations above the 45 degree line. In the Appendix, Figure E.1 shows that this pattern is driven by young and small firms that tend to have a higher productivity rank as compared to their AKM-CHK rank. We also show that, as

[78] The poaching index is computed by comparing the number of workers poached from other firms to all hires at the firm level. We compute the poaching rank in the IEB data on a yearly basis. We then rank firms using the firm-level mean of the time-varying poaching index.

Figure 5: Comparison with Alternative Firm Rankings



Notes: The size of the circles is proportional to the number of matches in which the firm has the respective combination of the AKM-CHK firm effect (15 bins) in Panel (a) and the BL poaching index (15 bins) in Panel (b). Source: BHP, EP, IEB.

one would expect, low-wage (high-wage) firms are always ranked low (high) with AKM, even though both groups of firms largely overlap in terms of productivity, see Panels (a) and (b) of Figure E.1.

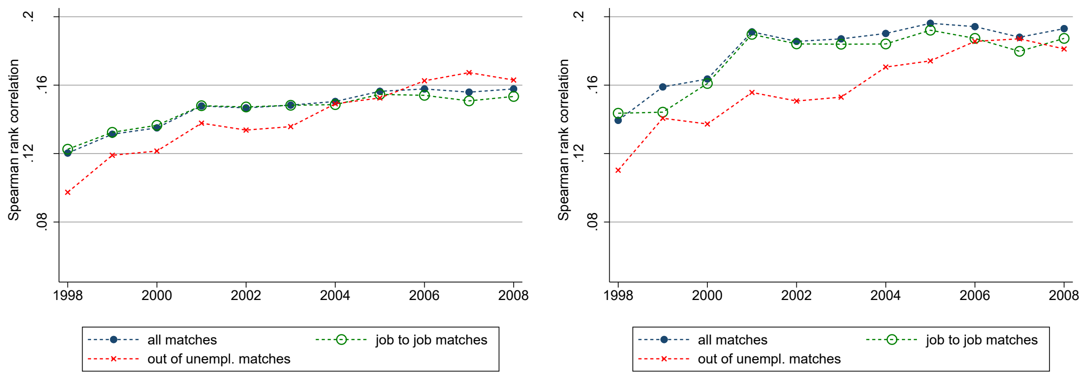
The correlation between productivity ranks and BL poaching ranks is higher, and deviations are somewhat less systematic, see Panel (b) of Figure 5. The poaching rank tends to be a bit lower than the estimated productivity rank. In other words, the most productive firms are not the ones that hire most of their workers from other firms. Again, we analyze the relation of these two rankings in more detail in Figure E.2. Old firms poach more relative to their productivity than young firms. Small firms have either a very low or a very high poaching rank but are almost uniformly distributed across productivity bins. Large firms are concentrated in the upper middle of the poaching rank distribution but are virtually never at the top, so large firms are hiring many workers out of unemployment. Finally, looking at the difference between poaching and productivity ranks through the lens of wages, we observe that high-wage firms are not at the top of the BL ranking. Again, there is a concentration in the upper middle (so these are likely also large firms). The firms that poach the most pay low wages on average and come from all parts of the estimated productivity distribution. All in all, our findings suggest that firm ranks based on wages and observed worker mobility imply rankings that are systematically different from a ranking that is based on the firms' estimated productivity.

5. Labor Market Sorting in Germany

We find evidence for PAM in the German labor market. As compared to earlier studies of labor market sorting using German data, we estimate a lower degree of sorting. The reason for this difference is that earlier studies, particularly CHK and HLM for Germany, rely primarily on wage and worker mobility data to rank both workers and firms. Our focus is on measuring the

Figure 6: Spearman Rank Correlations Coefficients over Time (1998-2008)

(a) All matches **(b)** New matches



Source: BHP, EP, IEB, LIAB.

correlation between wage-based worker types and productivity-based firm types, because productivity is the underlying firm characteristic that drives labor market sorting in theory and, as we have shown, high wages are not necessarily a reflection of high productivity.

CHK use the log-linear AKM model and interpret the correlation of estimated worker and firm effects in the data as a measure for sorting. They find correlations of 0.17 (1996-2002) and 0.25 (2002-2009) and conclude that sorting is positive and increasing. HLM study the German labor market through the lens of a structural sorting model with worker and firm heterogeneity, search frictions, and on-the-job search. The model allows them to identify the sign and strength of sorting without assuming a log-linear wage equation. Just like AKM/CHK, however, HLM rely primarily on wage data and worker mobility to estimate rankings of both workers and firms. HLM report a rank correlation of 0.76 for the years 1993–2007. This value suggests a high degree of PAM and severe misspecification of the log-linear AKM wage equation.

Correlating our worker and firm rankings across all years, we find a significant positive rank correlation coefficient (Spearman's ρ) of 0.146 in the sample with all matches starting in 1993 and of 0.186 in the sample that contains only new matches formed after 1998. This indicates that the matching process in the German labor market features positive sorting of workers to firms, albeit not to a very high degree. Our rank correlations are below but relatively close to the CHK-AKM result and much lower than the HLM result.

Figure 6 shows that the degree sorting is increasing over time in both samples we consider. The blue line plots rank correlation coefficients for all matches over time. We can distinguish between two types of matches: matches that were formed after an unemployment spell (red line) and matches of workers who switch between jobs without an intermittent unemployment spell (green line). In Panel (a), about 83 percent of matches are job-to-job moves. In Panel (b), this number is 71 percent.

For all matches (Panel (a)), the rank correlation rose from 0.12 in 1998 to about 0.16 in 2008 (blue line). The trend for the rank correlation of job-to-job matches (green line) is very similar to the blue line. Sorting out of unemployment has increased at a higher rate. The red line shows that this correlation was the lowest in 1998 (0.097) but the highest after 2006. So, the increasing degree of labor market sorting is driven by new matches out of unemployment to a large extent.

For new matches (Panel (b)), the trends are broadly the same, but the overall level of sorting is somewhat higher. Job-to-job matches were also "more sorted" in the beginning, but the green line is flat after 2001, it even decreases to some extent. The red line, however, increases almost uniformly. In 2008, all rank correlations are quite close to each other just below 0.2.[79]

We now study the empirical bivariate density of matches in our data to better understand the sorting patterns and how they changed over time. Figure 7 plots the estimated bivariate densities for all matches and new matches and, additionally, subdivided into two time periods: 1998–2002 and 2003-2008. We use these time periods because this allows a rough pre-post comparison in relation to the German labor market reforms, the implementation of which started in 2003.

For all matches, we observe that the allocation to some extent aligns with the 45 degree line, indicating PAM, but the dispersion of workers is huge. In the upper left and lower right corner, we observe very little density. Matches between high-ability workers and low-productivity firms are rare, as are matches between low-ability workers and high-productivity firms. We observe the highest density in the upper right corner, so there is a distinct tendency for high-ability workers to work in high-productivity firms. There is also a tendency for low-type workers to be matched with low-type firms in the lower-left corner of the plots. This has become more common over time.

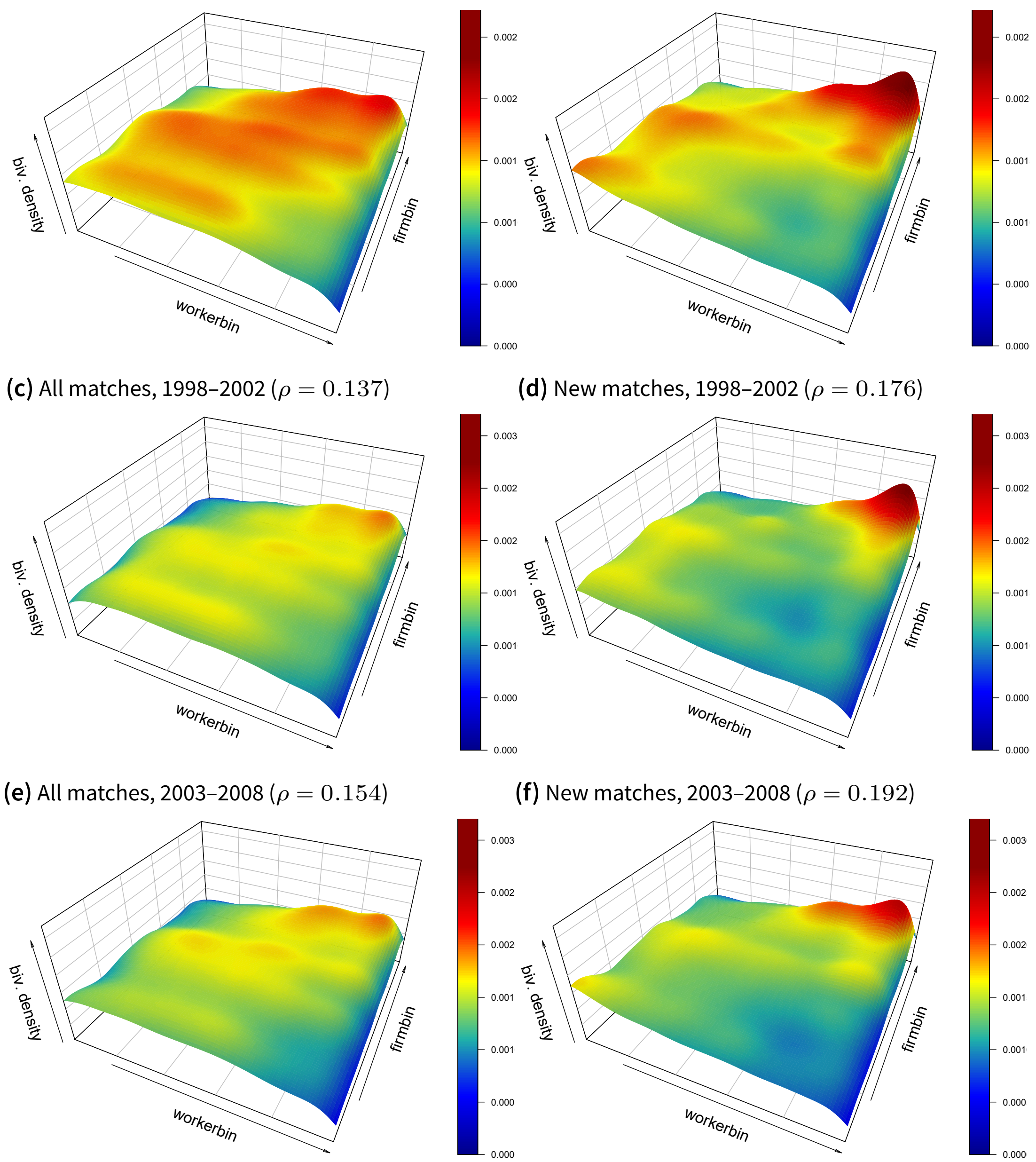
In Panels (c) and (e), still for all matches, we can see that sorting changed at the top but also at the bottom, for low-ability workers. These became more likely to be matched with low and medium-productivity firms in the second sub-period. High-productivity firms, however, have somewhat broadened the set of worker-ability types they are willing to match with. In Panels (d) and (f), for new matches only, this is much more visible. The distinct peak in the upper right corner in Panel (d) shrank a bit and became broader by expanding in the direction of lower workers ability. In Panel (f), we also see that a new peak appears in the lower left corner where low-ability workers are matched with low-productivity firms. A natural hypothesis appears to be that the growing number of matches at the bottom is related to the increasing trend of sorting out of unemployment, so this is what we analyze in more detail next.

5.1. Distributional Dynamics

We have established that labor market sorting in Germany has increased between 1998 and 2008. This increase is driven primarily by more sorting of new matches out of unemployment.

[79] Recall that the rank correlation for job-to-job switchers is computed on a selected sample of workers who were unemployed at least once or who we observe when they enter the labor market, otherwise we would not be able to rank them.

Figure 7: Empirical Bivariate Match Densities



(a) All matches ($\rho = 0.149$)

(b) New matches ($\rho = 0.186$)

(c) All matches, 1998–2002 ($\rho = 0.137$)

(d) New matches, 1998–2002 ($\rho = 0.176$)

(e) All matches, 2003–2008 ($\rho = 0.154$)

(f) New matches, 2003–2008 ($\rho = 0.192$)

Notes: Two-dimensional kernel density estimations with an axis-aligned bivariate normal kernel, evaluated on a grid with dimensions 50×15 (#worker bins $\times$ #firm bins). Source: BHP, EP, IEB, LIAB.

Moreover, the estimated bivariate densities reveal that the number of matches between low-ability worker and low-productivity firms increased while sorting at the top became less pronounced. We now study those developments in more detail, focusing on the sample of new matches. Results for the sample of all matches, which lead to no different conclusions, can be found in Appendix E.

To understand which worker-firm type combinations contribute most to the observed trends, we analyze how the univariate distributions of different worker types across firms change over time, both for matches out of unemployment and for job-to-job switches. This allows us to precisely track for which worker types the distribution across firms led to more sorting, where sorting decreased, and where no significant change occurred. Figuratively, we slice through the empirical bivariate density of matches depicted in Figure 7 and compare these "density slices" for different time intervals.

Figure 8 shows the estimated univariate density functions for worker bins 1, 10, 25, 40, and 50. We compare the first half of our sample, 1998-2002 (red line), to the second half, 2003-2008 (black line), and show estimated densities for new matches out of unemployment, job-to-job switches, and the sum of both, along with 95 percent confidence intervals.[80] Notably, the estimated densities change significantly for low-type workers, see Panels (a)–(c).[81] These workers became significantly more likely to be observed in matches with low-productivity firms below firm bin 5 both out of unemployment and when switching jobs and significantly less likely to be matched above firm bin 5.[82] Accordingly, the overall density shifted to the left, in line with more sorting between low-ability workers and low-productivity firms.

For workers of higher ability, the density functions largely lie on top of each other and cannot be distinguished statistically. This is true for all matches of bin 10 workers. Starting in bin 25, however, we observe significant changes for job-to-job matches. Workers in bins 25 and 40 have become more likely to form new matches with high-productivity firms above bin 10 when switching jobs. The same worker types have become less likely to be matched with low-productivity firms. There are no significant changes for the same worker types out of unemployment, however. For the most able workers in bin 50, there are no significant changes, even though the point estimates of the density suggest that the hiring of these high-type workers might have somewhat decreased at the most productive firms. We will look into this possibility by switching to the firms' perspective.

Figure 9 shows the result of the same exercise for five different firm bins.[83] We estimate univariate density functions for firm bins 1, 4, 8, 12, and 15 and test where the density of workers has changed significantly over time. First, we confirm that the largest changes occur for the "extreme" worker types of very low or very high ability. The lowest productivity firms (bin 1) have significantly increased their hiring of low-ability workers and significantly reduced their

[80] For the corresponding plots based on the sample of all matches, see Figure E.3.

