The productivity effect of migrants
Wage cost advantages and heterogeneous firms

Michael Lucht
Anette Haas

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Michael Lucht (IAB)
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Abstract

Empirical evidence for the US shows that migrants increase the productivity of regions. To explain the impact of migrants on the average firm productivity we construct a general equilibrium model with monopolistic competition a la Melitz (2003). We consider heterogeneous firms with different productivity levels and imperfect substitutability between migrants and natives. This gives rise to wage differences between natives and migrants. As a consequence, firms with a higher share of migrants realize wage cost advantages. The heterogeneous distribution of migrants in our model fosters regional disparities. In equilibrium, it depends on the migrant share which kind of firms survives in the market. The only firms to stay in the market are those which are highly productive or able to compensate a lower productivity level through wage cost advantages. We show that a higher migrant share may explain a higher average productivity in a region. The welfare effects for natives are ambiguous.

Keywords: immigration, firm heterogeneity, skills, tasks, regional labour markets

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1 Introduction

Recently, Peri (2012) shows that there is a significant positive effect of immigration on the productivity of US States. What are the implications on immigration policy? What are the boundaries for this increase in productivity? To answer such questions, it is necessary to understand the underlying mechanism that drives the effect. This paper develops a general equilibrium framework to explain the productivity effect of immigrants.

Peri conjectures that the increase may be caused by a higher grade of specialization: on average, migrants focus on manual jobs and thus push natives into more productive jobs. The explanation lacks a general equilibrium foundation. Why did the natives not switch to more productive jobs before the immigration shock? Workers react on wage changes on the labour market. If different equilibria at the labour market cause the reallocation of workers, it is doubtful that the individual firm productivity changes. Instead, we show that migrants push natives into more productive firms, as their presence changes the conditions of the inter-firm competition.

We extend the heterogeneous firm model of Melitz (2003) so that firms differ in their relative demand for migrant labour in their workforce. It reflects that some firms may be more successful in integrating migrants because of their concept, e.g. fast-food restaurants like McDonalds, or the presence of a migrant entrepreneur, or product heterogeneity generally causing differences in the tasks needed in production. The demand differences imply that firms react differently on an immigration shock: Migrant-intense firms gain a competitive advantage and are thus pushing less migrant-intense low-productivity firms out of the market. The average firm productivity increases if the number of low-productivity firms that are pushed out of the market is higher than the number of previously unprofitable firms that are able to remain in the market after the shock. We show in a simulation that plausible distributions of firm parameters lead to an increase in productivity as a reaction to a migration shock.

A key mechanism in our model is the wage reaction of the economy to an immigration shock. Recently, many researchers found no or even positive effects of immigration on the wages of natives (Card 2001, 2009a, 2009b; Brücker/Jahn 2011; Südekum et al. 2014; D’Amuri/Ottaviano/Peri 2010). One explanation is that migrants do not compete with natives on exactly the same jobs. This can be interpreted as imperfect substitutability. Several studies use this assumption to estimate the elasticity of substitution between migrants and natives (Card/Lemieux 2001; Card 2009a; D’Amuri/Ottaviano/Peri 2010; Brücker/Jahn 2011; Haas/Lucht/Schanne 2013). Card (2009a) notes that both cross-section and time-series estimations are consistent with imperfect substitutability with a high but finite elasticity of substitution. While these studies are conducted at the aggregate level, Martins/Piracha/Varejão (2012) analyze the timing of recruitment and layoff of migrants and natives at the individual firm level. They find that, if anything, migrants are complements, not substitutes.
The heated debate on the implications of immigration on natives’ wages (e. g. Borjas 2003; Card 2009a) encouraged economists to search for more refined answers on the matter of labour market absorption of immigrants. Gonzales/Ortega (2011) find that industries adapt to changes in the skill mix. Dustmann/Glitz (forthcoming 2015) investigate effects of changes in local labour supply on firms and identify the creation and destruction of firms as an important economic channel of adjustment. Our model shows that the magnitude of the effects depends on the intra- and inter-firm adaptions to the changes in the labour market.

We simulate both a migration shock and cross-region comparison. In the latter, no scale effects occur. We compare different specifications on how much firms adapt to changes in the labour supply. Productivity effects are stronger the less adaptation takes place within firms.

The rest of this paper is organized as follows: The next two sections describe the theoretical model and show simulation results with different productivity levels. The fourth section covers the welfare analysis and the fifth section concludes with positive welfare effects for natives.

2 Model description

Our framework builds on the closed-economy setup of the Melitz (2003) trade model. Our contribution is that we model the production process in a less simple manner: Migrants are imperfect substitutes for natives. Furthermore, we extend the basic framework by allowing for differences in the demand for each kind of labour. Some firms are more likely to employ migrants than others. We simulate the model for several different parameter values to show that it replicates the relative wage and productivity effects of immigrants and to demonstrate their implications for the welfare of the workers. The next paragraphs explain our choices and discuss some of the results.

