Capital mobility, imperfect labour markets, and the provision of public goods

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Abstract

This paper examines equilibrium tax rates and provision levels of public goods in an international tax competition setting with imperfect labour markets. While earlier research mainly reexamined the result of underprovision of public consumption goods in the decentralised equilibrium, this paper focuses also on the provision of public intermediate goods with different sets of policy instruments available for governments, including a labour tax. In the tax game assuming symmetric jurisdictions, public inputs may also be overprovided if unemployment is caused by a fixed wage above the competitive wage rate. In detail, overprovision of public inputs may occur if governments have a head tax only, or head and capital taxes at disposal. Using comparative static analysis, the paper investigates further the sources of the differences between governmental provision of public consumption goods and public inputs.

Zusammenfassung


JEL classification: H21, H71, J51

Keywords: Capital mobility, unemployment, provision of public goods and public inputs
1 Introduction

In the simplest models of international tax competition, the economy is assumed to consist of a large number of small and identical jurisdictions. One central finding of the early models of symmetric tax competition is that if decentralised jurisdictions do not coordinate the use of policy instruments, capital tax rates and provision levels of public goods may be too low from an efficiency point of view, i.e. underprovision with public goods occurs. The standard tax competition model (see, for instance, Zodrow/Mieszkowski 1986 (in the following ‘ZM 1986’) and Wilson 1986) has been extended in various directions. In a more recent development in the tax competition literature, imperfect labour markets have been considered, relaxing the standard assumption of full employment in all jurisdictions. With high and persistent unemployment, especially in some European states, this is no doubt a realistic assumption. Almost all contributions following this approach relate to the first part of the classic Zodrow/Mieszkowski (1986) paper dealing with governmental provision of public consumption goods in the context of international capital mobility.\(^1\) The present paper extends the tax competition literature as it assumes imperfect labour markets, and considers a public good that is either a public input in the production process of firms (scenario A) or a public consumption good (scenario B).

In general, tax competition literature still focuses mainly on public goods in the sense of consumption goods rather than intermediate goods in production. The reasons to consider public inputs in tax competition models are obvious and have already been discussed extensively in the work assuming competitive labour markets. Policy makers, for instance, are aware of the productivity-enhancing effect of infrastructure provision, and that foreign direct investments can be attracted not only by decreasing capital tax rates but also by increasing public input provision. In the literature on tax competition assuming imperfect labour markets, Aronsson/Wehke (2008) analyse a tax game, where governments provide both public consumption goods and public inputs. Within their approach with source-based capital taxes and profit taxation, a coordination of tax rates increases welfare even in the presence of labour market imperfections. The authors deal also with the important research area of the efficient mix of public expenditures between public consumption goods and public inputs, such as pioneered by Keen/Marchand (1997).\(^2\)

In the analysis of Aronsson/Wehke (2008) the wage rate is a result of a bargaining process between unions and firms. Boeters/Schneider (1999) examine in detail the structure of the Nash game between the government and a union, where in addition to capital and labour a third production factor is considered, which generates a rent to the firms in the first instance. This production factor is assumed to be land or infrastructure. In a more recent approach Eichner/Upmann (2012) extend the framework of Ogawa/Sato/Tamai (2006) by allowing for various institutional settings of the labour market. Ogawa/Sato/Tamai (2006),

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1 See, for instance, Ogawa/Sato/Tamai (2006) and Eichner/Upmann (2012). Some of the earlier studies include Fuest/Huber (1999), Richter/Schneider (2001), Leite-Monteiro/Marchand/Pestieau (2003), and Lozachmeur (2003). The original ZM-framework has also been extended in other directions. Recently Becker/Runkel (2012) accounted for transport costs in the initial ZM (1986) model and found that the level of public (consumption) goods in the decentralised equilibrium with capital taxes (and no head taxes) is efficient under the presence of transport costs.

2 ZM (1986) were again the first to examine simultaneous provision of public inputs and public consumption goods in a tax competition framework.
in contrast, concentrate on a fixed wage above the competitive wage rate as a source of unemployment, while both examine governmental provision of public consumption goods in the context of international tax competition. In the analysis of Ogawa/Sato/Tamai (2006), overprovision of public goods is one possible outcome. This result is challenged by Eichner/Upmann (2012), who show that the results derived by ZM (1986) remain valid for an efficient bargaining solution on the labour market. As a further difference, Eichner/Upmann (2012) consider leisure in the utility function of individuals and assume a positive reservation wage rate.

Although related to some degree to our setting, the present analysis differs in some important aspects from the earlier literature. The paper examines symmetric tax competition with unemployment caused by a fixed wage above the competitive wage rate, in which the analysis of Ogawa/Sato/Tamai (2006) and Eichner/Upmann (2012) is extended to a situation where governments provide public inputs. Therefore, as a major difference, for the determination of tax rates and provision levels of public goods in the decentralised equilibrium, the focus is also on the second part of the ZM (1986) framework, which examines the efficient provision of public inputs. Unlike in the scenario when public consumption goods are provided, head taxes can then be shown to affect equilibrium values of capital, and, in addition, of the production factor labour if the labour market is not competitive. So far, tax games, where decentralised regions provide public inputs, have been examined in detail only for competitive labour markets. However, as argued in the literature assuming competitive labour markets, it is useful to think of public goods also as public intermediate goods, e.g. productive infrastructure. In contrast to Boeters/Schneider (1999), productive infrastructure is considered as a public input (scenario A), for which funding with tax revenues is required.