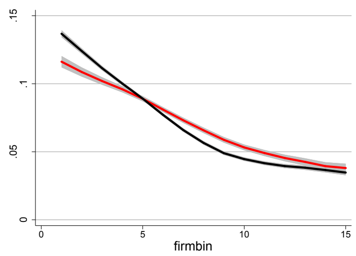
[81] We report statistical significance based on the overlap of confidence intervals which is a conservative approach: it is always true that with non-overlapping confidence intervals, two statistics are significantly different from each other. However, an overlap of the confidence intervals does not necessarily imply an insignificant difference.

[82] Recall the bivariate densities in Figure 7 where more new matches appeared in the lower-left corner.

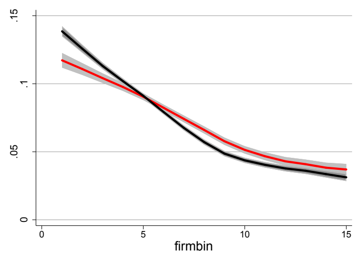
[83] For the corresponding plots based on the sample of all matches, see Figure E.4.

Figure 8: Estimated Density Functions, Distribution of Worker Types across Firm Bins, New Matches: 1998-2002 (red) vs. 2003-2008 (black)

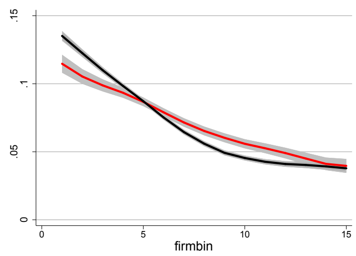
(a) Worker Bin 1



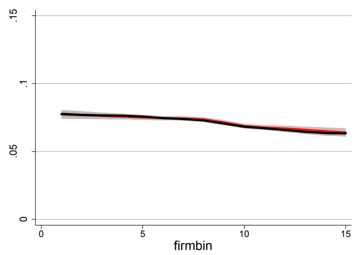
(b) Worker Bin 1, out of Unemp.



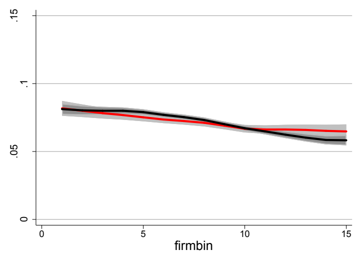
(c) Worker Bin 1, Job-to-Job



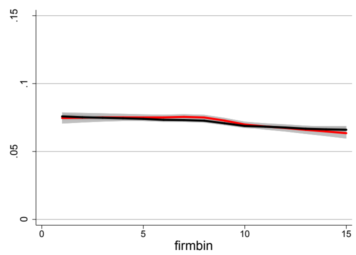
(d) Worker Bin 10



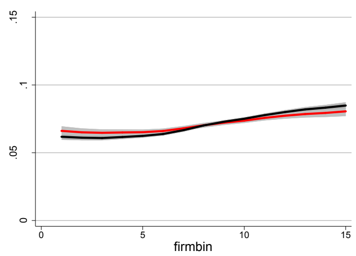
(e) Worker Bin 10, out of Unemp.



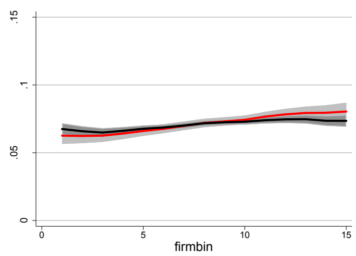
(f) Worker Bin 10, Job-to-Job



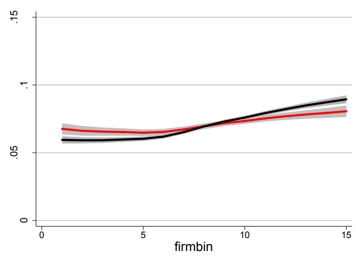
(g) Worker Bin 25



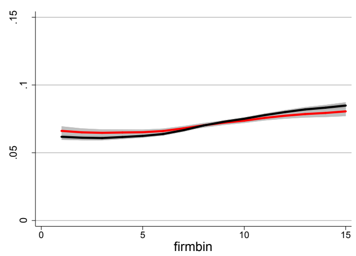
(h) Worker Bin 25, out of Unemp.



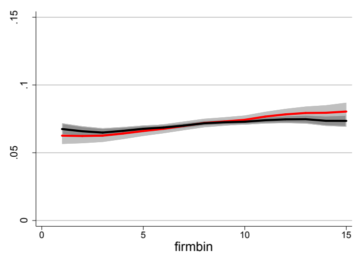
(i) Worker Bin 25, Job-to-Job



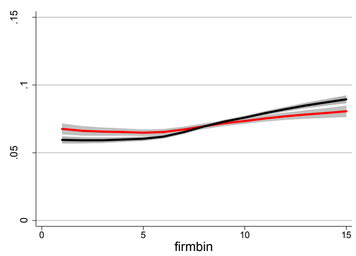
(j) Worker Bin 40



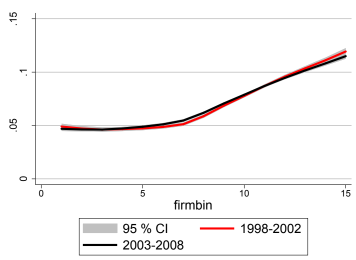
(k) Worker Bin 40, out of Unemp.



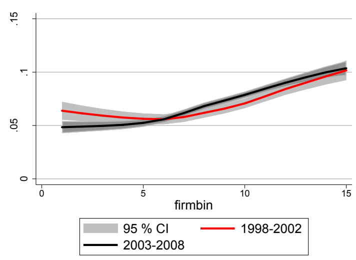
(l) Worker Bin 40, Job-to-Job



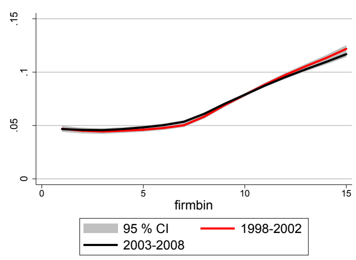
(m) Worker Bin 50



(n) Worker Bin 50, out of Unemp.

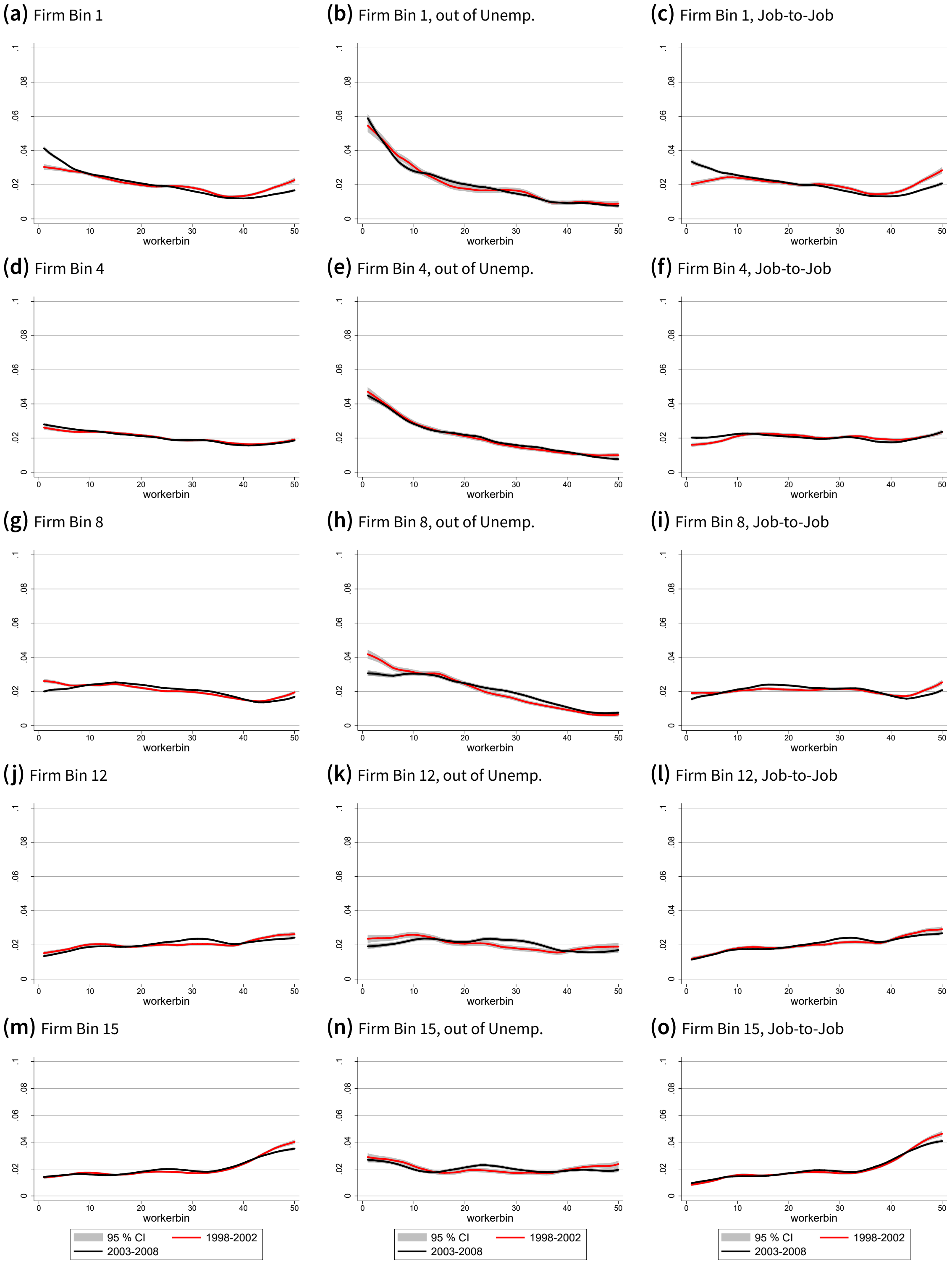


(o) Worker Bin 50, Job-to-Job



Notes: Estimated univariate kernel densities of all new matches conditional on worker bins, time, and match type. The kernel is estimated using an Epanechnikov kernel function. The bandwidth is calculated by Silverman's rule of thumb. Pointwise confidence intervals are calculated using a quantile of the standard normal distribution. Source: BHP, EP, IEB, LIAB.

Figure 9: Estimated Density Functions, Distribution of Firm Types across Worker Bins, New Matches: 1998-2002 (red) vs. 2003-2008 (black)



Notes: Estimated univariate kernel densities of all new matches conditional on worker bins, time, and match type. The kernel is estimated using an Epanechnikov kernel function. The bandwidth is calculated by Silverman's rule of thumb. Pointwise confidence intervals are calculated using a quantile of the standard normal distribution. Source: BHP, EP, IEB, LIAB.

hiring of high-ability workers, see Panel (a). Interestingly, the firm side perspective reveals that this change is driven by job switchers, see Panel (c). There is also a small but significant positive change towards more low-ability job switchers in firm bin 4, see Panel (f). In firm bin 8, Panels (g)–(i), we observe that firms have significantly reduced their hiring of low-ability workers and instead increased the hiring of medium-ability workers. The latter applies also to firm bin 12 firms, see Panels (j)–(l). Finally, the most productive firms in bin 15 have indeed significantly reduced the share of high-ability workers in their workforce, see Panel (m). This is mainly driven by less high-ability worker poaching from other firms (Panel (o)). We also observe a small but significant positive increase of the density for medium-ability workers out of unemployment in those firms, see Panel (n). Both observations are in line with the decreasing bivariate density at the top, recall Figure 7, Panels (d) and (f). Worker sorting to the top of the firm-productivity distribution has decreased, and high-ability workers have to some extent been replaced by medium-ability workers.

6. Wages and Inequality

6.1. Wages Across Worker and Firm Bins

Wages are an important determinant of how workers select jobs. Therefore, the logical next step in our analysis is to check how wages have changed across worker and firm types over time and to what extent these changes are in line with the observed distributional shifts. Moreover, our analysis is motivated by the theory of labor market sorting. Recall that, according to the theory discussed in Section 2, worker ability and firm productivity are complements in production. Their interaction determines output and the wage. An implication of this theory is that, for a given worker type, moving to a highly productive firm does not necessarily lead to a higher wage. The reason is option value compensation: in a match with a highly productive firm and high output, the worker has to compensate the firm for not waiting longer to hire a worker of higher ability and accepts a lower wage. Workers' wages, thus, evolve non-monotonically across firm types: if workers move from lower to higher productivity firms over time, wages may start falling at some point.

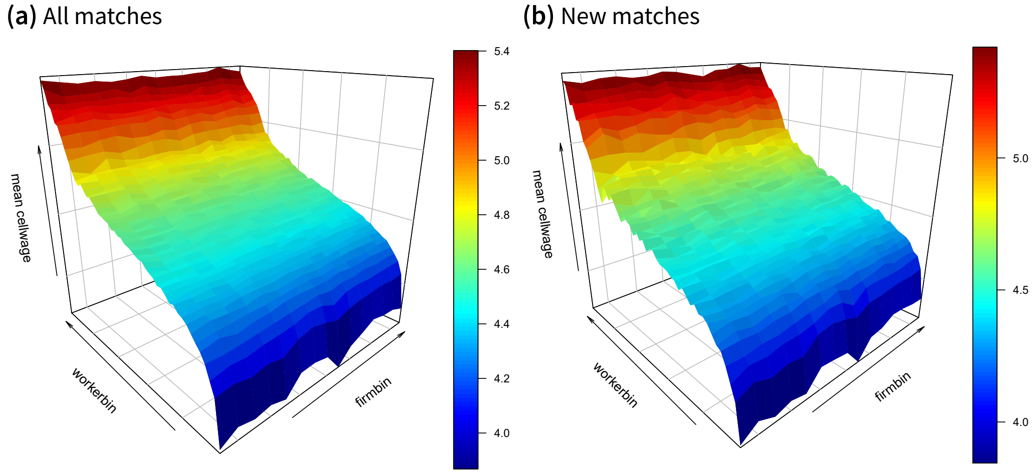
As many authors have emphasized before us, the implied non-monotonicity of wages is at odds with the log-linear AKM model, which by construction assumes monotonicity of wages in the estimated firm effects.[84] Our approach separately identifies worker ability and firm productivity and does therefore not restrict the interaction of worker and firm heterogeneity. This allows us to test for non-monotonicities in the data by simply studying observed wages through the lens of our estimated worker and firm ranks. This test reveals how prevalent non-monotonic wage patterns actually are in the data.