Labour market and firm production

The only difference among the workers in our model is whether they are migrants or natives. To replicate the empirical results (see introduction) on the reaction of wages to a change in labour supply we assume a CES (constant elasticity of substitution)-production function framework similar to Card/Lemieux (2001), where migrants and natives are imperfect substitutes. The assumption is related to Peri/Sparber (2009) who find that migrants tend to choose different types of jobs than natives. They argue that migrants may have a comparative advantage in manual jobs. Unlike Card/Lemieux (2001), the production function is used at the firm level. In a CES-production function framework, firms need both types of labour to produce the final good. There are three parameters to the production function: the total factor productivity, the relative factor productivity and the elasticity of substitution.

The relative factor productivity determines how much a firm uses each type of labour. If migrants pick other jobs than natives this may be due to the production of
some products requiring a more intensive use of those jobs. So, if the migrant share is relatively high in manual production jobs, the relative migrant productivity of a manufacturing firm might be higher than that of a sales firm, which mostly offers jobs where interaction with customers is important. Firms are very different, e.g. some manufacturing firm might use machines intensively, so that most jobs in that firm are office jobs. Even firms producing similar products may have different relative productivity parameters and thus their profit reacts differently to relative changes in the labour supply.

Within the CES-production framework an increase in the relative supply of one group decreases the relative wage of that group, while the relative wage of the other group increases.

A firm that intensively uses the work of migrants therefore gains an advantage relative to other firms from immigration. The elasticity of substitution determines how much a firm is able to react on changes in the supply. There are two extreme cases: an elasticity of 0 and an infinite elasticity of substitution. The first is called the Leontief case, and it implies that the migrant share equals the relative productivity parameter. Firms can only react on changes in the labour supply by increasing or decreasing overall production. The higher the elasticity, the more firms can react on supply changes by changing the share of migrants in the firm. With an infinite elasticity, migrants and natives are perfect substitutes.

Lastly, there is the total factor productivity which varies between firms. This assumption reflects heterogeneity in entrepreneurial decisions.

Firm competition
There is heterogeneity between the firms regarding the total factor productivity and the relative factor productivity. It is modelled similar to Melitz (2003) with the assumption that the firm founding process involves risk. What our model adds is the heterogeneity in the relative factor productivity.

3 A Heterogeneous Firm Model with Wage Cost Advantages – Basic Framework

3.1 Households and final goods production
The households maximize utility by the consumption of a final good. They supply labour inelastically and do not save money. The aggregated amount of the final good is named $Q$ and is sold at price $P$. The aggregate production function of the perfectly competitive final goods sector is a CES-aggregate over a continuum of intermediate product varieties indexed by $\omega \in \Omega$:
\[ Q = \left( \int_{\omega} q(\omega)^\rho d\omega \right)^{1/\rho} \]  

(1)

with \( 0 < \rho < 1 \) and an elasticity of substitution between intermediate firms \( \sigma = \frac{1}{1-\rho} > 1 \). As in the well-known Dixit-Stiglitz model of monopolistic competition, the optimal demand for intermediate (product) variety \( \omega \) is then

\[ q(\omega) = Q \cdot P^\sigma \cdot p(\omega)^{-\sigma} \]  

(2)

with aggregate price index \( P = \left( \int p(\omega)^{1-\sigma} d\omega \right)^{1/(1-\sigma)} \) and firm-individual price \( p(\omega) \).

### 3.2 Intermediate firm behaviour

The labour force consists of migrants and natives. Firms have to choose the composition of their workforce and the firm size at the same time. Firms use the well-known CES-production function

\[ q_f = A_f \cdot \left( \beta_f^\gamma \cdot \left( l_1^f \right)^{\gamma-1} + (1 - \beta_f^\gamma) \cdot \left( l_2^f \right)^{\gamma-1} \right)^{\gamma^{-1}}, \]  

(3)

where \( \gamma > 0 \) is the elasticity of substitution between migrants and natives, \( q_f \) is the output, \( A_f \) the total factor productivity, \( \beta_f \in [0,1] \) is the parameter of intensity for the use of migrants in the workforce, and \( l_1^f, l_2^f \) are the labour demands for jobs one and two of the firm \( f \), respectively. Details on the derivation of the relevant equations in the CES-setting are explained in appendix A.

The production function is more complicated than in the Melitz (2003) model, but with the inverse marginal costs to produce one unit of an intermediate good

\[ \phi_f = \frac{A_f}{(\beta_f \cdot w_1^{1-\gamma} + (1 - \beta_f) \cdot w_2^{1-\gamma})^{1-\gamma}}, \]  

(4)

some analogous results can be deducted. In this expression \( w_1 \) is the wage of migrants (in job 1) and \( w_2 \) the wage of natives (in job 2). Note that, unlike the Melitz (2003) model, the marginal costs depend on the labour market equilibrium.

The factor \( \beta_f \) is drawn stochastically at the moment of firm formation from a known distribution \( G(\cdot) \) with density \( g(\cdot) \). This parameter determines the affinity of a firm to employ migrants: the larger \( \beta_f \), the more migrants the firm will hire in equilibrium.

We assume in this paper that the wage \( w_2 \) for natives exceeds the wage for migrants, \( w_1 \). Then a higher draw of the affinity to hire migrants, \( \beta_f \), implies lower mar-
ginal costs. This fact is important in this model as the firm survival only depends on
the marginal costs.

The firm founding involves sunken entry costs \( e \) which are paid in natives' labour.
Furthermore, every year the firm may incur a negative productivity shock with prob-
ability \( \delta \) that forces it to instantly leave the market. Additionally, there are per-period
fixed costs \( F \) which are also paid in native labour.