In general, when public intermediate goods are considered as additional inputs in production, the determination of the sign of the reactions of capital and labour to a change in policy parameters is not trivial. As a result, assumptions regarding the signs are often made in the respective literature. The present paper, instead, first derives equilibrium tax rates and public good provision levels for different combinations of tax instruments in the non-cooperative equilibrium, including a source-based tax on mobile capital, a head tax, and a labour tax. Then, in order to analyse the factors contributing to the derived public good provision levels and tax rates in the different tax games considered, the paper also examines in detail the reaction of a region’s capital endowment and the number of employed consumers to a change in policy parameters (both in general and in the symmetric equilibrium). Following Ogawa/Sato/Tamai (2006), a wage rate that is determined exogenously above the competitive wage rate serves as a source of unemployment. The fixed-wage model can, for instance, be appropriate to describe unemployment that is caused by a minimum wage.

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3 These are referred to as ‘business public services’ by ZM (1986).
4 This is a common assumption in most models with infrastructure. In general, as for the production of private goods, the production of infrastructure requires the employment of rare resources, i.e. capital and labour. Land, in contrast, does not require any productive processes.
5 This paper is not looking at the factors contributing to unemployment in detail. As in the public consumption good scenario, different results may be obtained if one allows for other institutional settings of the labour market.
The structure of the paper is as follows: The subsequent section introduces the basic model of tax competition where regions compete for mobile capital and with unemployment in each jurisdiction. Section 3 analysis the decentralised equilibrium of symmetric tax competition assuming the availability of head and or capital taxes. Section 4 extends the analysis of the previous section through the introduction of a labour tax. Section 5 summarises the results and concludes the paper.

2 The model

The following analysis distinguishes between two scenarios. In scenario A we consider a tax game between identical jurisdictions, where regional governments provide a public input (e.g. productive infrastructure) which is employed as an additional factor in production. In the second scenario (scenario B), jurisdictions provide a public consumption good, which enters the utility function of consumers in a region. In addition, imperfect labour markets are assumed in each jurisdiction. In this respect we follow Ogawa/Sato/Tamai (2006) and Eichner/Upmann (2012) who (also) assume a fixed wage above the competitive wage as a source of unemployment in international tax competition.

As in the literature examining the provision level of public consumption goods in the context of imperfect labour markets, we use the simple model of symmetric tax competition for the analysis involving public inputs. The total amount of capital ($\bar{K}$) in the economy is fixed and mobile between all ($n$) jurisdictions:

$$\sum_{i=1}^{n} K_i = \bar{K}. \quad (1)$$

As convenient in the context of numerous and rather small jurisdictions, it is assumed that regions have no possibility to favourably manipulate the (net-of-tax) interest rate ($r$) of the international capital market ($\bar{r}$). With this rather basic setting of international tax competition, we aim to concentrate on the effects that can be traced back to imperfections in the labour markets caused by the fixed wage, and public input provision. Also, the subsequent approach distinguishes between equilibrium tax and public good provision levels for governmental provision of public goods and public inputs.

**Households.** Each region has an identical number of $N$ individuals, which are assumed to be immobile between regions. For the sake of simplicity (and as distributional issues are not examined here), we assume that each individual possesses the same share of the capital stock ($\bar{K}$) of the economy. Both the $L$ employed and $(N - L)$ unemployed consumers receive (exogenous) income $\bar{y}$ from owned capital and are taxed at a rate $t$, i.e. $t$ is a lump sum tax or head tax. Employed consumers can, in addition, derive income from labour equal to the exogenously fixed wage rate $w$. Utility ($v$) of a consumer in a jurisdiction depends on the tax game considered, i.e. on the fact whether regional governments

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6 Eichner/Upmann (2012); also allow for other institutional settings of the labour market.

7 In detail: $\bar{y} = \bar{r}\bar{K}/(Nn)$. 

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provide public inputs (scenario \(A\)) or public consumption goods (scenario \(B\)), and on the fact whether the consumer is employed or unemployed. In scenario \(A\), the representative individual derives utility from private consumption only, whereas in scenario \(B\) he/she derives also utility \(U(G)\) from the public consumption good. As in Ogawa/Sato/Tamai (2006) and Eichner/Uppmann (2012) we assume, in addition, that firm profits \((\pi)\) accrue to all individuals as an additional source of income, and each individual obtains the same share of total firm profits. Utility of an employed \((v^e)\) and unemployed \((v^u)\) consumer in scenario \(A\) is:

\[
\begin{align*}
v^e &= w + \bar{y} - t + \pi \\
v^u &= \bar{y} - t + \pi.
\end{align*}
\]

(2a)

In scenario \(B\) we derive:

\[
\begin{align*}
v^e &= w + \bar{y} - t + \pi + U(G) \\
v^u &= \bar{y} - t + \pi + U(G),
\end{align*}
\]

(2b)

where ‘\(a\)’ and ‘\(b\)’ in equation (2) (and in all following equations) denote scenarios \(A\) and \(B\) in which governments provide a public input or a public consumption good.

**Government.** Following ZM (1986), we assume in a first setting that jurisdictional governments have a head tax \((t)\) and a source-based capital tax \((T)\) at disposal. If revenues from both instruments are used entirely for the provision of the public factor,

\[
TK + tN = i \quad \text{with} \quad i = B, G
\]

(3)

are the balanced budget constraints when governments provide a public input \((i = B)\) or a public consumption good \((i = G)\).