We first look at wages across all combinations of estimated worker ability and firm productivity types. Figure 10 suggests that, both for all matches and new matches, the mean of the log wage can be approximated well by a log-linear function of the worker and the firm type.[85]

[84] See, among others, Gautier/Teulings (2006), Eeckhout/Kircher (2011), Hagedorn/Law/Manovskii (2017), and Lopes de Melo (2018).

[85] Recall that our firm ranking is based on productivity. Monotonicity of wages in firm productivity does not

Figure 10: Mean Wages for all Worker-Firm Type Combinations (1998–2008)



Notes: The plots shows the mean of the log real daily wage for all combinations of worker and firm types, all matches and new matches, on a grid with dimensions 50×15 (#worker types $\times$ #firm types). Source: BHP, EP, IEB, LIAB.

Wages increase strongly in the worker dimension but are rather flat in the firm dimension, at least in Figure 10. This is consistent with the broader literature on wage dispersion: the high slopes in the worker dimension suggest that worker heterogeneity is the dominant source of wage dispersion.

To analyze the variation of wages in firm productivity, Figure 11 zooms in and plots wage profiles across firms for groups of 10 worker bins: for all matches in Panel (a) and for new matches in Panel (b). Holding the worker type fixed reveals quantitatively important deviations from monotonicity. These deviations are most pronounced at high-productivity firms where virtually all worker types receive lower wages as compared to matches with slightly less productive firms. CHK argue that systematic departures from the monotonicity assumption of the AKM model demand an in-depth analysis which we attempt to provide in the following.[86]

Wages are almost perfectly monotonic in the firm type for the lowest worker types in bins 1-10, although they become quite flat and slightly decrease at the most productive firms. Virtually all worker types above bin 10 earn significantly lower wages at the top of the firm productivity distribution: wages are typically maximized in firm bin 13 and decrease at more productive firms. This sizable non-monotonicity is present in both samples. To show that these non-monotonicities are indeed not observed using AKM-based firm types, Figure E.7 in the Appendix plots the same wage profiles for firm bins constructed from AKM firm-fixed effects. All wage profiles are monotonic in this case.

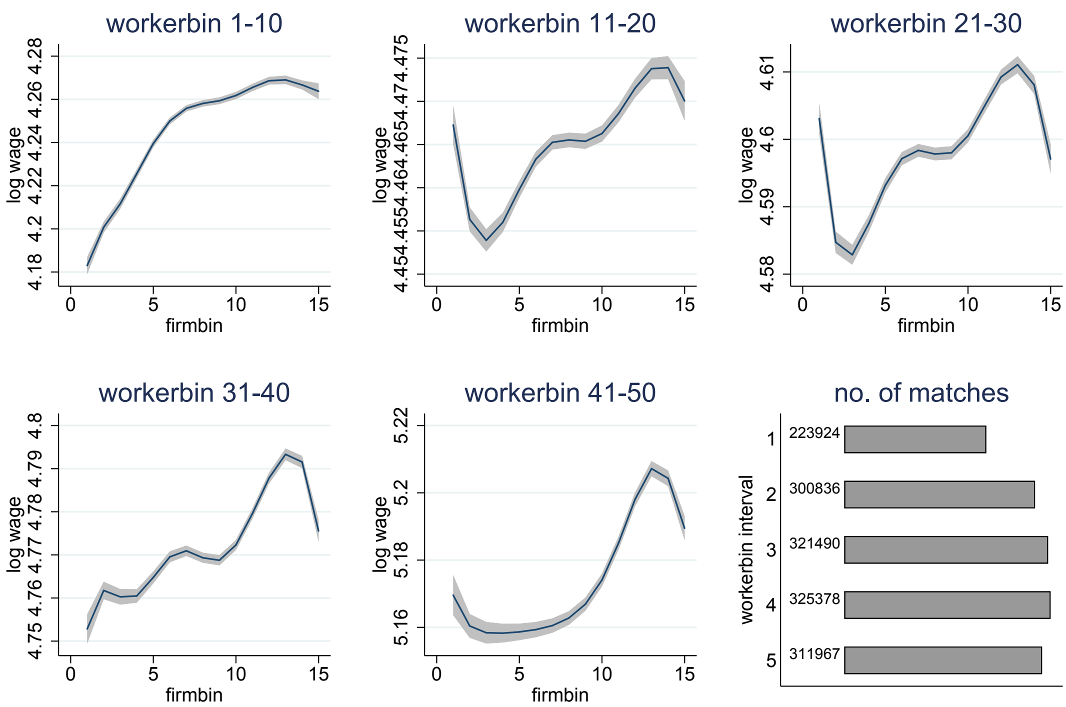
We interpret the observation of declining wages at the top as a "wage penalty" that workers incur when working at the most productive firms. The prevalence of these wage penalties in the data supports the idea that firms at the top, our potential "superstars", are special in

imply that output is also monotonic in firm productivity. That is, monotonic wage patterns should not be seen as a rejection of our theory by the data.

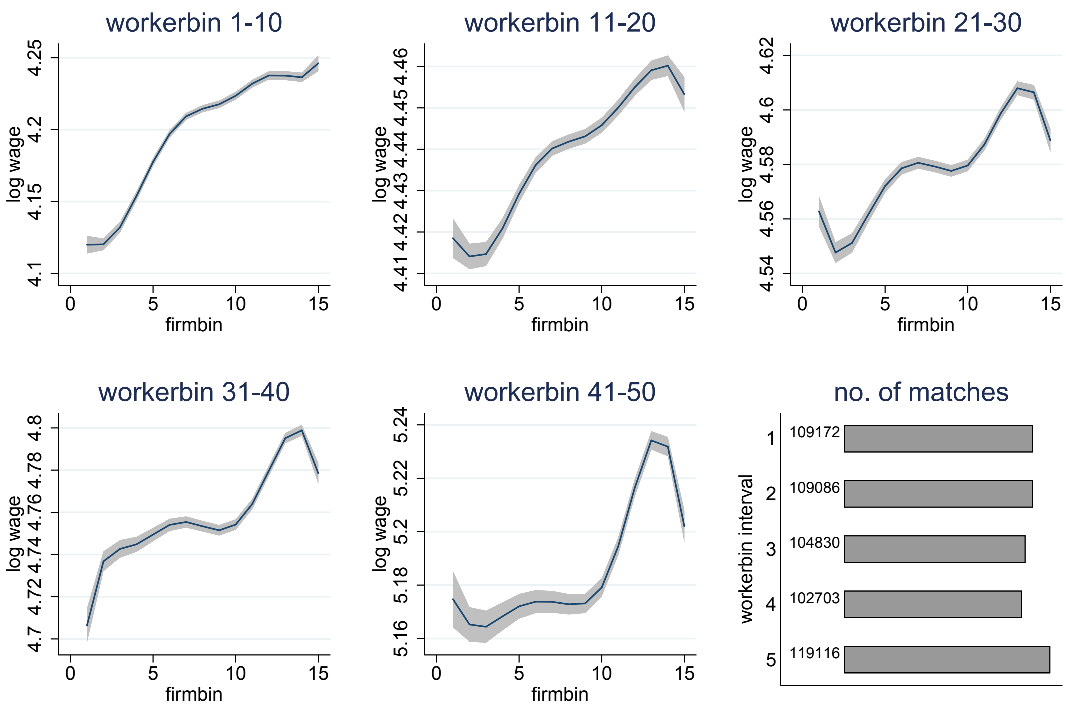
[86] See the related discussion on p. 996 in CHK.

Figure 11: Mean Wages across Worker Types

(a) All Matches



(b) New Matches



Notes: Plots show estimated wage profiles across firm bins for all matches (a) and new matches (b). The kernel is estimated using an Gaussian kernel function. The bandwidth is 2. 95 percent confidence bands in gray. Source: BHP, EP, IEB, LIAB.

some way. According to the sorting model, the lower wages at these firms could be explained by option value compensation.[87]

Quantitatively, the wage penalty is most pronounced for new matches of high-ability workers in bins 41-50. For these workers, the wage difference between being employed in a bin 14 firm and a bin 15 firm amounts to, roughly, 0.03 in terms of (deflated) log daily wages. On a yearly basis, this translates into a wage loss of about 2,015 euros or 3 percent of the annual wage.[88] We suspect that the wage penalty at the most productive firms has contributed to the observed decrease of sorting between high-ability workers and high-productivity firms documented in Section 5.1.

For medium worker types in bins 21-40, we also observe higher wages in medium-productivity firms as compared to wages at firms that are slightly more productive in firm bins 7-9. This non-monotonicity is quantitatively small and hardly significant. At the bottom, however, we find that some medium-type workers face significantly lower wages in firm bins 3-5 as compared to the least productive firms in bin 1. For workers in bins 21-30, this wage difference can amount to up to 0.01 (new matches) and 0.02 (all matches) in terms of (deflated) log daily wages. A log difference of 0.02 for these workers translates into a yearly wage loss of about 719 euros or 2 percent of the annual wage when moving out of the least productive firms.[89]

Recall that we documented in Figure 3 that the least productive firms pay large shares of their value added out to their workers, resulting in relatively high wage bills. Figure 11 reveals which workers receive those high wages at the least productive firms. For brevity, we relegate a decomposition of the wage profiles into matches out of unemployment and job-to-job moves to Appendix E, see Figures E.5 and E.6. The broad patterns do not depend on this distinction.

The wage profiles in Figure 11 depict mean wages across firms for a given set of worker types. It is interesting to check whether workers that transition between firm bins actually move along these wage profiles. If workers moved in reaction to the observed wage differences, mobility decisions would not be exogenous. Suppose that this was indeed the case. One would expect that observed transitions out of the most productive firms—the reason for decreasing productivity sorting at the top—are accompanied by wage gains, at least if the target firm of the transition is not too unproductive. Similarly, moving into the most productive firms should yield wage gains or wage losses, depending on which firm bin the transitioning worker comes from. Figure 12 shows that this is indeed what we see in the data. The wage changes of transitioning workers coincide with the wage differences depicted in the wage profiles. This is suggestive evidence of endogenous mobility between different firm productivity types based on wages.

We regress log-wage differences[90] for all individual workers who move between firm bins on

[87] A related but slightly more general explanation could be that these firms exercise some form of monopsony power, allowing them to pay lower wages. Of course, we can also not rule out that these firms offer some kind of amenity that makes worker willing to accept lower wages.

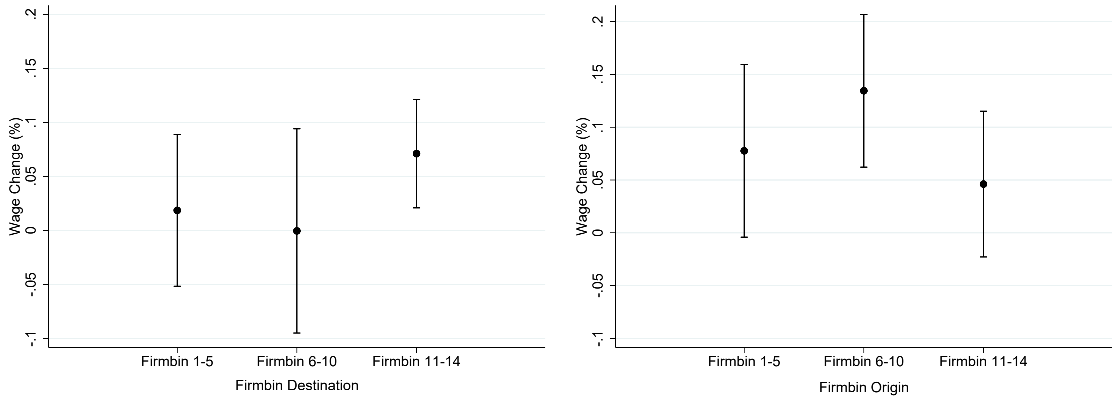
[88] The average yearly wage of a bin 41-50 worker in a bin 14 firm is approximately 68,179 euros.

[89] The average yearly wage of a bin 21-30 worker in a bin 1 firm is approximately 36,311 euros.

[90] We measure the difference between the wage in the last spell in the pre-transition firm and the wage in the first spell in the post-transition firm.

Figure 12: Wage Changes for Observed Transitions

(a) Transitions out of Firm Bin 15 **(b)** Transitions into Firm Bin 15



Notes: The plots show estimated coefficients and 95 percent confidence intervals (robust standard errors) from a linear regression of individual-level wage differences on dummies for origin and destination firm bins. Source: BHP, EP, IEB, LIAB.

a set of origin and destination firm bin dummies to construct Figure 12. Panel (a) shows estimated coefficients for transitions out of the most productive firms in bin 15 into three categories of destination firm bins, low (1-5), medium (6-10), and high productivity (11-14).[91] As one would expect from the observed wage profiles, workers that transition out of bin 15 into other high-productivity firms effectively increase their wages. We estimate a significantly positive wage effect of around 7 percent. Transitions out of bin 15 into medium and low-productivity firms yield no significant wage change, which is again in line with the wage profiles in Figure 11. Note that for many worker types, the wage in bin 15 firms is actually not that different from the wage in medium-productivity firms due to the aforementioned non-monotonicity.

Panel (b) shows estimated coefficients for transitions into the most productive firms in bin 15 for workers who come out of three categories of origin firm bins: low (1-5), medium (6-10), and high productivity (11-14). Coming from other high-productivity firms, transitions into bin 15 yield no significant wage change, although the point estimate is positive at around 5 percent. Transitions from medium-productivity firms to bin 15 lead to a significant wage increase of more than 10 percent. The point estimate for transitions out of low-productivity is also relatively large and positive but only marginally significant.

The lesson from studying worker mobility is that wage changes from downward transitions, that is, movements out of the most productive firms, do indeed follow the non-monotonic pattern: workers increase their wages by moving down the firm-productivity ladder. Upward transitions also lead to positive wage effects, although these are not always significant. In the end, workers appear to select jobs to maximize their wages, as one would expect. How-

[91] We only use three categories of firm bins to increase the number of observed transitions used for estimating the single coefficients.

ever, transitions towards higher wages can move the worker both up and down the firm-productivity ladder.