Profit maximization implies the individual firm pricing behaviour

\[
p(\phi_f) = \frac{1}{\rho \cdot \phi_f}
\]

(5)

As in Melitz (2003), the prices \( p \), quantities \( q \) and revenues \( r \) of two firms with in-
verse marginal costs \( \phi_f \) and \( \phi_g \) are related to each other via:

\[
\frac{p(\phi_f)}{p(\phi_g)} = \frac{\phi_g}{\phi_f}; \quad \frac{q(\phi_f)}{q(\phi_g)} = \left( \frac{\phi_f}{\phi_g} \right)^\sigma; \quad \frac{r(\phi_f)}{r(\phi_g)} = \left( \frac{\phi_f}{\phi_g} \right)^{\sigma - 1},
\]

(6)

and for the profit of a firm \( \pi \) it holds that:

\[
\pi(\phi_f) = \frac{r(\phi_f)}{\sigma} - w_2 \cdot F.
\]

(7)

From this equation it follows that for given wages \( w_1, w_2 \) the short-run decision of a
firm to remain in the market only depends on the marginal costs of that firm. A firm
immediately leaves the market if profits \( \pi(\phi_f) \) are negative.

3.3 Equilibrium without heterogeneity in total factor productivity

In this part we develop the equilibrium properties of the outlined model while ignor-
ing the heterogeneity of the total factor productivity in this step. Thus, we describe
the mechanism implied by the differences in the job composition. Therefore it holds
that \( A_f = 1 \) for all firms.

As in Melitz (2003), we define a weighted average of the inverse marginal costs for
a symmetric good as:

\[
\bar{\phi}(w) = \left( \int_0^1 \phi(\beta, w_1, w_2) \mu(\beta) d\beta \right)^\frac{1}{\sigma - 1}
\]

(8)

with \( \mu(\beta) \) being the density of firms in the market. \( M \) being the number of
firms the price index \( P \), summed output \( Q \), revenue \( R \) and firm profit \( \Pi \) can then be
stated as:

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\[ P = M^{\frac{1}{1-\sigma}} \cdot p(\phi); \quad Q = M^{\frac{1}{\rho}} \cdot q(\phi) \]

\[ R = P \cdot Q = M \cdot r(\phi) =: M \cdot \tilde{r}; \quad \Pi = M \cdot \pi(\phi) =: M \cdot \tilde{\pi}. \]

So far, the only difference between firms is the share of job 1 in total production. As the wage of migrants by necessity is not higher than the wage of natives, a firm has cost advantages relative to a firm with a lower share of job 1. Therefore, there may be a minimum share parameter \( \beta^* \), as firms with a share parameter of job 1 below this bound are forced to immediately exit the market because they are not able to generate any profit. Such a \( \beta^* \) need not necessarily exist, because it may be the case that even a firm with a share equal to zero is profitable. But if such an \( \beta^* \) between zero and one exists with \( \pi(\beta^*) = 0 \), it holds that

\[ \mu(\beta) = \frac{1}{1 - G(\beta^*)} \cdot g(\beta), \quad \text{for} \quad \beta \geq \beta^*, \quad \mu(\beta) = 0 \quad \text{else.} \]

This leads to the zero-cut-off-condition (using (6) & (7))

\[ \tilde{\pi} = w_2 \cdot F \cdot \left( \left( \frac{\phi(\beta^*)}{\phi^*} \right)^{\sigma-1} - 1 \right), \]

where \( \phi^* = \phi(\beta^*, w_1 w_2) \) is the minimum inverse marginal cost relating to \( \beta^* \), and \( F \) are fixed costs.

In the steady state the profit of a firm is constant over time, so the expected lifetime profit of a new firm is given by:

\[ E(\pi_f^{l(e)}) = -e \cdot w_2 + \sum_{i=0}^{\infty} (1 - \delta)^i \cdot E(\pi_f) = \frac{E(\pi_f)}{\delta} - e \cdot w_2 \]

with entry costs \( e \) and expected per period short-run profit (ignoring entry costs) \( E(\pi_f) \).

The expected per period profit of a firm is given by

\[ E(\pi_f) = 0 \cdot G(\beta^*) + (1 - G(\beta^*)) \cdot \tilde{\pi} \]

and thus the free-entry-condition

\[ \tilde{\pi} = \frac{e \cdot \delta \cdot w_2}{1 - G(\beta^*)} \]

combined with the zero-cut-off-condition lead to ((14) & (11), (8) & (10)): 

\[ \text{(9)} \]
\[
\frac{e \cdot \delta}{F} = \int_\chi(\beta^*) \left( \frac{(\beta^* + (1 - \beta^*) \cdot \frac{w_2}{w_1})^{1 - \gamma}}{(\beta + (1 - \beta) \cdot \frac{w_2}{w_1})^{1 - \gamma}} \right)^{\frac{\sigma - 1}{1 - \gamma}} - 1 \right) g(\beta) d\beta
\]

(15)

with clamp function \( \chi(\beta) = \min(1, \max(0, \beta)) \). In the Melitz (2003) model the zero-
cut-off-condition and free-entry-condition determine the equilibrium. This is not the
case here, because the equation (15) depends on the relative wages. As the right
side is decreasing in \( \beta^* \) and increasing in the relative wage \( \frac{w_2}{w_1} \), the deduced implicit
function \( \beta^* \left( \frac{w_2}{w_1} \right) \) is increasing (see proof in Appendix B). A higher difference in wages therefore implies tighter competition in terms of firms that are forced to exit.