**Production.** Firms in each jurisdiction produce the final good \(X\). One unit of the numeraire good \(X\) can be transformed into one unit of the private consumption good or into one unit of the public factor (i.e. the public input or the public consumption good). In addition to the mobile factor capital, there is an immobile factor of production, labour \((L)\). Depending on the fact whether governments provide a public input or a public consumption good, aggregate production in a representative jurisdiction is:

\[
X = F(L, K, B)
\]

(4a)

or

\[
X = F(L, K).
\]

(4b)
Regarding the production technology we assume decreasing returns to scale in private factors of production, capital and labour (scenarios A and B).\footnote{Diminishing returns to scale in private factors of production are, for instance, also assumed in Fuest/Huber (1999) and Boeters/Schneider (1999), whereas Ogawa/Sato/Tamai (2006) assume constant returns to scale in private factors (without explicitly accounting for a possible third production factor in their model). In contrast, Eggert/Goerke (2004) allow for both possibilities. Decreasing returns to scale in private factors give rise to profits in the first instance if factor prices of capital and labour equal their marginal products, whereas for constant returns to scale firm profits are zero. In the following it is assumed as in Aronsson/Wehke (2008) that firm profits are not generated by the public input (scenario A), so that the number of firms can be normalised to one.} First order derivatives are assumed to be positive, and from the second order derivatives cross derivatives (e.g. $\partial^2 F / (\partial K \partial B) \equiv F_{KB}$) are assumed to be positive:

$$\frac{\partial F}{\partial q} = F_q, \quad \frac{\partial^2 F}{\partial q \partial v} = F_{qv} > 0, \quad \frac{\partial^2 F}{(\partial v)^2} = F_{vv} < 0,$$

(5)

where $q, v = L, K \text{ (and } B\text{)},$ and $q \neq v.$ From Euler’s theorem we derive

$$F > F_K K + F_L L,$$

(6)

and

$$-F_{KK} K > F_{LK} L, \quad -F_{LL} L > F_{LK} K.$$

(7)

In addition, one obtains:

$$F_{KK} F_{LL} > (F_{LK})^2.$$

(8)

If governments provide a public input (scenario A), we derive

$$F_B > K F_{KB} + L F_{LB}.$$

(9)

The sign in inequalities (6) - (9) follows from the assumption of diminishing returns to scale in private factors of production.\footnote{Inequalities (6) - (8) are derived for scenarios A and B. Inequalities (7) and (8) are, for instance, also derived in Eggert/Goerke (2004).} In addition, one derives the profit maximising conditions

$$F_K(\cdot) = \bar{r} + T,$$

(10)

and

$$F_L(\cdot) = w.$$

(11)

### 3 Decentralised equilibrium with head and / or capital taxes

**Capital and head taxes.**

In a first scenario considered it is assumed that decentralised governments have the capital tax and a head tax on all consumers at disposal. The utilitarian welfare function of the government is:

$$W = L v^e + (N - L) v^u.$$

(12)

$L$ and $v^e$ ($N - L$ and $v^u$) are the number and the utility of the employed (unemployed) individuals in a jurisdiction. Taking into account (2), the utilitarian welfare function of the
government reads:

\[ W = Lw + N(\bar{y} + \pi - t) \]  

or

\[ W = Lw + N(\bar{y} + \pi - t) + NU(G). \]

In scenario \( A \) where individuals do not derive utility from public consumption goods (and individuals spend all income for consumption) governments can concentrate on the maximisation of individual consumption. In scenario \( B \), governments also need to account for the utility a representative individual derives from the provision of public consumption goods. With the public budget constraint (3), the maximisation problem of a representative government can be expressed as:

\[
\max W = F(K, L, TK + tN) - (\bar{r} + T)K + N(\bar{y} - t) 
\]

in the public input case, and

\[
\max W = F(K, L) - (\bar{r} + T)K + N(\bar{y} - t) + NU(TK + tN) 
\]

should regional governments provide a public consumption good. Accounting for the reaction of endogenous variables to a change in policy parameters: \( K = K(T, t) \) and \( L = L(T, t) \), which can be taken from the comparative statics results, the first order conditions for a maximum are (for an interior solution):

\[
Z = \frac{N - F_L L_T + TK_t}{N + TK_t} \tag{15}
\]

\[
Z = \frac{K - F_L L_T}{K + TK_T} \tag{16}
\]

with \( Z = F_B \) if governments provide a public input (scenario \( A \)) and \( Z = NU_G \) in the public consumption good case (scenario \( B \)). Conditions (15) and (16) need to be fulfilled simultaneously in order for the maximum welfare to be obtained. When public inputs are provided we derive the optimal capital tax rate:

\[
T = \frac{F_L(L_TN - L_TK)}{F_L(L_TK_T - L_TK_t) + KK_t - NK_T}. \tag{17}
\]

The optimal tax rate in the case that governments provide public consumption goods is obtained from (17) with \( L_T = K_t = 0 \). In both scenarios, the optimal capital tax rate can

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13 See Appendix.

14 See also Eichner/Upmann (2012) on the public consumption good scenario. Note that reactions of capital and labour to a change in policy parameters \( (L_T, K_T, L_t, K_t) \) are not identical in scenarios \( A \) and \( B \).

15 Modifying the first order conditions appropriately, we arrive at the results derived in the classic ZM (1986) model for full employment (i.e., competitive labour markets). In this case the terms \( L_T \) and \( L_t \) in first order conditions (15) and (16) are zero. As a result, with availability of the head tax (and \( K_T < 0 \)) the optimal capital tax is zero, and the public good is provided according to the first-best rule, i.e., \( NU_G = 1 \) or \( F_B = 1 \).