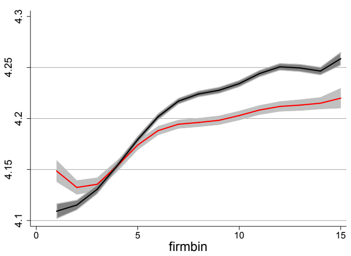
6.2. Wage Growth and Inequality

Have the wage profiles changed over time in a way that is consistent with the changing sorting patterns? In Figure 13, we again plot log wages across firm bins for groups of 10 worker bins, but now we show the wage differences between the two sub periods considered earlier, 1998–2002 (red) and 2003–2008 (black).[92] We observe large differences in wage growth across worker and firm bins. For high-type workers above bin 30, wages shift upwards almost in parallel. This reflects growth rates between 10 and 15 percent which are largely independent of the firm type. The non-monotonic wage humps at the most productive firms become less pronounced over time. We suspect that this is related to decreasing sorting at the top.
For low-type workers, wage growth is to a large extent driven by the productivity of the firm they work at. At the top firms, the wages of these workers also grew by 5–10 percent. Low-ability workers at low-productivity firms, however, experienced stagnating or even shrinking wages. For workers in the lowest ten bins, wages did not increase in firms up to bin 5 and even declined at the least productive firms. This is remarkable. It implies that increasing sorting out of unemployment at the bottom is not driven by higher wages. On the contrary, sorting increased despite negative wage growth. We suspect that the creation of these new matches at the bottom, driving increasing sorting out of unemployment, might be related to the German labor market reforms implemented between 2003 and 2005, which have contributed to the creation of a sizable low-wage sector in the German labor market.[93]
In the related literature on the sources of wage dispersion, Song et al. (2019) show for the U.S. that two thirds of the rise in wage inequality can be attributed to rising between-firm wage inequality. Using the AKM approach, they decompose this contribution into increasing sorting of high wage workers into high wage firms and increasing segregation of workers. Both components contribute roughly equally to increasing between-firm wage inequality. In the final step of our analysis, we check whether we can identify the same patterns in German data and to what extent this finding depends on the way one measures firm types. In Figure 14, Panel (a) replicates the primary finding of Song et al. (2019). We decompose the variance of wages into the respective shares explained within and between establishment identifiers and observe that, just like in the U.S., the within-firm contribution to wage dispersion is higher in levels, but the between-firm contribution is growing by almost 10 percent over time.
Taking into account firm productivity and worker ability heterogeneity, however, changes this picture. Panel (b) presents a similar decomposition based on our estimated worker and firm bins. From this perspective, the between-firm(bin) contribution to wage dispersion (blue

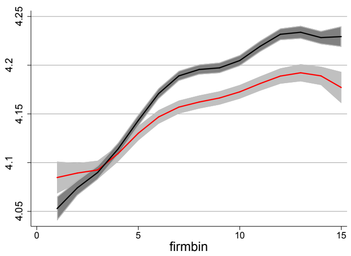
[92] Figure 13 shows wage differences for new matches. Figure E.8 in the Appendix contains the same plots for all matches.
[93] Dustmann/Ludsteck/Schönberg (2009) report that already between 1990 and 2000, real wage growth for full-time working men in Germany was negative below the 18th percentile of the wage distribution. Thus, the labor market reforms are certainly not the only reason for low wage growth in Germany during this period of time, see also Dustmann et al. (2014).

Figure 13: Wages across Firm Bins, New Matches: 1998-2002 (red) vs. 2003-2008 (black)

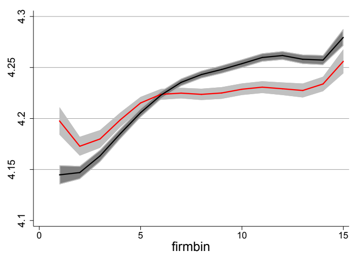
(a) Worker Bins 1-10, all Matches



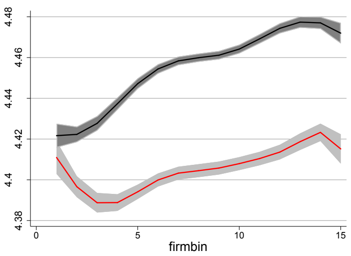
(b) Worker Bins 1-10, out of Unemp.



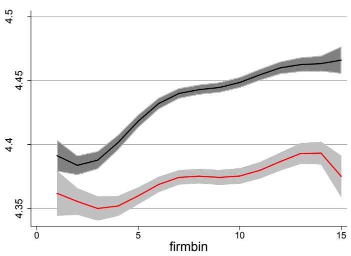
(c) Worker Bins 1-10, Job-to-Job



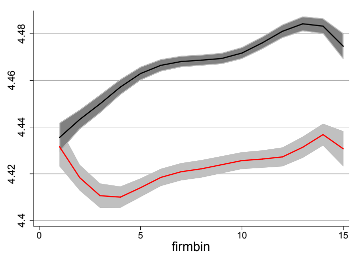
(d) Worker Bins 11-20, all Matches



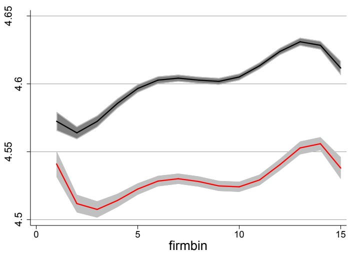
(e) Worker Bins 11-20, out of Unemp.



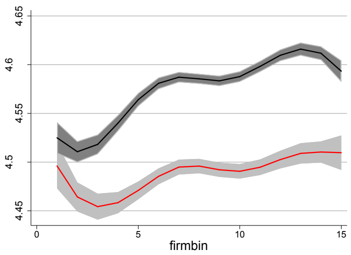
(f) Worker Bins 11-20, Job-to-Job



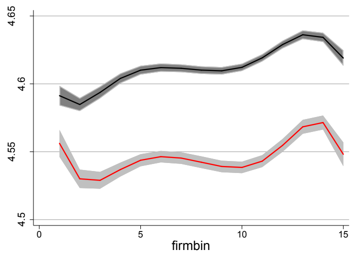
(g) Worker Bins 21-30, all Matches



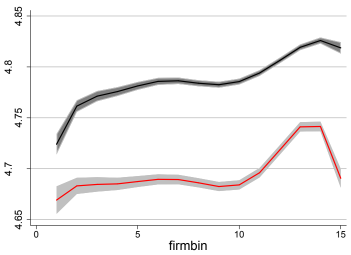
(h) Worker Bins 21-30, out of Unemp.



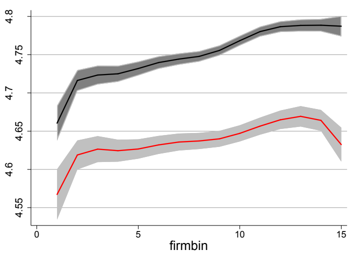
(i) Worker Bins 21-30, Job-to-Job



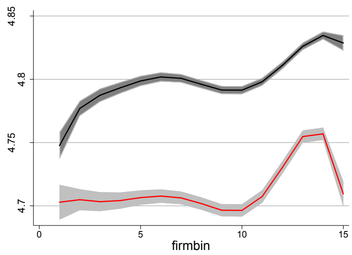
(j) Worker Bins 31-40, all Matches



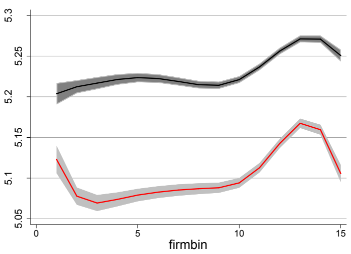
(k) Worker Bins 31-40, out of Unemp.



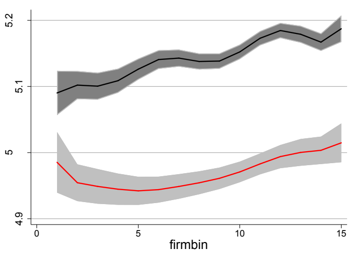
(l) Worker Bins 31-40, Job-to-Job



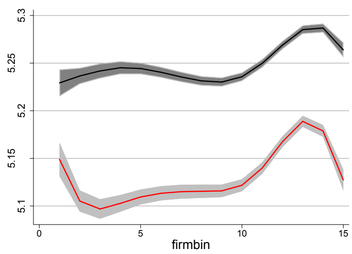
(m) Worker Bins 41-50, all Matches



(n) Worker Bins 41-50, out of Unemp.



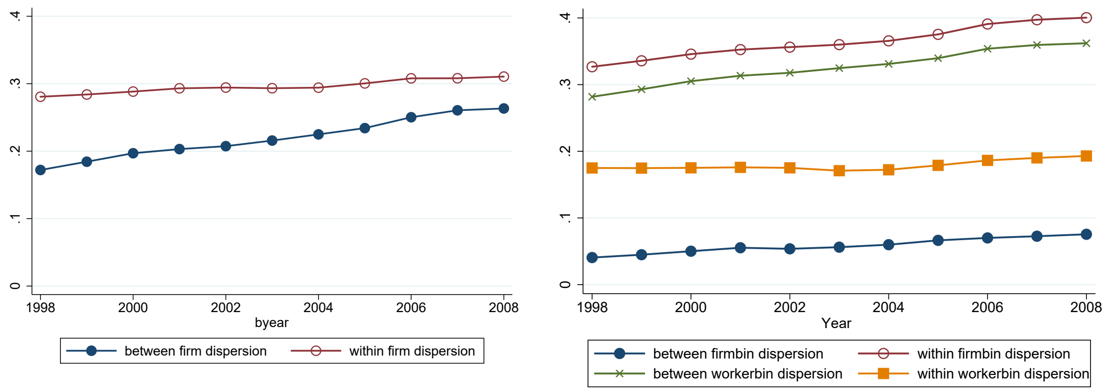
(o) Worker Bins 41-50, Job-to-Job



Notes: Plots show estimated wage profiles across firm bins and over time for new matches. The kernel is estimated using an Gaussian kernel function. The bandwidth is 2. 95 percent confidence bands in gray. Source: BHP, EP, IEB, LIAB.

Figure 14: Decomposition of Wage Dispersion over Time

(a) Song et al. (2019) replication **(b)** Our approach



Notes: Panel (a) shows yearly decompositions of wage dispersion using establishment identifiers. Panel (b) shows yearly decompositions of wage dispersion into the respective contributions within and between estimated worker bins and within and between estimated firm bins. Source: BHP, EP, IEB, LIAB.

line) has also increased but contributes much less to increasing wage dispersion, only about 4 percent. We find that the rising dispersion is mainly driven by dispersion *within firm bins* (red line) and *between worker bins* (green line). That is, we find that rising segregation of workers is the major contributor to inequality. Increasing sorting of worker ability types into specific firm productivity types, however, appears to be of minor importance. This finding is in line with our positive but relatively low estimated degree of productivity sorting in Germany.

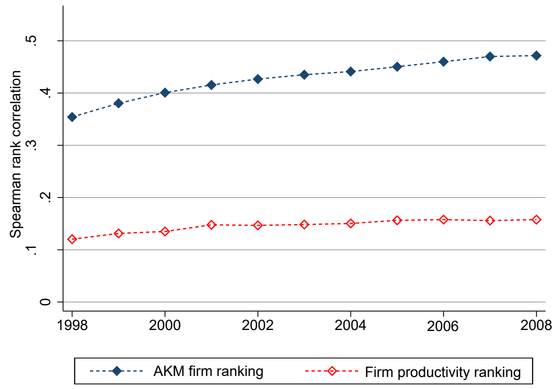
Figure 15 underlines this conclusion. It presents our measure of labor market sorting using worker-ability and firm-productivity types (red) for both all matches and new matches and compares it to a measure of sorting based on our worker-ability ranking and estimated AKM firm-fixed effects (blue).[94] The difference is striking: the AKM firm ranking yields much higher rank correlations and a steeper increase over time, particularly in the sample of all matches. Thus, a decomposition of increasing wage dispersion using wage-based firm types leads to the conclusion that increasing wage sorting is very important for rising inequality. Our measure of sorting, however, is lower and thus, mechanically, productivity sorting also contributes less to increasing wage inequality.

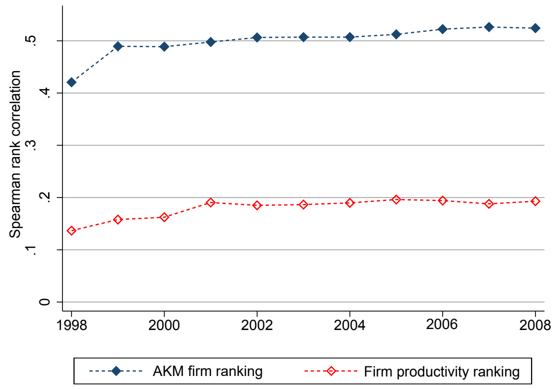
7. Conclusions

In this paper, we exploit a link between firm-level output, productivity, worker ability, workforce composition, and wages to measure the sign and strength of labor market sorting in a new way. Building on sorting theory, we estimate firm-level production functions and take into account that the contribution of heterogeneous worker ability to output is firm-specific and covaries with firm productivity due to complementarities at the match level. We use our estimated measure of firm productivity, net of workforce ability, to rank firms and study pro-

[94] For this reason, these correlations are also higher than what CHK report.

Figure 15: Different Measures of Labor Market Sorting





Notes: Panels (a) and (b) show different time series of Spearman rank correlation coefficients for all matches and new matches, respectively. The red line is our benchmark firm-productivity ranking. The blue line ranks firms using AKM firm-fixed effects. All estimated correlations use the same worker ranking (HLM). Source: BHP, EP, IEB, LIAB.

ductivity sorting in the German labor market.

Productivity sorting is positive and increasing. However, it is lower than available wage-based measures of sorting for Germany (CHK, HLM). The reason is that workers tend to move towards higher wages, but these moves do not necessarily lead up the firm-productivity ladder. The contribution of productivity sorting to increasing wage inequality is therefore smaller compared to wage sorting. We argue that our approach is a useful complement to wage-based methods whenever it is possible to link detailed firm-level information to matched employer-employee data.

Our analysis reveals a number of novel empirical facts. Increasing productivity sorting is driven by low-ability workers that match with low-productivity firms out of unemployment. At the most productivity firms, sorting is decreasing as high-ability workers become more likely to be matched with slightly less productive firms that pay higher wages. Thus, we find that wages are not everywhere monotonically increasing in firm productivity. They decrease at the most productive firms. Workers take this into account as evidenced by observed transitions towards higher wages and lower firm productivity.

It is important to know which firms pay high wages and why to understand the firms' contribution to increasing wage inequality. We find that the highest wages are not paid by the most productive firms; in fact, some very unproductive firms pay relatively high wages, perhaps to grow or retain workers. The firms at the top of the estimated productivity ranking have relatively low wage bills, low labor shares, extremely high labor productivity, and they are not big. This finding constitutes a link between the literature on increasing wage inequality and labor market sorting, on the one hand, and the literature on firm performance, falling labor shares, rising market concentration and increasing market power, on the other hand. Analyzing this link more deeply is a fascinating avenue for future research.