With arguments similar to Melitz (2003), it can be shown that the aggregated revenue of the economy must equal the sum of all wage payments:

\[
R = w_1 L_1 + w_2 L_2.
\]

(16)

The number of firms that remain in the market then is:

\[
M = \frac{R}{\bar{F}} = \frac{w_1 L_1 + w_2 L_2}{\sigma \cdot w_2 \cdot \left( \frac{e \cdot \delta}{1 - G(\beta^*)} + \bar{F} \right)} = \frac{L_1 + \frac{w_2}{w_1} \cdot L_2}{\frac{w_2}{w_1} \cdot \sigma \cdot \left( \frac{e \cdot \delta}{1 - G(\beta^*)} + \bar{F} \right)}
\]

(17)

The native workforce splits up into three parts that are constant in size in the long-
run equilibrium: workers that work to cover the firm entry costs \( L_2^E \), the fixed costs \( L_2^F \) and usual production workers \( L_2^D \). Migrants only work in usual production so that in the equilibrium \( L_1 = L_1^D \). The demand for migrant production workers is (using the partial derivative of the profit maximum with regard to \( l_1 \)):

\[
L_1^D = \frac{1}{1 - G(\beta^*)} \left[ M \cdot l_1(\beta) g(\beta) d\beta \right] = \frac{MQ P^\sigma \rho^\sigma w_1^{-\gamma}}{1 - G(\beta^*)} \cdot \left[ \frac{1}{\beta} \cdot \left( \frac{w_2}{w_1} \right)^{-\gamma} \frac{\gamma - \sigma}{1 - \gamma} g(\beta) d\beta \right]
\]

(18)

and the demand for natives excluding the labour demand to cover fixed cost and market entry costs is

\[
L_2^D = \frac{MQ P^\sigma \rho^\sigma w_2^{-\gamma}}{1 - G(\beta^*)} \cdot \left[ (1 - \beta) \cdot \left( \frac{w_2}{w_1} \right)^{-\gamma} \frac{\gamma - \sigma}{1 - \gamma} g(\beta) d\beta \right].
\]

(19)
Thus, the relative labour demand is given by:

\[
\frac{L_1^D}{L_2^D} = \left( \frac{w_2}{w_1} \right)^{\gamma} \cdot \frac{\int_\beta^1 \beta \cdot \left( \beta + \left( \frac{w_2}{w_1} \right)^{1-\gamma} \cdot (1 - \beta) \right)^{\frac{\gamma-\sigma}{1-\gamma}} g(\beta) d\beta}{\int_\beta^1 (1 - \beta) \cdot \left( \beta + \left( \frac{w_2}{w_1} \right)^{1-\gamma} \cdot (1 - \beta) \right)^{\frac{\gamma-\sigma}{1-\gamma}} g(\beta) d\beta}
\]

(20)

It can be seen that the right side is increasing\(^1\) both in \(\beta^*\) and \(\frac{w_2}{w_1}\). The labour demand to cover fixed costs and market entry cost is given by:

\[
L_{2e}^F = \frac{e \cdot \delta}{1 - G(\beta^*)} \cdot M + F \cdot M,
\]

so that the use of the general equilibrium firm number equation leads to:

\[
\frac{L_1^D}{L_2^D} = \frac{L_1}{L_2 - M \cdot \left( \frac{e \cdot \delta}{1 - G(\beta^*)} + F \right)} = \frac{L_1}{L_2 - \frac{1}{\sigma} \left( \frac{L_1}{w_2} w_1 + L_2 \right)} = \rho - \frac{1}{\sigma} \cdot \frac{L_1}{L_2} \left( \frac{L_1}{w_2} w_1 \right)
\]

(22)

The right side is increasing in \(\frac{L_1}{L_2}\), because the denominator is positive, and decreasing in \(\frac{w_2}{w_1}\). The left side is increasing\(^2\) in \(\beta^*\) and in \(\frac{w_2}{w_1}\), especially considering the monotonically increasing implicit function \(\beta^* \left( \frac{w_2}{w_1} \right)\). Therefore, the implicit function \(\frac{w_2}{w_1} \left( \frac{L_1}{L_2} \right)\) is increasing, which is the result one would expect (as e.g. a relative increase of the supply of migrants leads to a relative decrease of the wage of migrants, and vice versa).

The equations (15), (20) and (22) relate the relative labour supply \(\frac{L_1}{L_2}\) to the labour demand for production \(\frac{L_1^D}{L_2^D}\), the relative wage \(\frac{w_2}{w_1}\) and the cut-off intensity \(\beta^*\). Thus, there are no scale effects in these two values, unlike the number of firms \(M\), as follows from equation (17). The effect is that an increase in the relative supply of migrants increases the wage differences. These differences give an advantage to firms which use a migrant workforce extensively. Firms mostly relying on natives are affected detrimentally. Thus the cut-off increases and a larger share of new firms fail.