16 The optimal capital tax rate in the public consumption good case is derived in Fuest/Huber (1999), 15, (26), Ogawa/Sato/Tamai (2006), 354, (10) and Eichner/Upmann (2012), 207, (10). Equation (17) is the corresponding tax rate for governmental provision of public inputs.
be stated as

\[ T = \frac{F_L F_{KL}}{F_{LL}}, \]  

which is negative if the wage rate is above the competitive wage rate, and for the assumptions on the production technology (see (5)). The first order conditions for the optimal provision of the public goods are

\[ F_B = \frac{F_{LL} + F_L F_{LB}}{F_{LL}}, \]  

(19a)

if governments provide public inputs, and

\[ NU_G = 1 \]  

(19b)

when governments provide public consumption goods. Efficiency condition (19b) is a replication of the result derived in Ogawa/Sato/Tamai (2006) and Eichner/Upmann (2012) who show that also in the presence of imperfect labour markets due to a fixed wage, decentralised regions provide an efficient level of public consumption goods if a head tax is available.\(^{17}\)

Although the optimal capital tax rate can be simplified to (18) in both scenarios, the initial ZM (1986) result of an efficient provision of public goods can not be extended to the scenario when public inputs are provided. In contrast, as a glance at (19a) shows, \( F_B < 1 \) holds in the decentralised equilibrium, i.e. an overprovision with the public input may occur.\(^{18} 19\)

We can sum up these results as follows: For a fixed wage above the competitive wage rate, symmetric jurisdictions subsidise the mobile production factor capital according to (18), if governments have a source based capital tax and a head tax at disposal. This is true for both scenarios when decentralised jurisdictions provide public consumption goods or public inputs. In the former case, the provision level of the public consumption good is efficient. In contrast, in the latter case, regional governments tend to overprovide public inputs in the decentralised equilibrium.

In order to inspect the causes of overprovision with the public input in greater detail, we take a closer look at the reaction of a region’s capital endowment \( (K) \) and the number of employed consumers \( (L) \) to a change in policy parameters (both in general and in the symmetric equilibrium). Applying Cramer’s rule on (A-1) from the comparative static analysis, we derive for the reaction of a region’s capital endowment to a change in capital taxes:\(^{20}\)

\[ K_T = \frac{F_{LL}(1 - F_{KB}K) + F_{KL}F_{LB}K}{E}, \]  

(20)

and, by applying the homogeneity assumptions of the production function according to (7), and (9):

\[ K_T < \frac{F_{LL}(1 - F_{KB}K - F_{LB}L)}{E} < \frac{F_{LL}(1 - F_B)}{E}. \]  

(21)

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\(^{17}\) As will be shown below, this is a direct consequence of \( K_t = L_t = 0 \).

\(^{18}\) Technically, this is due to the fact that increasing the public input provision increases the marginal product of labour \( (F_{LB} > 0) \), and the wage rate has been assumed to be above the competitive wage \( (F_L = w > 0) \).

\(^{19}\) The derived results are obtained as long as the head tax is not increased up to a point where full employment occurs in all jurisdictions, as in that case \( L_t = 0 \), and the provision level of the public input is efficient.

\(^{20}\) \( E \) is taken from (A-2) with \( j = 0 \) (see Appendix).
As a consequence:

\[
F_B \leq 1 \quad \Rightarrow \quad K_T < 0
\]
\[
F_B > 1 \quad \Rightarrow \quad K_T \geq 0.
\] (22)

Should decentralised jurisdictions provide public inputs, the reaction of the capital endowment to a change in capital taxes \((K_T)\) depends on the provision level of the public input. In case of an overprovision \((F_B < 1)\) raising capital taxes drives out capital \((K_T < 0)\). A negative capital reaction holds, in addition, for an efficient provision level of the public input \((F_B = 1)\). In case of an underprovision \((F_B > 1)\), the sign of the capital reaction is ambiguous. Note that as the first order condition \((19a)\) indicates an overprovision of public inputs for the tax game with capital and head taxes, a negative capital reaction is obtained in this case.\(^\text{21}\)

Similarly to the reaction of the production factor capital to a change in capital taxes, the sign of the reaction of labour to a change in capital taxes is not immediately obvious when governments provide public inputs. The variation in the number of employed individuals to a change in the capital tax rate can be derived as

\[
L_T = \frac{F_{KL}(F_KB - 1) - F_{LB}(T + F_{KK})}{E},
\] (23)

which can again be simplified with help of the homogeneity assumptions on the production function (see \(I7)\):

\[
L_T > \frac{F_{KL}(F_KB + F_{LB}L - 1) - F_{LB}T}{E}.
\] (24)

Whether an increase in the capital tax rate increases or decreases the number of employed individuals, depends also on the level of the tax rate. The higher the latter the more likely it is that a negative sign is obtained. Evaluation in the equilibrium with capital and head taxes, using the homogeneity assumptions, and the first order condition \((19a)\), the reaction of labour to a change in capital tax rates can be simplified to:

\[
L_T > \frac{F_{KL}(F_KB + F_{LB}L - F_B)}{E} < 0.
\] (25)

As a result, the sign of the reaction of the factor labour to a change in capital taxes remains ambiguous, and, unlike the reaction of capital to a change in capital taxes, does not directly hinge on the provision level of the public input.\(^\text{22}\)

Finally, from \((A-1)\), we are able to derive positive reactions of capital and labour to a change in head taxes:

\[
K_t = \frac{N(F_{LB}F_{KL} - F_{KBF_L})}{E} > 0,
\] (26)
\[
L_t = \frac{N(F_{KBF_{KL}} - F_{LB}F_{KK})}{E} > 0.
\] (27)

For an explanation of the overprovision of the public input in the tax game with head and capital taxes, it may prove useful to compare \((20), (23), (26)\) and \((27)\) with the comparative

\(^{21}\) Note that for a 'moderate' underprovision, a negative reaction of capital to an increase in the capital tax \((K_T < 0)\) may still occur because of \(I7\) and \(9\) which have been used for the derivation of \(21\).