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A. Details of the Sorting Model

Following Shimer/Smith (2000), we introduce search frictions to the Becker (1973) model and assume random search. Meeting rates are governed by a standard Cobb-Douglas matching function with constant returns to scale. We assume a functional form for the production function at the match level $f(i,j)$, a shorthand for $f(a(i),\omega(j))$, that features log-supermodularity of the function itself, its first derivatives, and cross-derivatives to ensure the existence of a search equilibrium.[95] Only unemployed workers search. Agents are willing to form matches whenever the surplus is high enough to compensate both parties for the foregone option value of continued search for a better match. We assume Nash bargaining and the match surplus is defined as

$$\mathcal{S}(i,j) = \mathcal{P}(i,j) - \mathcal{V}(j) + \mathcal{E}(i,j) - \mathcal{U}(i), \tag{A.1}$$

which depends on four option value equations defined below. These capture the values of a producing firm, a vacant job, an employed worker, and an unemployed worker, respectively, for all (i,j) combinations. Matches are formed in case of positive surplus. An indicator function capturing this is

$$\mu(i,j) = \begin{cases} 1 & \text{if } \mathcal{S}(i,j) > 0 \\ 0 & \text{if } \mathcal{S}(i,j) \leq 0. \end{cases} \tag{A.2}$$

Thus, $\mu(i,j)$ equals 1 whenever a firm of type j is willing to match with a worker of type i and vice versa. We indicate that $\mu(i,j) = 1$ ($\mu(i,j) = 0$) by writing $\mu^+(i,j)$ ($\mu^-(i,j)$). The value of employment is

$$\mathcal{E}(i,j) = W(i,j) + \underbrace{\beta\delta\mathcal{U}(i)}_{\text{separation}} + \underbrace{\beta(1-\delta)\max\{\mathcal{E}(i,j),\mathcal{U}(i)\}}_{\text{continued employment}}. \tag{A.3}$$

The value of unemployment is

$$\begin{aligned} \mathcal{U}(i) = b(i) + &\underbrace{\beta(1-q_u(\theta))\mathcal{U}(i)}_{\text{no meeting}} + \underbrace{\beta q_u(\theta)\int_0^1 \frac{g_v(j)}{V}\mu^+(i,j)\mathcal{E}(i,j)\mathrm{d}j}_{\text{successful match}} \\ &+ \underbrace{\beta q_u(\theta)\mathcal{U}(i)\int_0^1 \frac{g_v(j)}{V}\mu^-(i,j)\mathrm{d}j}_{\text{meet unacceptable firm}}. \end{aligned} \tag{A.4}$$

The value of a producing firm is

$$\mathcal{P}(i,j) = F(i,j) - W(i,j) + \underbrace{\beta\delta\mathcal{V}(j)}_{\text{separation}} + \underbrace{\beta(1-\delta)\max\{\mathcal{P}(i,j),\mathcal{V}(j)\}}_{\text{continued production}}. \tag{A.5}$$

[95] That is, for any $i' > i$ and $j' > j$, $f(i',j')f(i,j) \geq f(i',j)f(i,j')$, $f_i(i',j')f_i(i,j) \geq f_i(i',j)f_i(i,j')$, and $f_{ij}(i',j')f_{ij}(i,j) \geq f_{ij}(i',j)f_{ij}(i,j')$.

The value of a vacant firm is

$$
\begin{aligned}
\mathcal{V}(j) = -c(g_v(j)) + \underbrace{\beta(1-q_v(\theta))\mathcal{V}(j)}_{\text{no meeting}} + \underbrace{\beta q_v(\theta)\int_0^1 \frac{g_u(x)}{U}\mu^+(i,j)\mathcal{P}(i,j)\mathrm{d}i}_{\text{successful match}} \\
+ \underbrace{\beta q_v(\theta)\mathcal{V}(j)\int_0^1 \frac{g_u(i)}{U}\mu^-(i,j)\mathrm{d}i}_{\text{meet unacceptable worker}}\,.
\end{aligned}
\tag{A.6}
$$

With Nash bargaining, the wage becomes

$$
\begin{aligned}
w(i,j) &= \alpha f(i,j) + (1-\alpha)b(i) \\
&+ (1-\alpha)\beta\alpha\left[q_u(\theta)\int_0^1 \frac{g_v(j)}{V}\mu(i,j)\mathcal{S}(i,j)\,\mathrm{d}j\right],
\end{aligned}
\tag{A.7}
$$

which can be rewritten in the following way to reveal the three components of the wage in the sorting model:

$$
\begin{aligned}
w(i,j) \quad = \quad \alpha &\left(\underbrace{f(i,j)}_{\text{Match output}} + c(v(j))\underbrace{\left[\frac{\int_0^1 g_v(j)\mu(i,j)\mathcal{S}(i,j)\,\mathrm{d}j}{\int_0^1 g_u(i)\mu(i,j)\mathcal{S}(i,j)\,\mathrm{d}i}\right]}_{\text{Ratio of option values}}\right) \\
+ \quad (1-\alpha) &\quad \underbrace{b(i)}_{\text{Flow value of unemployment}} \quad .
\end{aligned}
\tag{A.8}
$$

B. The Firm Ranking

We demonstrate the link between the match-level sorting model presented in Section 2 and the firm-level production function we estimate to infer firm productivity in Section 4.2. Firms employ multiple workers of various ability types, but we assume there are no complementarities between worker types within the same firm. As in the basic sorting model, there is a complementarity between firm productivity and worker ability. Thus, the contribution to production of every single match depends on both the firm's productivity and the worker's ability. Time indices are omitted for brevity.

Match-level output is determined by a function of worker ability and firm productivity, see also Appendix A. For illustrative purposes, we assume the simple weakly log-supermodular form

$$
f(a(i),\omega(j)) = a(i)\times\omega(j), \tag{A.9}
$$

where $a(i)$ is the ability of the worker and $\omega(j)$ is the firm's productivity. The match-level output is aggregated up and forms a composite labor input at the firm level, which is then used to produce the final output in combination with the firm's capital stock. We assume that the firm combines the different labor inputs using a simple CES aggregator:

$$\left(\sum_{i=1}^{L_j} f(a(i),\omega(j))^\rho\right)^{\frac{1}{\rho}} = \omega(j)\left(\sum_{i=1}^{L_j} a(i)^\rho\right)^{\frac{1}{\rho}} = \omega(j)L_j^*, \tag{A.10}$$

where L_j is the size of the workforce (in heads) of firm j and ρ determines the elasticity of substitution between different worker types. Since the assumed aggregator function is homogeneous of degree one, we can simply write firm j's productivity, $\omega(j)$, in front of the sum, which we redefine as the firm's composite labor input in units of worker ability, L_j^*. The contribution of the composite labor input to firm-level output thus depends on the firm's productivity, similar to the match-level sorting model.

The firm-level production function is assumed to be of the Cobb-Douglas type:

$$Y_j = \left(\sum_{i=1}^{L_j} f(a(i),\omega(j))^\rho\right)^{\frac{\beta_l}{\rho}} K_j^{\beta_k}. \tag{A.11}$$

The composite labor input and the firm's capital stock, K_j, jointly determine the firm's output Y_j. β_l and β_k are the output elasticities of the composite labor input and capital, respectively. Rewriting this using (A.10) yields

$$Y_j = \left(\omega(j)L_j^*\right)^{\beta_l} K_j^{\beta_k}, \tag{A.12}$$

or in logs (denoted by lower-case symbols)

$$y_j = \beta_l(\ln(\omega(j)) + \tilde{l}_j) + \beta_k k_j. \tag{A.13}$$

Separately identifying firm productivity and the output elasticity of labor in this setting is difficult because worker ability and firm productivity interact at the match level. A special case that recovers the "textbook" Cobb-Douglas production function in which $\omega(j)$ takes the role of TFP involves assuming that the capital stock also interacts directly with firm-productivity and that (A.12) has constant returns to scale.

$$Y_j = \left(\omega(j)L_j^*\right)^{\beta_l}\left(\omega(j)K_j\right)^{\beta_k} = \omega(j)L_j^{*\beta_l}K_j^{\beta_k}, \text{ iff } \beta_l + \beta_k = 1. \tag{A.14}$$

Apart from this special case, the composite labor input of the firm always interacts with firm productivity, and so does the marginal product of labor of a worker with ability $a(i)$. The marginal value of one unit of worker ability varies with the productivity of the firm. The reason is that units of worker ability are not comparable across firms. To overcome this problem, we suggest to construct a measure of workforce ability conditional on firm productivity by adjusting observed labor inputs with the wage bill ratio, as described in the main text.

C. The HLM Worker Ranking

HLM propose an algorithm to merge within-firm wage rankings into a global ranking of workers by solving a Kemeny-Young rank aggregation problem. Rank aggregation is an ancient problem in social choice theory. Kemeny-Young rank aggregation solves the problem of aggregating inconsistent preference rankings of different voters by minimizing the number of disagreements, see Kemeny/Snell (1962). The approach here is essentially the same. In the HLM application to the labor market, firms are "voters" and individual workers are the "voting alternatives". A within-firm worker ranking based on wages can be understood as a preference ranking of voters in the social choice context. The rank aggregation algorithm minimizes the number of disagreements between potentially inconsistent within-firm worker rankings. These within-firm rankings are linked due to job mobility in the labor market as workers show up at multiple firms over time.

Suppose for illustrative purposes that the economy consists only of two workers, A and B, as well as two firms, 1 and 2. Over time, both workers happen to work at both firms. Firm 1 pays a higher wage to worker A while firm 2 pays a higher wage to worker B. The two within-firm rankings are thus inconsistent but the fact that we observe both workers at both firms means that the two rankings can be compared statistically and aggregated up.

The computational algorithm used by HLM effectively maximizes the likelihood of the correct global ranking of workers as proven by Kenyon-Mathieu/Schudy (2007). The input of the algorithm are the workers' residual wages, $\tilde{w}_{it}$, that is, wages net of observables as presented in Section 3.3. The algorithm is initialized by ranking workers according to a simple wage statistic which needs to be monotonically increasing in the unobserved worker type.[96] Using a Bayesian approach with a normal prior, HLM show how to compute the probability of worker i being ranked higher than worker j given wage histories at firm k in the presence of measurement error:

$$c(i,j) = P(\tilde{w}_{i,k} > \tilde{w}_{j,k}) = \Phi\left(\frac{\bar{\tilde{w}}_{i,k} - \bar{\tilde{w}}_{j,k}}{\frac{\sigma^2}{n_{i,k}} + \frac{\sigma^2}{n_{j,k}}}\right). \tag{A.15}$$

Φ is the standard Normal CDF. Observed (residual) wages are assumed to follow a noisy process: $\tilde{w}_{i,k,t} = \tilde{w}_{i,k} + \epsilon_t$, with σ^2 being the variance of ϵ. Intuitively, the difference of the average residual wages $\bar{\tilde{w}}_{i,k} - \bar{\tilde{w}}_{j,k}$ at firm k is weighted by the wage variance σ^2 in proportion to the number of wage observations for workers i and j at firm k, $n_{i,k}$ and $n_{j,k}$. The more available observations, the smaller is the potential impact of measurement error on the average wage of worker i at firm k and the more plausible is the ranking implied by the wage observations at this firm, resulting in a higher value of $c(i,j)$.[97] Note that σ^2 is the overall wage variance and not firm-specific because HLM make the assumption that all variation in

[96] HLM prove that, in the context of their model, the reservation wage, the maximum wage, and the adjusted average wage of a worker are monotonically increasing in the unobserved type. Importantly, average wages, sometimes used to rank workers in empirical applications, are not monotonically increasing in the type because they do not factor in the values of workers' interjacent unemployment spells.

[97] For details of the derivation of $c(i,j)$, see Appendix III.1 in Hagedorn/Law/Manovskii (2017).

wages for a specific job stems from measurement error only.[98] The probability $c(i,j)$ is defined for worker pairs employed at the same firm. In case a pair of workers is observed at more than one firm, the wage observations are considered to be independent and the probabilities are simply multiplied. By comparing the initial ranking with the ranking implied by the posterior probabilities $c(i,j)$, the algorithm iteratively increases the value of the objective function and, hence, maximizes the likelihood of the global ranking:

$$\sum_{i>j} \left[c(i,j)\, \Pi(i,j) + c(j,i)\, \Pi(j,i) \right]. \tag{A.16}$$

$\Pi(i,j)$ ($\Pi(j,i)$) is an indicator function that takes on the value 1 in case worker i (j) is ranked higher than j (i) and 0 otherwise. Whenever $c(i,j) > c(j,i)$ but $\Pi(i,j) = 0$ and $\Pi(j,i) = 1$, the values of the indicator functions are swapped and the value of the objective increases. The procedure continues until no further swap of workers increases the value of the objective. It runs on the set of worker pairs who are employed by the same firm at some point in time. The employment spells do not have to overlap.[99] We choose the "LIAB Mover Model" version of German matched employer-employee data because the sampling procedure maximizes the numbers of observed coworker pairs in our data, an ideal environment for the outlined computational procedure to run. Importantly, we do not need to observe all workers of a given establishment to compute $c(i,j)$. The pairwise comparison of residual wages of two workers at the same firm is not affected by a potential wage premium (or firm-fixed effect) because both workers receive it.

D. Details of Data Preparation

D.1. Sampling

The sampling of the LIAB Mover Model data set is based on the IAB Establishment Panel. In the first step, establishments are selected that employ at least one employee who is also employed by at least one other surveyed establishment of the IAB Establishment Panel at some point in time. In the second step, up to 500 additional employees per establishment are drawn randomly. The sampling procedure includes a robustness check regarding the number of employees in a certain establishment, i.e. whenever the information in the IAB Establish-

[98] While this assumption is consistent with period-by-period wage bargaining in their model, from an empirical perspective it could be desirable to allow for heterogeneity of the within-firm wage distributions beyond the mean. Imagine a firm using different contracts to discriminate between worker types: two workers could have different slopes in their wage profile over time because tenure is remunerated differently. Such patterns could be due to history dependence, as evidenced by Bauer/Lochner (2019), or due to the coexistence of wage bargaining and wage posting, as evidenced by Gartner/Holzner (2015) (both for Germany). Ranking the two workers based on their mean wage in this setting might not yield the correct ranking. In contrast, the k-means clustering technique proposed by Bonhomme/Lamadon/Manresa (2019) allows for heterogeneity of within-firm wage distributions even beyond the second moment. However, the computational complexity of this method increases quickly with the number of moments to be estimated, hence the number of clusters/types is limited. The HLM method, in turn, allows for (almost) unique worker and firm ranks. The researcher faces a trade-off: to allow for more heterogeneity of the within-firm wage distributions, the number of types to be identified must be smaller.

[99] Recall that residual wages are deflated and net of time effects.

ment Panel survey data deviates by more than 50 percent from the information in the register data the establishment is excluded.

D.2. Education Imputation

The employee education information is reported by employers after every year and whenever a job ends. Its quality may suffer because employers do not face consequences for non- and misreporting. However, the existence of a reporting rule allows for corrections. It prescribes that only the highest educational degree of an employee needs to be reported. Therefore the individual educational attainment should not decline over consecutive job spells. The imputation procedure (IP1) suggested by Fitzenberger/Osikominu/Völter (2006) exploits this reporting rule by assuming that there is any over-reporting in the data.