Practically, the equations are solved backwards: for a given relative wage \(\frac{w_2}{w_1}\) equation (15) is solved for the unique \(\beta^*\), the result is plugged into (20) to get the unique

---

\(^1\) At least if \(\gamma \leq 1\) or \(2 \cdot \gamma \geq \sigma\), so \(\sigma \leq 2\) is a sufficient condition.

\(^2\) See footnote 1.
relative demand \( \frac{L_1}{L_2} \) and (22) then solves for a unique relative labour supply \( \frac{L_1}{L_2} \). To justify \( \frac{w_2}{w_1} \geq 1 \) we assume that job 1 can be performed by both migrants and natives. Therefore, if the wage for job 1 would exceed the wage for job 2 natives would also enter the market for job 1 until the wages are equal.

4 Productivity Differences

In the next step the model is expanded by introducing productivity differences. This is modelled by drawing independently a second stochastic parameter at firm foundation, namely the total factor productivity. For simplicity, only two different levels are possible: \( A_h \) and \( A_l \), with \( A_h > A_l \). The probability \( Ph = Pr(A_f = A_h) \) that a firm \( f \) draws the high productivity level is known to the investors. As noted before, the decision of a firm to continue production depends on the marginal costs. Therefore, the critical (inverse) marginal costs level \( \phi^* \) can be related to the parameters of both classes of firms via:

\[
\phi^* = \frac{A_l}{w_1 \cdot \beta_l^* + w_2 \cdot (1 - \beta_l^*)} = \frac{A_h}{w_1 \cdot \beta_h^* + w_2 \cdot (1 - \beta_h^*)},
\]

(23)

where \( \beta_l, \beta_h \) are the critical share-parameter levels for low and high productive firms respectively. From \( w_2/w_1 > 1 \) follows that \( \beta_h^* < \beta_l^* \), so that low productive firms are more likely to fail.

The combination of the zero-cut-off-condition and the free-entry-condition then looks like:

\[
\frac{e \cdot \delta}{F} = (1 - Ph) \cdot \frac{1}{\chi(\beta_l^*)} \left( \frac{\beta_l^* + (1 - \beta_l^*) \cdot \left( \frac{w_2}{w_1} \right)^{1-\gamma} \frac{\sigma-1}{1-\gamma}}{\beta + (1 - \beta) \cdot \left( \frac{w_2}{w_1} \right)^{1-\gamma}} - 1 \right) g(\beta) d\beta
\]

\[
+ Ph \cdot \frac{1}{\chi(\beta_h^*)} \left( \frac{\beta_h^* + (1 - \beta_h^*) \cdot \left( \frac{w_2}{w_1} \right)^{1-\gamma} \frac{\sigma-1}{1-\gamma}}{\beta + (1 - \beta) \cdot \left( \frac{w_2}{w_1} \right)^{1-\gamma}} - 1 \right) g(\beta) d\beta
\]

(24)

with \( \chi(\beta) = \min(1, \max(0, \beta)) \). The case that some \( \beta^* \) is negative or larger than one is explicitly valid. Negative means that all firms with that productivity level remain in the market, regardless of the \( \beta_f \) draw. Larger than one means that no such firm survives.

The share among the workforce is small, but we assume nonetheless that the wage of migrants is smaller than the wage of natives. To replicate this scarcity of natives
we need to assume a very strongly left-weighted distribution for $\beta$. The density we use for the numerical example is

$$
g(\beta) = \frac{1}{\log \left( \frac{z + 1}{z} \right) \cdot (\beta + z)}, \quad \text{for } 0 \leq \beta \leq 1
$$

(25)

with $z = 0.01$. This density has most mass at very low values, so that many firms employ very few migrants and only a few firms employ many migrants. Now it is possible to calculate the resulting minimum shares of job 1 of both productivity groups for a given relative wage $\frac{w_2}{w_1}$. We start with a relative wage of one, which means there is no wage difference between natives and migrants. This mainly corresponds to the Melitz (2003) setup, where a critical productivity cut-off exists. Only two cases are possible. In the first case only the high productive firms are able to stay in the market (the cut-off productivity level is between $A_l$ and $A_h$), in the second case all firms will stay in the market (the cut-off productivity level is below $A_l$. In the following, only parameter constellations where the second case holds are shown, as there are usually low-productivity firms present even if the supply of migrants in a region is low.
In a second step, we insert the relative wage and the respective share parameters $\beta_1^*, \beta_2^*$ into the labour demand equation. The results of this simulation are shown in figure 1. The different colours signify distinct values for the firms’ elasticity of substitution of migrants and natives $\gamma$. Among the empirical papers listed in the introduction (see especially Card (2009a) for an overview) that estimate the elasticity of substitution between migrants and natives, most studies find elasticities even above 10. But note that these studies estimate aggregate production functions. The elasticity used here is on the firm level. Firm level elasticities are hard to estimate because many firms do not employ migrants at all and the CES-framework cannot cope with that. We hope our firm heterogeneity approach sheds some light on this topic. Martins/Piracha/Varejão (2012) estimate substitutability, but they can only show that migrants and natives are more likely complements than substitutes. This indicates that elasticities might be lower than 10 on the firm level.
Note further that if $\gamma = 0$ the production function of the intermediate firms is the so-called Leontief production function. This implies that the share of migrants on the workforce is fixed for every firm by the share parameter $\beta_f$. All adaptations to changes in labour supply then take place between firms.