\(^{22}\) This follows also from \(9\).
statics results obtained in the situation where governments provide public consumption goods as considered by Ogawa/Sato/Tamai (2006) and Eichner/Upmann (2012). In the public consumption good scenario an increase in the capital tax rate generally decreases capital endowment and employment in a region:  

\[ K_T = \frac{F_{LL}}{E'} < 0, \quad L_T = -\frac{F_{KL}}{E'} < 0 \quad \text{with} \quad E' = F_{KK}F_{LL} - (F_{KL})^2 > 0. \]  

(28)

Also, in the public consumption good case, the head tax has no effect on capital and labour, i.e. \( K_t = L_t = 0 \), and, as a direct consequence, the public consumption good is always provided at an efficient level when head taxes are available.

In the public input case, an increase in the head tax increases a region’s capital endowment. This effect can also be found for a provision of public inputs and no imperfections in labour markets. When labour markets are imperfect, regional governments can, in addition, increase local employment through the implementation of head taxes, which increases incentives to use head taxes further. Also, by increasing the share of employed households in the population through higher head taxes, jurisdictions increase the marginal productivity of capital \( (F_{KL} > 0) \) and the capital endowment in a region. In contrast, regional capital endowment falls for increasing capital taxes, which reduces incentives to tax the mobile production factor. As a result, head taxes and the level of the public input are set too high (i.e. \( F_B < 1 \)) compared to the first-best policy.

Note that revenues from head taxes must be sufficiently high to cover both the subsidisation of capital and the (over)provision of the public good. This requires that individuals generate enough income from capital, (labour) and firm profits in the first instance, which may be a problematic assumption, especially for the \((N - L)\) unemployed individuals, which do not receive income from labour. Therefore, in the following, we also consider the case that the capital tax is the only source of income for regional governments.

**Capital taxes only.**

In addition to the tax game between identical jurisdictions providing public consumption goods using income from capital and head taxes, Eichner and Upmann (2012) have also analysed in detail the tax game when only capital taxes are available as a source of income. Rather than an overprovision of the public consumption good in the symmetric equilibrium, underprovision is argued to be the more plausible case. Next to some other modeling assumptions similar to our setting, this result is obtained when positive cross derivatives of production factors are assumed (in detail \( F_{KL} > 0 \)).

In the present approach, when governments provide public inputs, (20) and (23) are derived as reactions of capital and labour to a change in capital taxes. Inserting both into the first order condition (16) yields after some modifications:  

\[ F_B > K\left(\frac{F_{KK}F_{LL} - F_{KL}^2}{K^2} - F_{L[L}F_{KL}(F_{KB} - 1) - F_{LB}(KF_{KK} + T)}\right). \]  

(29)

\[ F_{KB} = F_{LB} = 0 \quad \text{and} \quad F_B = NU_G. \]  

For the effects in the public consumption good case, see also Boeters/Schneider (1999), 176 and Ogawa/Sato/Tamai (2006), 353.

\[ \text{See Appendix.} \]
With a glance at (23), we find that the provision level of the public input depends also on the sign of the reaction of the production factor labour to a change in capital taxes ($L_T$):

$$F_{KL}(F_{KB}K - 1) - F_{LB}(KF_{KK} + T) \leq 0 \iff L_T \leq 0 \Rightarrow F_B > 1$$

$$F_{KL}(F_{KB}K - 1) - F_{LB}(KF_{KK} + T) > 0 \iff L_T > 0 \Rightarrow F_B \geq 1.$$  \hspace{1cm} (30)

In addition we have from (29) and (23)

$$F_B \leq 1 \Rightarrow L_T > 0,$$  \hspace{1cm} (31)

which states that an overprovision or efficient provision of the public input comes together with a positive reaction of employment to capital taxes. According to (30) and (29) this outcome is rather unlikely. From the results above it can be shown that the tax rate that guarantees $L_T > 0$:

$$T < \frac{F_{KL}(F_{KB}K - 1)}{F_{LB}} - F_{KK}K < -F_{KL}L - F_{KK}K + \frac{F_{KL}}{F_{LB}}(F_B - 1)$$  \hspace{1cm} (32)

can be considered too low for a positive level of the public input or an efficient provision (or even overprovision) of the public input.\textsuperscript{25} If $F_B \leq 1$ is considered, it becomes obvious from (32) that the capital tax rate is required to be at a very low (and possibly negative) value for $L_T > 0$ to occur. Within (32) the only possibility for a positive capital tax rate to occur arises from the first two terms on the right-hand side ($-F_{KL}L - F_{KK}K > 0$), i.e. directly from homogeneity assumption (7). Note, however, that this effect is close to zero, when returns to scale in private factors of production are close to one. As a result, efficient provision or even overprovision are rather unlikely outcomes, leaving underprovision of the public input the more realistic outcome. According to the analysis, the underlying causes of underprovision of the public input can also be attributed to the increase in the number of unemployed individuals resulting from an increase in the capital tax. As a consequence, due to reduced incentives to use capital-tax financing for the provision of the public input, capital tax rates and provision levels of the public input tend to be too low in the decentralised equilibrium.

In the public consumption good case, the situation is far less complicated. For the first order conditions one derives:\textsuperscript{26}

$$NU_G = \frac{F_L F_{KL} + K F_{KK} F_{LL} - K F_{KL}^2}{T F_{LL} + K F_{KK} F_{LL} - K F_{KL}^2}.$$  \hspace{1cm} (33)

As $T > 0$ and $K_T < 0$, it is immediately obvious from (33) that the public consumption good $G$ is underprovided relative to the first-best situation where $NU_G = 1$. The result of an underprovision can be derived quickly because both the negative employment effect arising from an increase in the capital tax ($L_T < 0$), and the negative capital reaction ($K_T < 0$) work towards an underprovision of the public consumption good (see equation (16)).