The original education variable distinguishes the following four different educational degrees: high school, vocational training, technical college and university. By imputing following the IP1 procedure we extrapolate both back and forwards and do some additional adjustments using individual information on age and occupational status. As a result we get six education categories which can be ranked in increasing order. However, we still observe missing entries of about 2 percent of the initial data after imputation. We drop these observations because we simply cannot make any statement about their true educational background.

D.3. Wage Imputation

In the LIAB data earning are right censored at the contribution assessment ceiling ('Beitragsbemessungsgrenze'). We use the pension insurance of workers and employees. This earning limit is given by the statutory pension fund and is adjusted annually due to changes in earnings. First we deflate daily wages by using the CPI with base year 2005. Then we identify censored wage observations by comparing wages with the contribution assessment ceiling. We define a wage observation as censored whenever the reported wage is higher than 99 percent of the censoring threshold. On average about 13 percent among all wage observations are censored according to our definition.

Following Dustmann/Ludsteck/Schönberg (2009), we fit a series of Tobit regression on age-education-year-combinations to impute the right tail of the wage distribution. In all regressions we control for eight five-year age-categories, six education categories, and all possible interactions. This assumes that the error term in the Tobit regression is normally distributed but and each education and age category can have different variance. For each year, we impute censored wages as the sum of the predicted wage and a random component which is computed based on standard error of the forecast. This component is drawn from separate normal distributions with mean zero and the different variances for each education and age category. Table D.1 shows moments of the imputed wage distributions compared to the censored wage distribution. Table D.2 shows additional wage variance decompositions that follow from running the wage regression (2) either without top-coded wages (Panel (a)) or with additional occupational controls (Panel (b)).

Table D.1: Summary Statistics of the Wage Distribution (1998-2008)

	Mean	Std. Dev.	Min	Max
Censored	4.582	0.393	2.411	5.153
Imputed	4.618	0.455	2.411	7.132

Notes: Summary statistics of the distribution of daily real log wages. Source: LIAB.

Table D.2: Additional Variance-Covariance Matrices

(a) Without Top-Coded Wages

	$\ln w_{it}$	$x'_{it}\hat{\gamma}$	$\hat{\alpha}_i$	$\hat{r}_{it}$
$\ln w_{it}$	0.126			
$x'_{it}\hat{\gamma}$	0.006	0.005		
$\hat{\alpha}_i$	0.091	0.002	0.089	
$\hat{r}_{it}$	0.029	0.000	0.000	0.029

Notes: Variance-Covariance matrix of regression model 2 without imputation of the censored part of the wage distribution. Top-coded wages are dropped. The variance of log wages ($\ln w_{it}$) is decomposed into the variance of observable characteristics ($x'_{it}\hat{\gamma}$), the person-fixed effect ($\hat{\alpha}_i$), and the residual ($\hat{r}_{it}$). Rounded to three decimal places. Source: LIAB.

(b) Including Occupational Controls

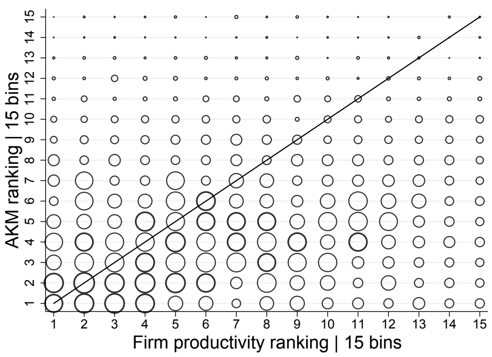
	$\ln w_{it}$	$x'_{it}\hat{\gamma}$	$\hat{\alpha}_i$	$\hat{r}_{it}$
$\ln w_{it}$	0.207			
$x'_{it}\hat{\gamma}$	0.031	0.016		
$\hat{\alpha}_i$	0.143	0.014	0.128	
$\hat{r}_{it}$	0.034	0.000	0.000	0.034

Notes: Variance-Covariance matrix of regression model 2 with 32 additional occupational controls, interacted with education and time effects. The variance of log wages ($\ln w_{it}$) is decomposed into the variance of observable characteristics ($x'_{it}\hat{\gamma}$), the person-fixed effect ($\hat{\alpha}_i$), and the residual ($\hat{r}_{it}$). Rounded to three decimal places. Source: LIAB.

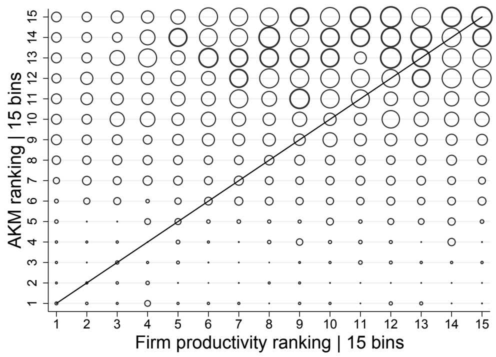
E. Additional Results

Figure E.1: Comparison of Productivity-based Firm Ranking and AKM-based Firm Ranking (Firm Effects) by Wages, Age, and Size

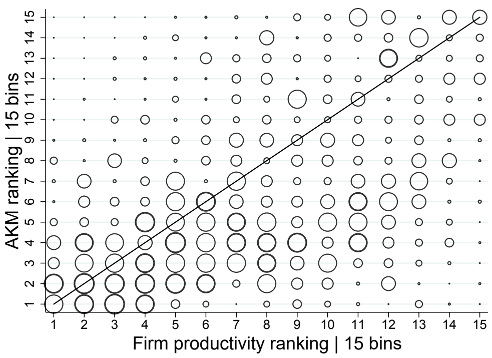
(**a**) Low-wage firms



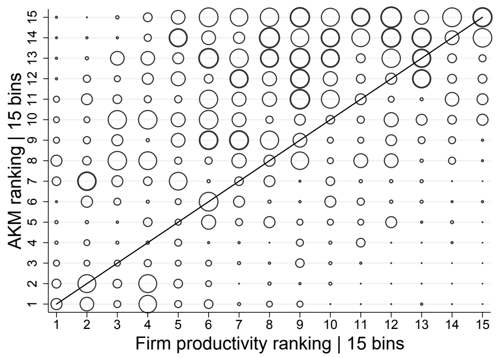
(**b**) High-wage firms



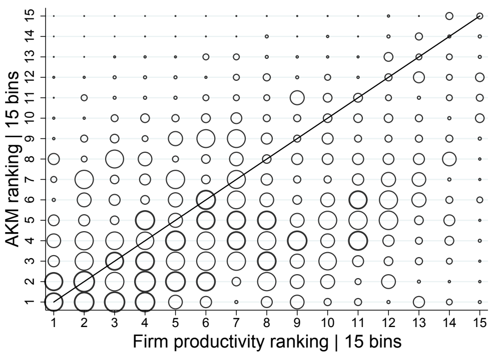
(**c**) Young firms



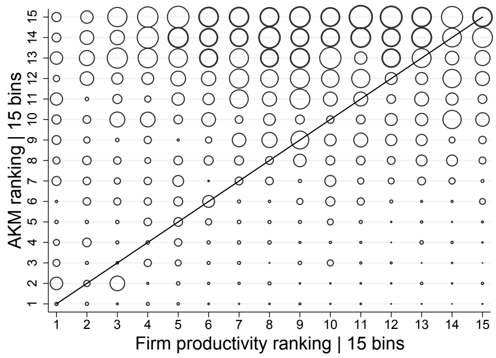
(**d**) Old firms



(**e**) Small firms



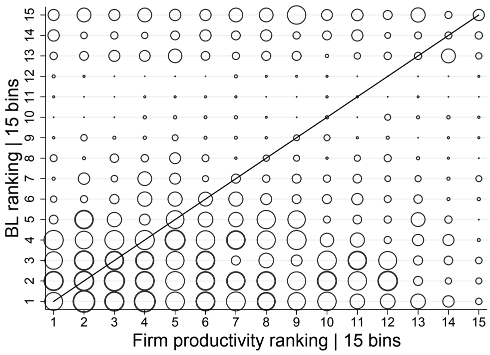
(**f**) Large firms



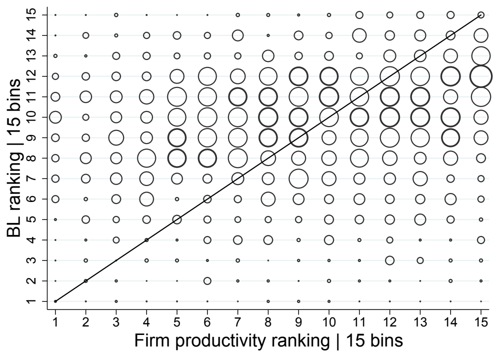
Notes: The size of the circles is proportional to the number of matches in which the firm has the respective combination of the AKM-CHK firm effect (15 bins) and our estimated productivity-type (15) bins. In Panels (a) and (b), high-wage firms pay more than the grand mean of all firm-level mean wages and low-wage firms pay less. In Panels (c) and (d), the age of young firms is less than 15 years, old firms are 15 years and older. In Panels (e) and (f), small firms have less than 100 employees, large firms have more. Source: BHP, EP, IEB.

Figure E.2: Comparison of Productivity-based Firm Ranking and BL-based Firm Ranking (Poaching Rank) by Wages, Age, and Size

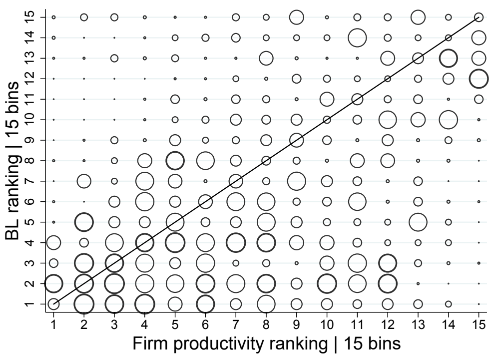
(a) Low-wage firms



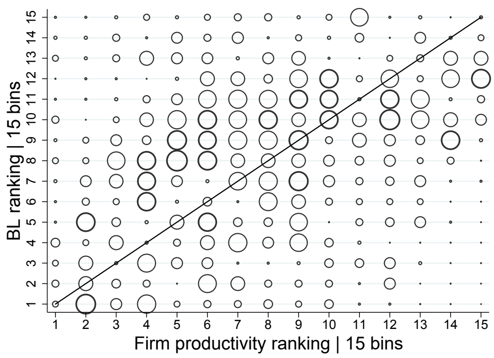
(b) High-wage firms



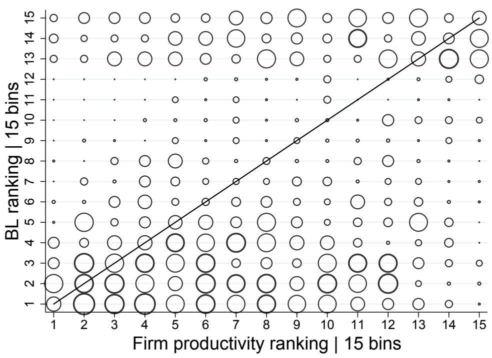
(c) Young firms



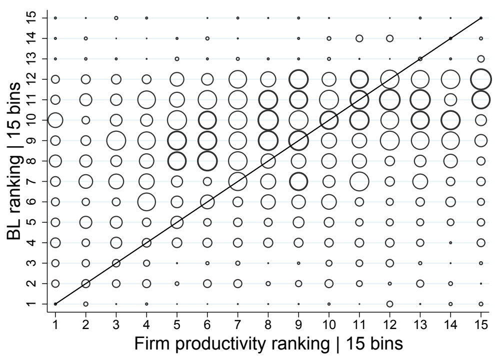
(d) Old firms



(e) Small firms



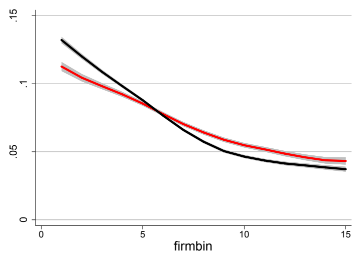
(f) Large firms



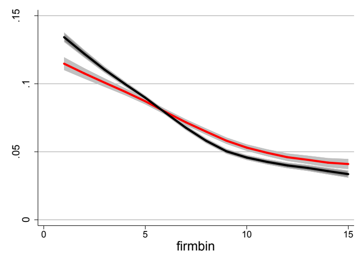
Notes: The size of the circles is proportional to the number of matches in which the firm has the respective combination of the BL poaching index (15 bins) and our estimated productivity-type (15) bins. In Panels (a) and (b), high-wage firms pay more than the grand mean of all firm-level mean wages and low-wage firms pay less. In Panels (c) and (d), the age of young firms is less than 15 years, old firms are 15 years and older. In Panels (e) and (f), small firms have less than 100 employees, large firms have more. Source: BHP, EP, IEB.

Figure E.3: Estimated Density Functions, Distribution of Worker Types across Firm Bins, All Matches: 1998-2002 (red) vs. 2003-2008 (black)

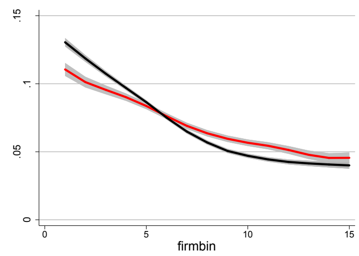
(a) Worker Bin 1



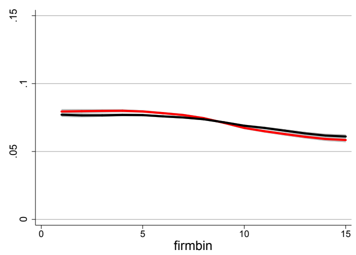
(b) Worker Bin 1, out of Unemp.



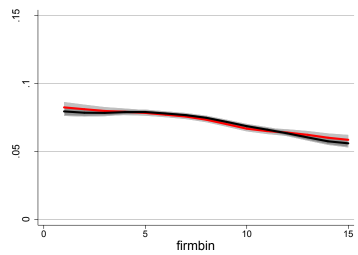
(c) Worker Bin 1, Job-to-Job



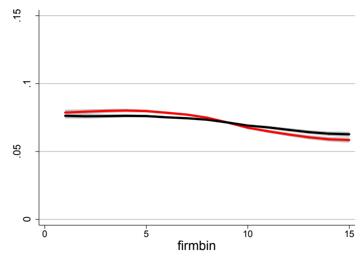
(d) Worker Bin 10



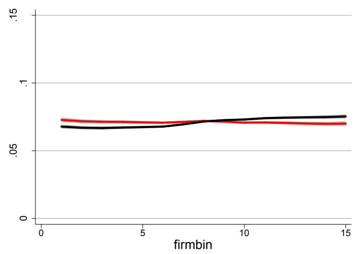
(e) Worker Bin 10, out of Unemp.