The upper right part of figure 1 shows that the labour demand curve is well behaved. The average productivity of firms in the market increases in the relative supply of migrants (figure 1, upper left). The better the firms are able to substitute workers, the less steep is the increase. This productivity increase is driven by the minimum share parameter $\beta_l^*$, which reaches zero in all cases and is increasing in the relative supply (figure 1, lower left). Therefore, unproductive firms have to leave the market if their share parameter is below this value. The minimum share parameter for the high-productivity firms $\beta_h^*$ does not reach zero in the relevant range of the relative supply of workers (figure 1, lower right).

Overall, the effects are stronger the less firms adapt to the changes in labour demand. To understand this effect, note that an input to the model is the density of the share parameter $\beta_f$. The larger this density parameter, the larger is the share of migrants among the workforce of this particular firm. However, the share of migrants in a firm also depends on the relative wage $w_2/w_1$ and the elasticity of substitution $\gamma$. First consider the Leontieff case $\gamma = 0$. When both cutoff share-parameters $\beta_i^*, \beta_h^*$ are negative, all firms stay in the market. The density of firms with respect to the migrant share is then identical to the density of firms with respect to the share parameter. The latter is identical to the density from which the share parameter is drawn. When the migrant share increases, the cut-off share $\beta_i^*$ increases above 0, so that some firms shut down. All firms with a lower share parameter are then high-productivity firms. Thus, the share of firms with a low migrant share gets smaller. The density of firms with respect to the migrant share flattens. The flattening is even stronger when the elasticity of substitution $\gamma$ is larger than 0. As soon as wage differences appear, firms start to substitute workers with each other. To summarize, this effect occurs because we calibrate the model to replicate identical densities of the migrant share of firms when no wage differences occur. At larger migrant shares on the aggregated workforce, there is a cut-off and an in-firm flattening effect on this density.

5 Welfare analysis

The capital market is balanced, so the aggregate firm profits are used to pay the entry costs of new firms. Thus, the welfare of a worker $i$ only depends on how much he can buy of the final good. It is given by:

$$U_i = \frac{w_i}{\bar{P}} = \frac{\sigma - 1}{\sigma} \cdot M^{-\frac{1}{\sigma - 1}} \cdot \Phi \cdot w_i$$  \hfill (26)
The utility depends positively on the average firm productivity $\bar{\phi}$, the wage and the number of firms. The latter shows the love for variety, which is common in this type of models.

Similar to equation (19), the number of firms in the economy $M$ can be calculated in by:

\[
M = \frac{L_1 + \frac{w_2}{w_1} \cdot L_2}{\frac{w_2}{w_1} \cdot \sigma \cdot \left( \frac{e \cdot \delta}{1 - \Gamma(\bar{\phi}^*) + F} \right)}
\]

(27)

where $1 - \Gamma(\phi^*)$ measures the probability that a newly founded firm is able to survive in the market, and it therefore holds that:

\[
\Gamma(\phi^*) = (1 - P_h) \cdot \int_{\beta_i^*}^{1} g(\beta) d\beta + P_h \cdot \int_{\beta_h^*}^{1} g(\beta) d\beta
\]

(28)

which can be calculated from the simulation results.

For the average productivity it holds that:

\[
\bar{\phi} = \left( \frac{1}{1 - \Gamma(\phi^*)} \right) (1 - P_h)
\]

\[
\int_{\beta_i^*}^{1} \left( \frac{A_i}{(\beta \cdot w_1^{1-\gamma} + (1 - \beta) \cdot w_2^{1-\gamma})^{\frac{1}{1-\gamma}}} \right)^{\sigma-1} g(\beta) d\beta + P_h \cdot \int_{\beta_h^*}^{1} \left( \frac{A_h}{(\beta \cdot w_1^{1-\gamma} + (1 - \beta) \cdot w_2^{1-\gamma})^{\frac{1}{1-\gamma}}} \right)^{\sigma-1} g(\beta) d\beta
\]

(29)

The number of firms and the welfare effects for both types of workers are calculated for two cases:

1. The migrant share increases, but the size of the labour force is fixed
2. Only the migrant labour force size increases, while the natives labour force size is fixed
In both cases all other parameters, like the fix costs, are fixed. The first case can be used to compare two regions or cities with the same number of workers but different migrant shares, while the second case investigates the impact of new immigration. The results of the first case are displayed in figure 2, the second case in figure 3.

In the first case (figure 2), otherwise identical cities or regions with differing migrant shares are compared. Therefore, no scale effects occur. The number of firms in a city or region is generally decreasing in the migrant share, until it reaches a low level. We call this the firm composition effect. Many firms, which do not employ many migrants, decrease their production. Some low-productivity firms have to leave the market because their share parameter is too low. A smaller share of firms, on the other side, gain advantage from the shift in wages: They employ many migrants, who receive a lower wage due to the labour supply changes. These firms increase production and make more profit. We restrict our approach to two productivity levels. One could add a third productivity level below the low level, such that firms with this
level do not produce when wage differences are small. Such firms, insofar as they also draw a very high share parameter (close to 1), would then enter the market when the supply of migrant workforce increases. This would dampen the composition and the productivity effect a bit.