\textsuperscript{25} This follows also from (9).

\textsuperscript{26} This is obtained again from inserting (20) and (23) in the first order condition (with $Z = NU_G$).
Head taxes only.
The reaction of the number of employed consumers to a change in the head tax and the relevant first order condition when head taxes are available only, are:

\[ L_t = \frac{N(F_{KB}F_{KL} - F_{KK}F_{LB})}{F_{KK}F_{LL} - F_{KL}^2} > 0 \] (34)

and

\[ F_B = \frac{N - F_LL_t}{N}. \] (35)

It becomes obvious that the provision level of the public input is above the optimal level, in a scenario where governments are restricted from using other taxes than head taxes. Formally this can be stated as:

\[ F_B = \frac{F_L(F_{KK}F_{LB} - F_{KB}F_{KL}) + F_{KK}F_{LL} - F_{KL}^2}{F_{KK}F_{LL} - F_{KL}^2} < 1. \] (36)

As already found in the tax game with capital and head taxes, regional employment and capital endowment react positively to an increase in head taxes \((L_t, K_t > 0)\) and governments have an incentive to increase head taxes. Then, in a situation with unemployment in all countries due to a fixed wage rate above the competitive wage rate, all jurisdiction raise the head tax above the optimal level, and the public input is overprovided \((F_B < 1)\).

In the tax game where regional governments provide public consumption goods, the public good is provided according to the first-best rule \((NUG = 1)\). This follows directly from the fact that the head tax has no effect on the equilibrium level of labour \((L_t = 0)\).

4 Decentralised equilibrium: An extension with labour taxes

As in Eichner/Upmann (2012) for public consumption goods (scenario B), we consider an extension of the framework by assuming that regional governments have (in addition) a labour tax \((j)\), i.e. a tax on the production factor labour, at their disposal. Accounting for the modified public budget constraint and profit maximisation conditions of firms due to the labour tax, the maximisation problem of regional governments can be expressed as (scenario A and B).\(^{27}\)

\[
\text{max} \quad W = F(K, L, TK + tN + jL) - (\bar{r} + T)K + N(\bar{y} + \pi - t) - jL \tag{37a}
\]

and

\[
\text{max} \quad W = F(K, L) - (\bar{r} + T)K + N[\bar{y} + \pi - t + U(TK + tN + jL)] - jL. \tag{37b}
\]

\(^{27}\) For the presentation of the maximisation problem it has been assumed that all three policy instruments are available.
**Labour taxes only.**

Assuming in a first scenario, that the labour tax is the only available policy parameter for jurisdictions, reactions of a region’s capital and labour endowment to a change in the labour tax can be derived as

\[ K_j = \frac{F_{KL}(F_{LB}L - 1) - F_{KB}(j + F_{LL}L)}{E} \]  
(38)

and

\[ L_j = \frac{F_{KK}(1 - F_{LB}L) + LF_{KL}F_{KB}}{E}. \]  
(39)

Due to the homogeneity properties of the production function we have:

\[ L_j \leq F_{KK}(1 - F_{LB}L - F_{KB}K) < F_{KK}(1 - F_{B}) \]  
(40)

implying:

\[ F_B \leq 1 \Rightarrow L_j < 0 \]
\[ F_B > 1 \Rightarrow L_j \geq 0. \]  
(41)

The first order condition, for the tax game with labour taxes only, reads:

\[ Z = \frac{L - (F_L - j)L_j}{L + jL_j} = 1 - \frac{F_LL_j}{L + jL_j}. \]  
(42)

From (41) and (42) it follows that an underprovision \((F_B > 1)\) together with a negative reaction of jurisdictional employment to a change in labour taxes \((L_j < 0)\) is the only possible outcome. Alternatively, the result of an underprovision is obtained from inserting (39) into the first order condition (42). After some short modifications one arrives at\(^{29}\)

\[ F_B > \frac{L(F_{KK}F_{LL} - F_{KL}^2) + jF_{KK} - F_LL_F_{KK}(F_B - 1)}{L(F_{KK}F_{LL} - F_{KL}^2) + jF_{KK}}, \]  
(43)

from which it becomes clear, again that the public input is underprovided in the equilibrium.

In the public consumption good case, the situation is again less complicated. The negative reaction of the jurisdictional employment to an increase in the labour tax \((L_j < 0)\) becomes immediately obvious\(^{30}\) implying an underprovision with the public consumption good in the equilibrium. Formally, one obtains:

\[ NU_G = \frac{L(F_{KK}F_{LL} - F_{KL}^2) + jF_{KK} - F_LL_F_{KK}}{L(F_{KK}F_{LL} - F_{KL}^2) + jF_{KK}} > 1. \]  
(44)

In both scenarios, due to the decline in the number of the employed individuals following an increase in the labour tax, regional governments have an incentive to underprovide public goods.