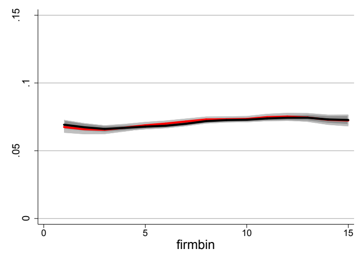
(f) Worker Bin 10, Job-to-Job



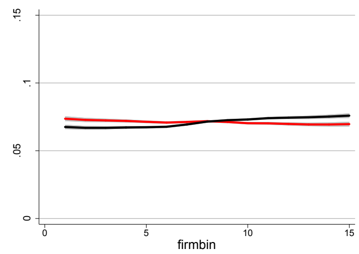
(g) Worker Bin 25



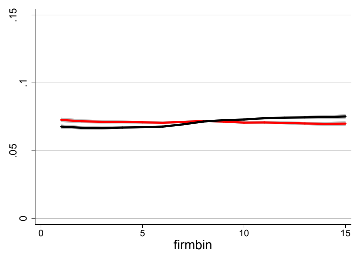
(h) Worker Bin 25, out of Unemp.



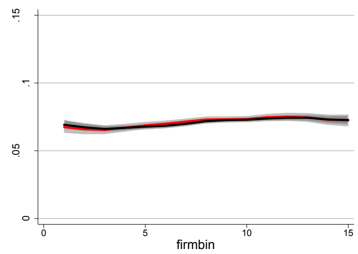
(i) Worker Bin 25, Job-to-Job



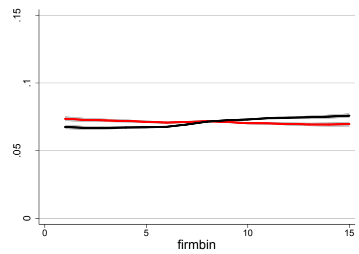
(j) Worker Bin 40



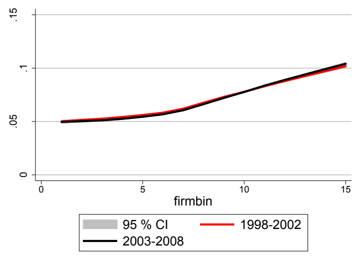
(k) Worker Bin 40, out of Unemp.



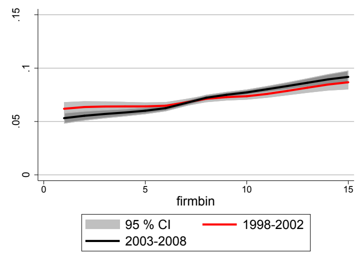
(l) Worker Bin 40, Job-to-Job



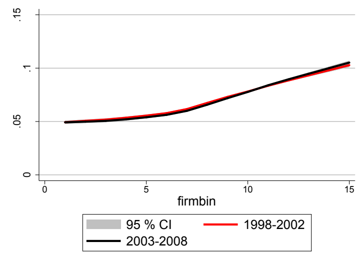
(m) Worker Bin 50



(n) Worker Bin 50, out of Unemp.



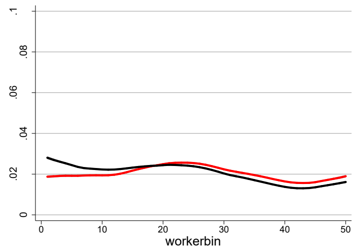
(o) Worker Bin 50, Job-to-Job



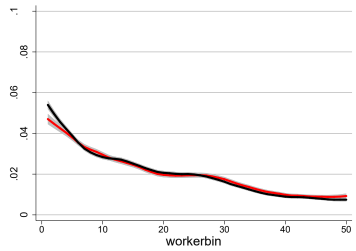
Notes: Estimated univariate kernel densities of all matches conditional on worker bins, time, and match type. The kernel is estimated using an Epanechnikov kernel function. The bandwidth is calculated by Silverman's rule of thumb. Pointwise confidence intervals are calculated using a quantile of the standard normal distribution. Source: BHP, EP, IEB, LIAB.

Figure E.4: Estimated Density Functions, Distribution of Firm Types across Worker Bins, All Matches: 1998-2002 (red) vs. 2003-2008 (black)

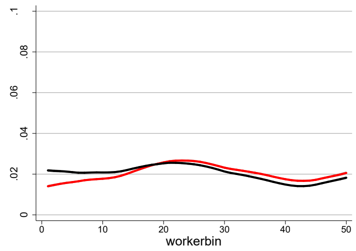
(a) Firm Bin 1



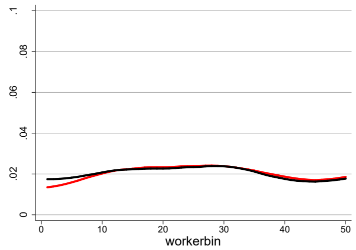
(b) Firm Bin 1, out of Unemp.



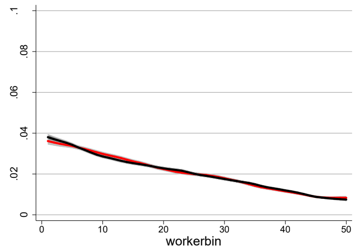
(c) Firm Bin 1, Job-to-Job



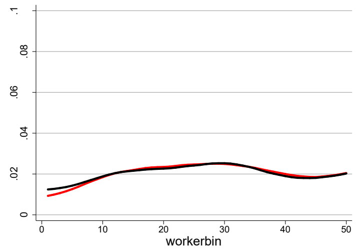
(d) Firm Bin 4



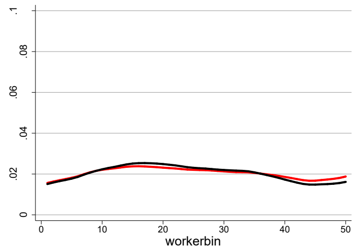
(e) Firm Bin 4, out of Unemp.



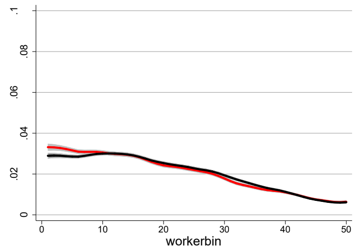
(f) Firm Bin 4, Job-to-Job



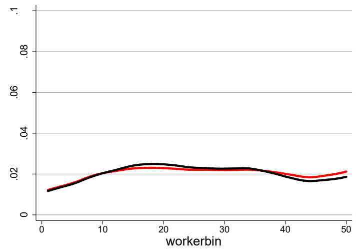
(g) Firm Bin 8



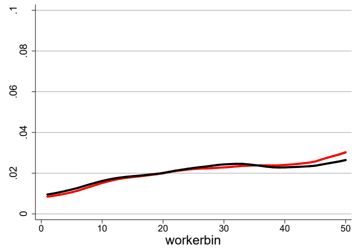
(h) Firm Bin 8, out of Unemp.



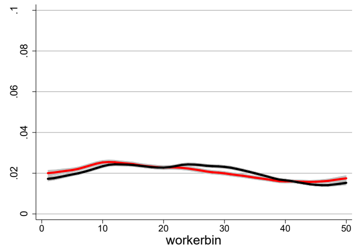
(i) Firm Bin 8, Job-to-Job



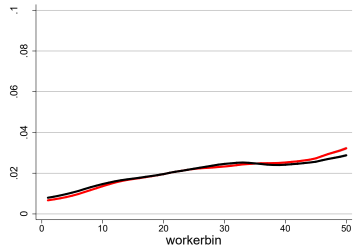
(j) Firm Bin 12



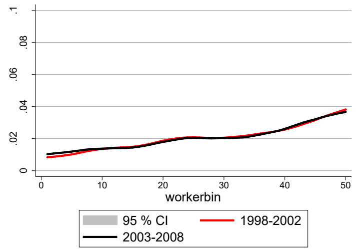
(k) Firm Bin 12, out of Unemp.



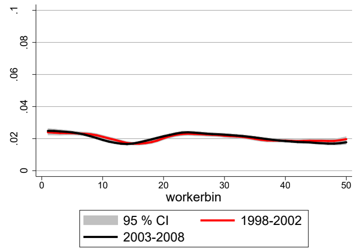
(l) Firm Bin 12, Job-to-Job



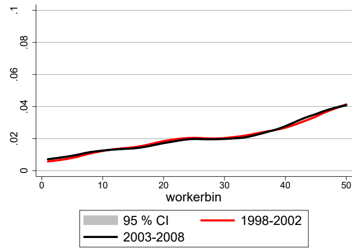
(m) Firm Bin 15



(n) Firm Bin 15, out of Unemp.



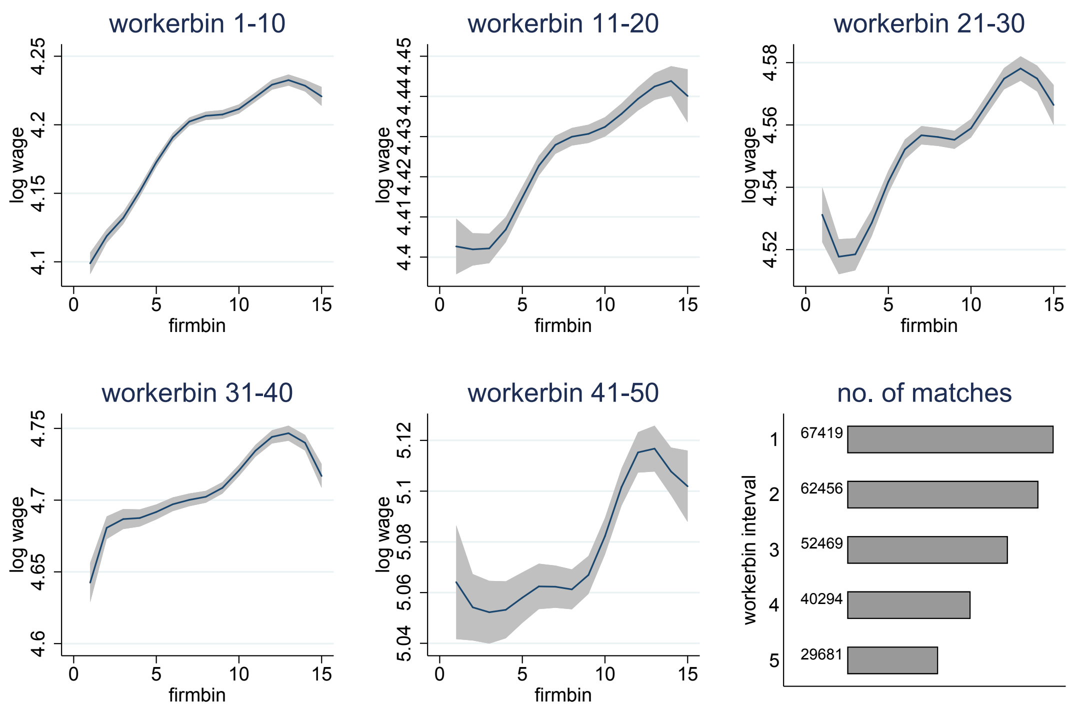
(o) Firm Bin 15, Job-to-Job



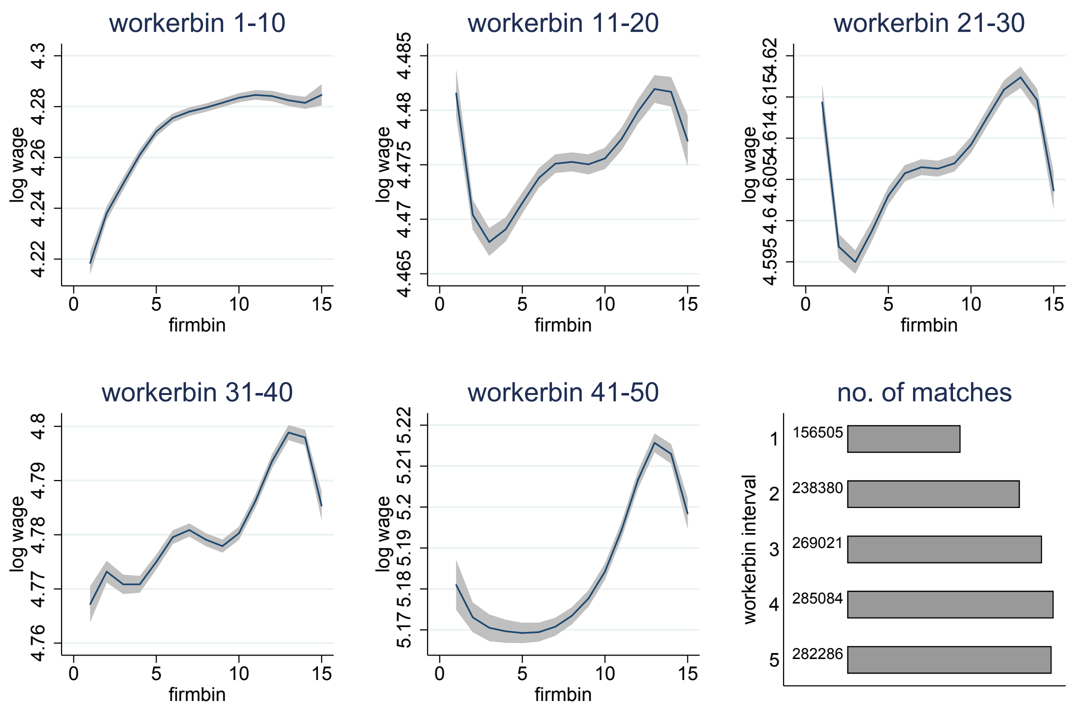
Notes: Estimated univariate kernel densities of all matches conditional on worker bins, time, and match type. The kernel is estimated using an Epanechnikov kernel function. The bandwidth is calculated by Silverman's rule of thumb. Pointwise confidence intervals are calculated using a quantile of the standard normal distribution. Source: BHP, EP, IEB, LIAB.