The welfare is influenced by three factors: the wage, the average firm productivity and the number of firms. For the natives, a higher migrant share has positive effects on wages and firm productivity but negative effects on the number of firms. The results for welfare show that for low migrant shares the negative effects on the number of firms dominate so that the welfare decreases. For higher shares, the wage effect pushes the welfare so that a high migrant share has positive effects on welfare. For migrants, the relative wage decreases in the migrant share so that the welfare of migrants always decreases in the migrant share, as well.

Figure 3
Number of firms and welfare - migration shock

Source: Own simulations.
In the second case (figure 3), the effects of a migration shock are studied. As in the first case, a migrant shock starting from a very low level can have negative effects on the number of firms if it is too weak. If the shock is sufficiently strong or the migrant share is sufficiently high before the shock, the scale effects dominate and the number of firms increases. For the natives, the wage and productivity effects dominate and their welfare always increases due to a migration shock. For migrants, the negative wage effects decrease the welfare unless there is quite perfect substitutability between migrants and natives.

6 Conclusion

Our model explains a mechanism by which migration increases the average firm productivity in a region. We do not need ad-hoc spillover effects. Instead, we show that wage cost differences matter for competing firms. Firms that employ many migrants can take advantage of these differences and push other, less productive firms out of the market. Therefore, the share of low-productivity firms in the market decreases with immigration.

So, is immigration good or bad? In the model, there are up- and downsides. First, an increased firm productivity generally means more output given the inputs. Thus, it raises consumption for all workers, which is clearly an upside. Second, the model builds on the imperfect worker substitutability framework. This implies that native’s wages increase with the share of migrants among the workforce. For migrants, the opposite holds. Third, it has implications for the number of firms, which matters due to love for variety in the model. In the model, the effects of immigration are ambiguous; there is a composition and a scale effect: the first means that a few firms which employ many migrants displace a lot of firms which employ mostly natives. This effect decreases the number of firms. The scale effect increases the number of firms, simply because immigrants increase the overall workforce. The composition effect describes that competition between workers not only takes place within firms, but also in between firms. We do not model unemployment in this paper, but the firm failure mechanism suggests that a selective group of workers, namely workers in low-productivity firms, are exposed to a higher risk of job loss due to immigration.

Summing up all up- and downsides of the impact of migrants, the simulation results – with minor exceptions – find positive welfare effects for natives. The positive wage and productivity effects mostly dominate the composition effect among the firms. For migrants who have already moved into the economy, new immigration only can have positive effects if the scale effect is very strong.
References


Appendix A

Profit maximization of the firm:

The index $f$ is dropped for convenience. For given wages $w_1, w_2$, elasticities $\sigma > 1$ and $1 \neq \gamma > 0$, aggregated quantity $Q > 0$ and price index $P > 0$, a firm with total factor productivity $A > 0$, fixed costs $F > 0$ and share parameter $0 < \beta < 1$ maximizes its profits:

$$
\pi = p \cdot q - \sum_{i=1}^{2} w_i \cdot l_i - w_2 \cdot F
$$

with respect to the price demand function:

$$
q = Q \cdot \frac{p^\sigma}{P^\sigma},
$$

and the production function:

$$
q = A \cdot \left( \beta_1^\gamma \cdot l_1^{\gamma-1} + \beta_2^\gamma \cdot l_2^{\gamma-1} \right)^{\frac{\gamma}{\gamma-1}}, \quad \beta_1 = \beta; \beta_2 = 1 - \beta.
$$

This is equivalent to the maximization of:

$$
\pi = PQ^\frac{1}{\sigma} \cdot q(l_1, l_2)^{\frac{\sigma-1}{\sigma}} - \sum_{i=1}^{2} w_i \cdot l_i.
$$

The first order conditions are for $i = 1, 2$:

(A):

$$
\left. w_i \right|_{i=1} = \rho \cdot PQ^\frac{1}{\sigma} \cdot q^\frac{1}{\sigma} \cdot A^{\frac{\gamma-1}{\gamma}} \cdot q^\frac{1}{\gamma} \cdot \beta_i^\frac{\gamma-1}{\gamma} \cdot l_i^\frac{1}{\gamma},
$$

so that:

$$
\left. w_i \cdot l_i \right|_{i=1} = \rho \cdot p \cdot A^{\frac{\gamma-1}{\gamma}} \cdot q^\frac{1}{\gamma} \cdot \beta_i^\frac{\gamma-1}{\gamma} \cdot l_i^\frac{1}{\gamma},
$$

which implies:

(B):

$$
\sum_{i=1}^{2} w_i \cdot l_i = \rho \cdot p \cdot A^{\frac{\gamma-1}{\gamma}} \cdot q^\frac{1}{\gamma} \cdot \sum_{i=1}^{2} \beta_i^\frac{\gamma-1}{\gamma} \cdot l_i^\frac{1}{\gamma}.$$
\[
\begin{align*}
\rho \cdot p \cdot A^{\frac{\gamma-1}{\gamma}} \cdot (q)_{\frac{\gamma-1}{A}} = \rho \cdot p \cdot q = \rho \cdot r, \\
\pi = r - \rho \cdot r - w_2 F = \frac{r}{\sigma} - w_2 F.
\end{align*}
\]
so that

\[
\pi = r - \rho \cdot r - w_2 F = \frac{r}{\sigma} - w_2 F.
\]