**Labour and capital taxes.**

The previous sections derived the result of an underprovision with the public good in the

---

28 With \(E\) from \(iA-2\) with \(T = 0\) and \(j \neq 0\): \(E = F_{LL}F_{KK} - (F_{KL})^2 + j(F_{KK}F_{LB} - F_{KB}F_{KL}).\)

29 See Appendix.

30 In this case \(L_j = \frac{F_{KK}}{(F_{KK}F_{LL} - F_{KL}^2)} < 0\) is obtained.
decentralised equilibrium when either the capital tax or the labour tax is the only policy parameter available for regional governments. This section analyses the situation in the non-cooperative equilibrium when both of these taxes are available. Should jurisdictions have a capital and a labour tax at disposal, the relevant first order conditions can be derived as:

\[ Z = L - (F_L - j)L_j \]
\[ Z = K - (F_L - j)L_T \]

and

\[ Z = L + TK_j + jL_j \]
\[ Z = K + TK_T + jL_T \]

Using the comparative statics results in the first order conditions, and solving for \( F_B \) and \( T \), we arrive at:

\[ F_B = \frac{F_{LL}L + F_{KL}K + j + F_L(LF_{LB} + KF_{KB} - 1)}{F_{LL}L + F_{KL}K + j} \]  
(47a)

in the public input case, and

\[ NU_G = \frac{F_{LL}L + F_{KL}K + j - F_L}{F_{LL}L + F_{KL}K + j} \]  
(47b)

when regional governments provide public consumption goods. It follows that whether the public good is over- or underprovided depends also on the the sign of the denominator in (47a) and (47b). The equilibrium capital tax rate is:

\[ T = \frac{F_L (L_T L - L_j K)}{KK_j - LK_T + j (L_T K_j - L_j K_T) + F_L (-L_T K_j + L_j K_T)} \]  
(48)

It can be also expressed by:

\[ T = \frac{F_L (L F_{KL} + K F_{KK})}{F_{LL}L + F_{KL}K + j - F_L} \]  
(49)

which is identical for both tax games, when governments provide public consumption goods, and when governments provide public inputs.

Using the comparative statics results in the first order conditions, and solving for \( T \) and \( j \) in the two scenarios instead, we have:

\[ T = \frac{(1 - F_B)(KF_{KK} + F_{KL}L)}{F_B - F_{KB}K - F_{LB}L} \]  
(50a)

and

\[ T = \frac{(1 - NU_G)(KF_{KK} + F_{KL}L)}{NU_G} \]  
(50b)

---

31 See also Eichner and Upmann (2012) for the public consumption good case.
32 The case that regional governments provide a public consumption good, is again obtained from setting \( F_{KB} = F_{LB} = 0 \) on the right-hand side of (47a), and from the substitution of \( NU_G = F_B \).
33 An identical expression for the optimal capital tax rate for scenarios \( A \) and \( B \) has also been derived in the tax game where capital and head taxes were available (see equation (18)).
Therefore, independent of the fact whether we are in scenario \( A \) or \( B \) we derive:

\[
Z \geq 1 \Leftrightarrow T \geq 0, \tag{51}
\]

i.e., for a potential over- (under-, efficient) provision of the public good, the capital tax rate is negative (positive, zero) and vice versa. For the labour tax we derive in the two scenarios:

\[
j = \frac{F_L(1 - LF_{LB} - KF_{KB}) - (LF_{LL} + F_{KLL})(1 - F_B)}{1 - F_B} \tag{52a}
\]

and

\[
j = \frac{F_L - (LF_{LL} + F_{KLL})(1 - NU_G)}{1 - NU_G}. \tag{52b}
\]

As a result,

\[
Z < 1 \Rightarrow j > F_L - (LF_{LL} + F_{KLL}) > 0; \quad Z > 1 \Rightarrow j < F_L - (LF_{LL} + F_{KLL}) \tag{53}
\]

states the magnitude of the labour tax rate for an under- and overprovision of the public good in both scenarios. From (51), (53) and (49) we derive for both scenarios:

\[
j \geq F_L - (LF_{LL} + KF_{KL}) \Leftrightarrow T \leq 0 \Leftrightarrow Z \leq 1. \tag{54}
\]

Should public goods be overprovided in the decentralised equilibrium, the optimal labour tax implies a negative capital tax rate, which can be taken from (49) together with (51). Eichner/Upmann (2012) argue that an underprovision of public consumption goods is likely in the tax game with labour and capital taxes since overprovision is a rather ‘pathological’ case. As a result, the capital tax rate is positive in this scenario. According to the analysis presented above, the result of an underprovision is also the more realistic scenario for governmental provision of public inputs.

### 5 Concluding remarks

The present paper examines international tax competition with imperfect labour markets, where unemployment is caused by a fixed wage above the competitive wage rate. The analysis of Ogawa/Sato/Tamai (2006) and Eichner/Upmann (2012) is extended to a situation where governments provide public inputs. As a major difference to a scenario where public consumption goods are provided, the head tax affects equilibrium values of labour and capital, and, as a consequence, the public good is not necessarily provided at an efficient level in tax games where head taxes are available.

Assuming that all production factors are complements in production, the following differences can be derived between scenarios with governmental provision of public inputs and public consumption goods: For an availability of head taxes and capital taxes, and when head taxes are available only, public inputs tend to be overprovided. In contrast, an efficient provision of public consumption goods occurs in this case in the decentralised equilibrium. If regional governments are restricted from using other instruments than a capital tax, both public inputs and public consumption goods tend to be underprovided. An underprovision
with both public goods is, in addition, obtained in the tax game with a labour tax as the only policy instrument available, and underprovision is also plausible for the tax game with a combination of capital and labour taxes. In general, the derivation of results is more challenging in the public input case compared to the public consumption good case, as in the latter the provision of the public good has no effect on marginal products of primary factors of production, capital and labour. There is a general trend towards higher provision levels of the public input compared to the public consumption good. This is also true for tax games where an underprovision with both public goods is found. In a similar result, Keen/Marchand (1997) find a relative overprovision with public inputs in a tax competition setting where governments provide both public inputs and public consumption goods, and where the labour markets are competitive. In analogy to the development of the basic tax competition literature, the consideration of asymmetric tax competition where regions are large enough to affect the interest rate may provide a fruitful model extension. Other promising areas for future research may be a reexamination of the obtained results for different sources of unemployment such as discussed in Eichner/Upmann (2012), or an extension of the model to a long-run analysis with free entry for firms.
APPENDIX