Figure E.5: Mean Wages across Worker Types, Out of Unemployment vs. Job-to-Job, All Matches

(a) Out of unemployment



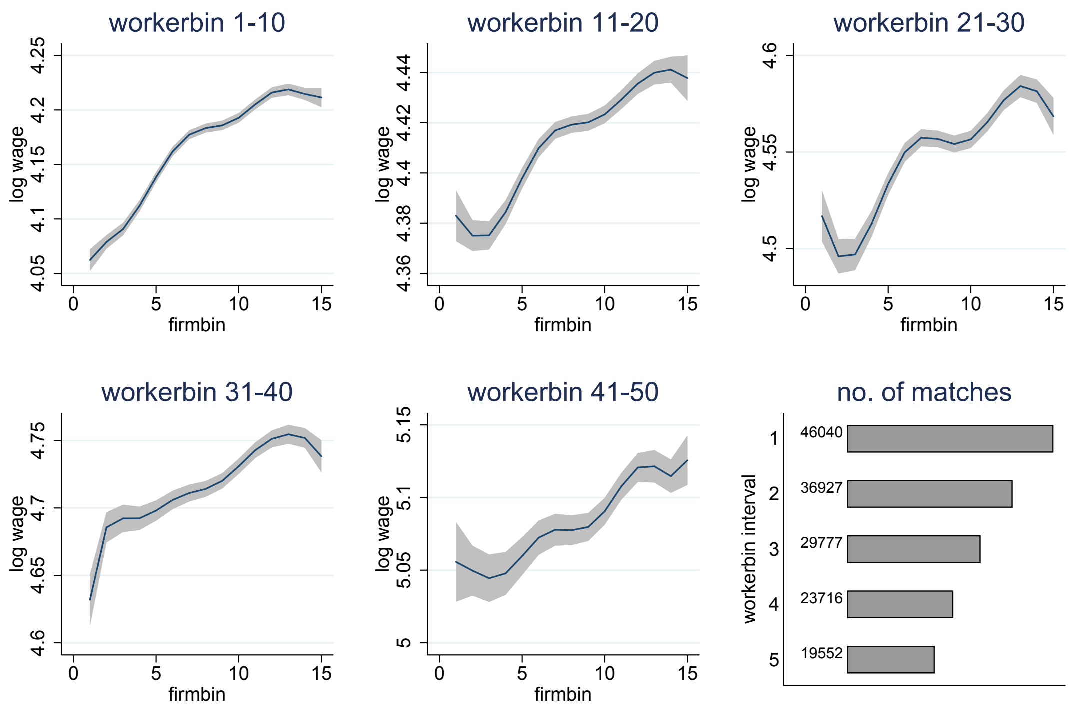
(b) Job-to-Job



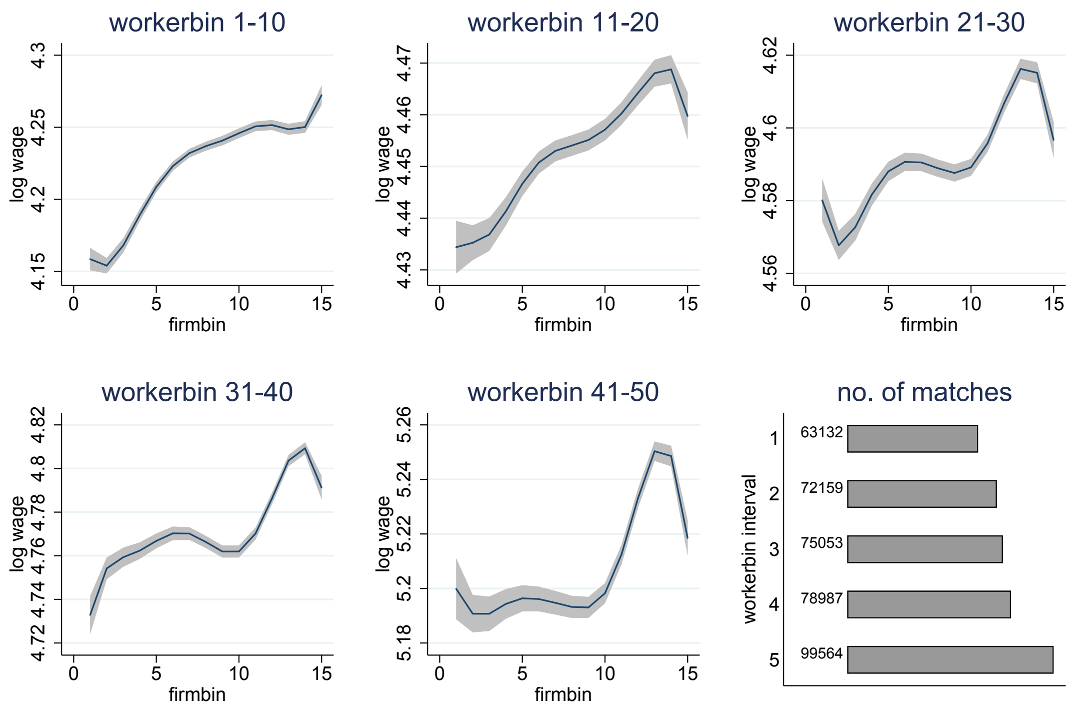
Notes: Plots show estimated wage profiles across firm bins for all matches out of unemployment and job-to-job. The kernel is estimated using an Gaussian kernel function. The bandwidth is 2. 95 percent confidence bands in gray. Source: BHP, EP, IEB, LIAB.

Figure E.6: Mean Wages across Worker Types, Out of Unemployment vs. Job-to-Job, New Matches

(**a**) Out of unemployment



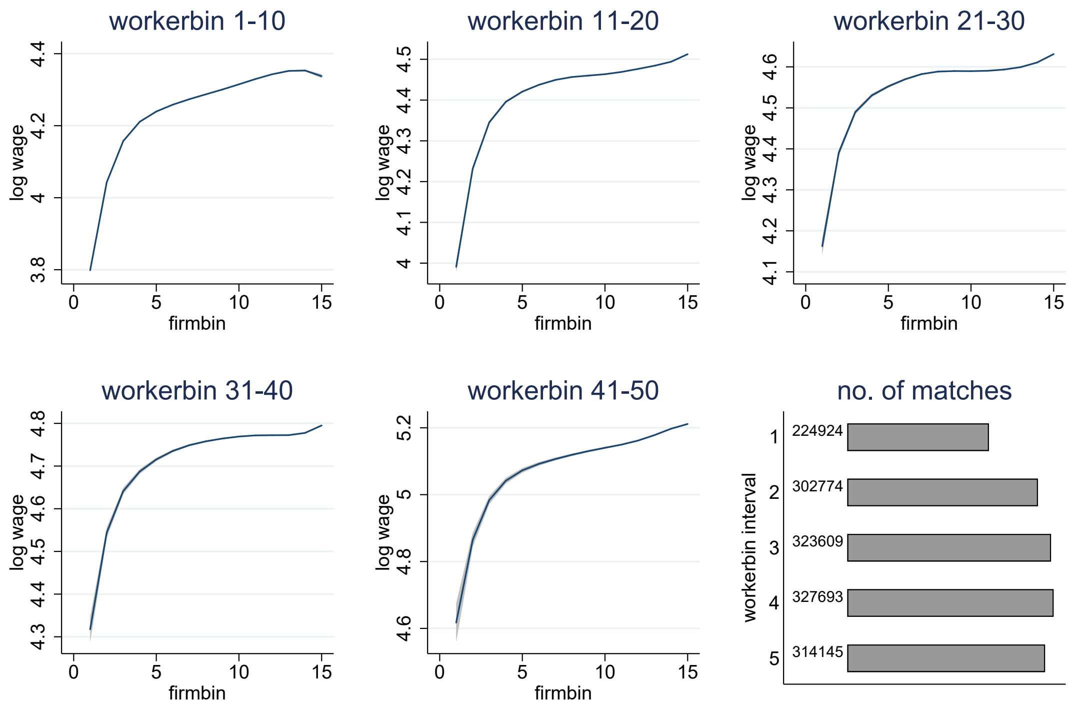
(**b**) Job-to-Job



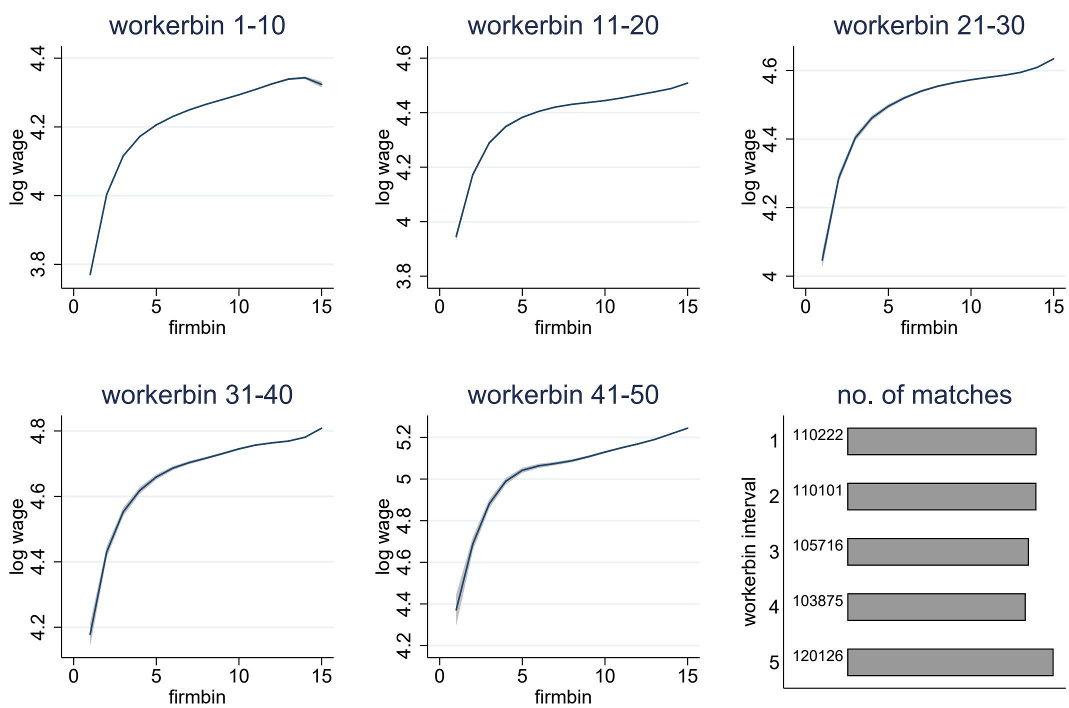
Notes: Plots show estimated wage profiles across firm bins for new matches out of unemployment and job-to-job. The kernel is estimated using an Gaussian kernel function. The bandwidth is 2. 95 percent confidence bands in gray. Source: BHP, EP, IEB, LIAB.

Figure E.7: Mean Wages across Worker Types with AKM-based Firm Ranking

(**a**) All Matches

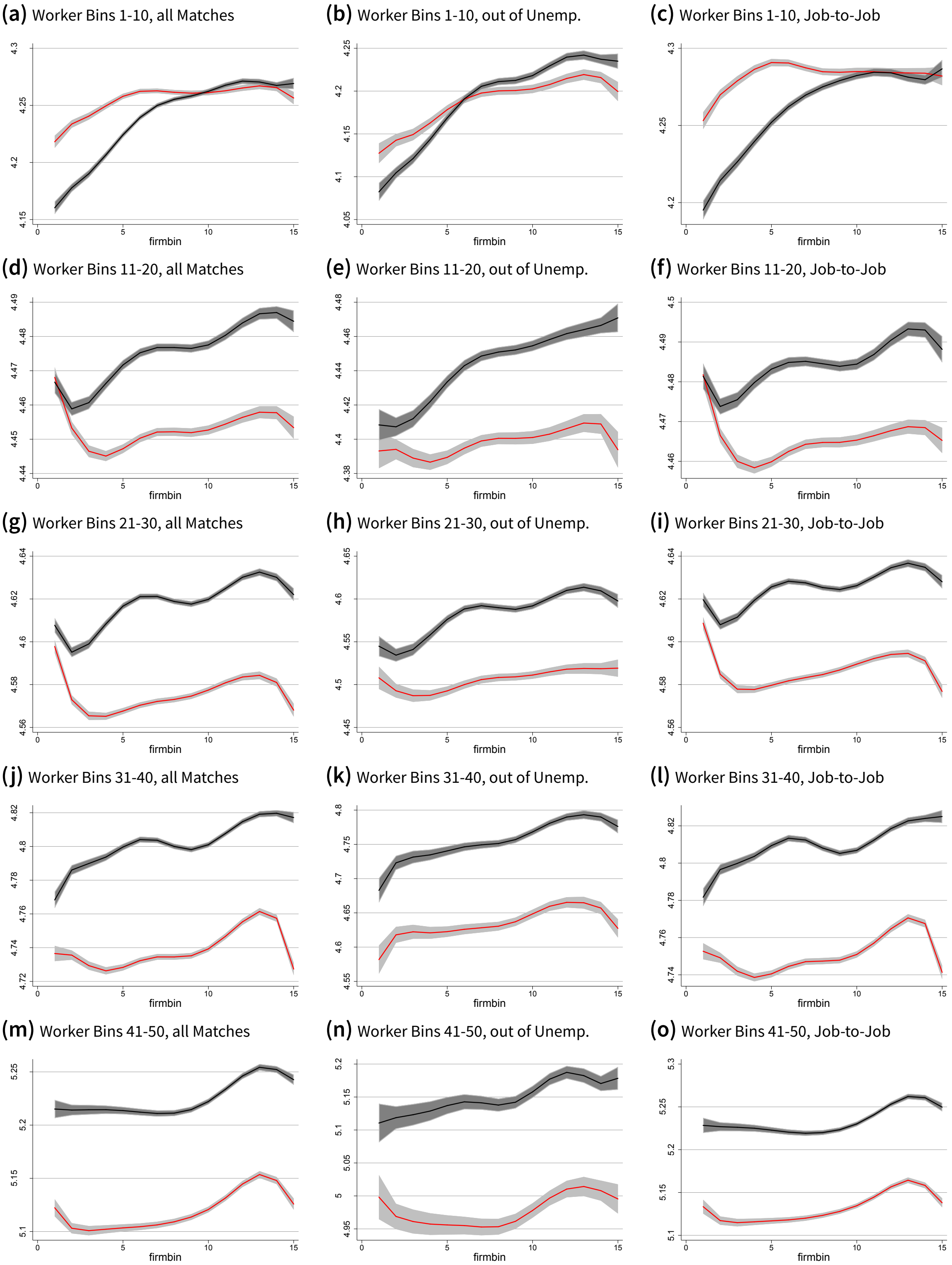


(**b**) New Matches



Notes: Plots show estimated wage profiles across firm bins constructed using AKM-CHK firm effects for all matches and new matches. The kernel is estimated using an Gaussian kernel function. The bandwidth is 2. 95 percent confidence bands in gray. Source: BHP, EP, IEB, LIAB.

Figure E.8: Wages across Firm Bins, All Matches: 1998-2002 (red) vs. 2003-2008 (black)



Notes: Plots show estimated wage profiles across firm bins and over time for all matches. The kernel is estimated using an Gaussian kernel function. The bandwidth is 2. 95 percent confidence bands in gray. Source: BHP, EP, IEB, LIAB.

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