Furthermore, from (A):

\[
\bar{l} = l_1 \cdot \frac{w_1^{\gamma}}{\beta_1} = \rho^{\gamma} \cdot p^{\gamma} \cdot A^{\gamma-1} \cdot q = l_2 \cdot \frac{w_2^{\gamma}}{\beta_2},
\]
so that, for every \( i = 1,2 \), it is:

\[
l_i = \bar{l} \cdot \frac{\beta_i}{w_i^{\gamma}}.
\]

It follows with (B):

\[
p = \frac{1}{\rho} \cdot \frac{1}{q} \cdot \sum_{i=1}^{2} w_i \cdot l_i = \]

\[
= \frac{1}{\rho} \cdot \frac{\sum_{i=1}^{2} w_i \cdot l_i}{A \cdot (\frac{1}{\beta_1} \cdot (l_1)_{\frac{\gamma-1}{\gamma}} + \frac{1}{\beta_2} \cdot (l_2)_{\frac{\gamma-1}{\gamma}})^{\frac{\gamma-1}{\gamma}}} = \frac{1}{\rho} \cdot \frac{\bar{l} \cdot \sum_{i=1}^{2} \beta_i \cdot w_i^{1-\gamma}}{A \cdot (\sum_{i=1}^{2} \beta_i \cdot w_i^{1-\gamma})^{\frac{\gamma-1}{\gamma}}} = \]

\[
= \frac{1}{\rho} \cdot \frac{(\sum_{i=1}^{2} \beta_i \cdot w_i^{1-\gamma})^{\frac{1}{1-\gamma}}}{A} = \frac{1}{\rho \cdot \phi}
\]
Appendix B

To show that the combination of the zero-cutoff and the free-entry condition has a unique solution, we here look at a more general case than in the text, namely that the firms draw the TFP (total factor productivity) from some distribution $H(.)$. Note that the firm profit in (9) only depends on the inverse marginal costs, so that the equation $\pi(\phi^*) = 0$ defines an implicit function $\beta^*(A, \phi^*)$, which is decreasing in $A$ and $\phi^*$. Using this, it needs to be shown that the right side of the generalized equation (15):

$$\frac{e \cdot \delta}{F} = \int_0^\infty \int_0^1 \left( \frac{\beta^*(A, \phi^*) + (1 - \beta^*(A, \phi^*)) \cdot \left( \frac{w_2}{w_1} \right)^{1-\gamma}}{\beta + (1 - \beta) \cdot \left( \frac{w_2}{w_1} \right)^{1-\gamma}} \right)^{\frac{\sigma-1}{1-\gamma}} \cdot g(\beta) d\beta dH(A)$$

is decreasing in $\phi^*$ and increasing in $\frac{w_2}{w_1}$.

**Proof**

First, note that for every $\beta < 1$ (which includes $\beta < 0$), $x > 1$ and $\gamma > 0$ the function

$$\psi(\beta, x) = (\beta + (1 - \beta) \cdot x^{1-\gamma})^{\frac{1}{1-\gamma}}$$

is decreasing in $\beta$ as long as $\beta + (1 - \beta) \cdot x^{1-\gamma} > 0$. Note furthermore that the last inequality is implied by $\phi^* > 0$ for every $\beta^*(A, \phi^*) < 0$. Then for $0 < \phi^* < \phi'$ it is

$$\int_0^\infty \int_0^1 \left( \frac{\psi(\beta^*(A, \phi^*), \frac{w_2}{w_1})}{\psi(\beta, \frac{w_2}{w_1})} \right)^{\frac{\sigma-1}{1-\gamma}} \cdot g(\beta) d\beta dH(A) > 0$$

$$\int_0^\infty \int_0^1 \left( \frac{\psi(\beta^*(A, \phi'), \frac{w_2}{w_1})}{\psi(\beta, \frac{w_2}{w_1})} \right)^{\frac{\sigma-1}{1-\gamma}} \cdot g(\beta) d\beta dH(A) > 0$$

$$\int_0^\infty \int_0^1 \left( \frac{\psi(\beta^*(A, \phi'), \frac{w_2}{w_1})}{\psi(\beta, \frac{w_2}{w_1})} \right)^{\frac{\sigma-1}{1-\gamma}} \cdot g(\beta) d\beta dH(A),$$

where the first inequality reduces the area to integrate over while the integrand is positive and the second inequality uses the monotonicity of the integral.

Note that for $\beta_1 < \beta_2 \leq 1$ the map
\[ \psi_{\beta_1, \beta_2}(x) = \frac{\psi(\beta_1, x)}{\psi(\beta_2, x)} \]

is increasing for \( x \in (1, \infty) \). With the monotonicity of the integral, it follows that the right side of the equation (17):

\[
\int_{0}^{\infty} \int_{0}^{1} \left( \psi_{\beta^{*}(A, \phi^{*}), \beta} \left( \frac{w_2}{w_1} \right)^{\sigma - 1} - 1 \right) g(\beta) \, d\beta \, dH(A)
\]

is increasing in \( \frac{w_2}{w_1} \).
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