Comparative statics

The comparative static results are derived for the case that jurisdictions select source-based capital taxes \( T \), head taxes \( t \), and labour taxes \( j \) as policy parameters. Profit maximisation conditions (10) and (11) and the public budget constraint (3) determine \( K \), \( L \) and \( B \) or \( G \) as functions of these policy instruments. In the public input case, reactions to a change in policy parameters can be obtained from:

\[
\begin{pmatrix}
-T & -j & 1 \\
F_{KK} & F_{KL} & F_{KB} \\
F_{KL} & F_{LL} & F_{LB}
\end{pmatrix}
\begin{pmatrix}
dK \\
dL \\
 dB
\end{pmatrix}
=
\begin{pmatrix}
K & N & L \\
1 & 0 & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
dT \\
dt \\
dj
\end{pmatrix}
\]  

(A-1)

The determinant of the equation system is:

\[
E = F_{LL}F_{KK} - (F_{KL})^2 + T(F_{LL}F_{KB} - F_{KL}F_{LB}) + j(F_{KK}F_{LB} - F_{KB}F_{KL}).
\]  

(A-2)

It is assumed that tax rates are selected in a way that we are on the left side of the Laffer-curve, i.e. \( B_T, B_J, B_j > 0 \), and that \( E > 0 \). Applying Cramer’s rule in (A-1), we derive:

\[
B_T = \frac{T F_{LL} + K F_{KK} F_{LL} - K F_{KL}^2 - F_{KL} j}{E},
\]

\[
B_J = \frac{-T F_{KL} + L F_{KK} F_{LL} - L F_{KL}^2 + F_{KK} j}{E},
\]

\[
B_I = \frac{N (F_{KL}^2 + F_{KK} F_{LL})}{E},
\]

\[
K_T = \frac{-F_{LB} j - F_{LB} K F_{KL} - F_{KL} L + K F_{KB} F_{LL}}{E},
\]

\[
K_J = \frac{-F_{KB} j + F_{KL} - F_{KL} F_{LB} L + F_{KB} L F_{LL}}{E},
\]

\[
K_I = \frac{N (F_{LB} F_{KL} - F_{KB} F_{LL})}{E},
\]

\[
L_T = \frac{-F_{KL} - F_{KK} F_{LB} K + F_{KL} F_{KB} K - F_{LB} T}{E},
\]

\[
L_J = \frac{F_{KB} T + F_{KK} - F_{KK} F_{LB} L + F_{KL} F_{KB} L}{E},
\]

\[
L_I = \frac{N (F_{KB} F_{KL} - F_{LB} F_{KK})}{E},
\]

which can be modified appropriately for the different tax games considered in sections 3 and 4.

\[34\] If labour taxes are available, profit maximisation condition (11) changes to \( F_L(\cdot) = w + j \).
Capital tax only - Derivation of equation (29)

Starting with:

\[ F_B = \frac{F_{KL}F_L(1 - F_{KB}K) + T(F_{KB}F_{LL}K - F_{KL}F_{LB}K) + K(F_{KK}F_{LL} - F^2_{KL}) + F_{LB}K(F_{KK} + T)}{F_{LL}T + K(F_{KK}F_{LL} - F^2_{KL})} \]  

(A-4)

Using (7) and (9), we derive:

\[ T(F_{KB}F_{LL}K - F_{KL}F_{LB}K) > TF_{LL}(F_{KB}K + F_{LB}L) > TF_{LL}F_B. \]  

(A-5)

As a result, we have:

\[ F_B > \frac{F_{KL}F_L(1 - F_{KB}K) + TF_{LB}F_B + K(F_{KK}F_{LL} - F^2_{KL}) + F_{LB}K(F_{KK} + T)}{F_{LL}T + K(F_{KK}F_{LL} - F^2_{KL})} \]  

\[ \iff \]

\[ F_B > \frac{F_{KL}F_L(1 - F_{KB}K) + F_{LB}K(F_{KK} + T) + K(F_{KK}F_{LL} - F^2_{KL})}{K(F_{KK}F_{LL} - F^2_{KL})} \]  

(A-6)

\[ \iff \]

\[ F_B > \frac{K(F_{KK}F_{LL} - F^2_{KL}) - F_L[F_{KL}(F_{KB}K - 1) - F_{LB}(K_{FKK} + T)]}{K(F_{KK}F_{LL} - F^2_{KL})}. \]  

(A-7)

Labour tax only - Derivation of equation (43)

The first order condition can be stated as:

\[ F_B = \frac{L(F_{KK}F_{LL} - F^2_{KL}) + jF_{KK} + F_{KL}F_{KK}(F_{LB}L - 1) - F_{LB}F_{KL}F_{KB}L}{L(F_{KK}F_{LL} - F^2_{KL}) + jF_{KK}}. \]  

(A-9)

With the homogeneity assumptions on the production function it follows:

\[ F_B > \frac{L(F_{KK}F_{LL} - F^2_{KL}) + jF_{KK} + F_{KL}F_{KK}(F_{LB}L + F_{KB}K - 1)}{L(F_{KK}F_{LL} - F^2_{KL}) + jF_{KK}} \]

\[ > \frac{L(F_{KK}F_{LL} - F^2_{KL}) + jF_{KK} + F_{KL}F_{KK}(F_{B} - 1)}{L(F_{KK}F_{LL} - F^2_{KL}) + jF_{KK}}. \]  

(A-10)
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