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# Decomposing Beveridge curve dynamics by correlated unobserved components

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## Abstract

Between 1979 and 2009, the German labour market moved along a Beveridge curve with changing slope that usually shifted outwards but once inwards. We employ an unobserved components model to simultaneously disentangle permanent and transitory components of matching efficiency and separation rate (shifting parameters) as well as unemployment and vacancies. Cointegration and identification are especially addressed. We find a steady overlay of structural and transitory shocks for both shifts of and movements along the curve. Thereby, the separation rate is more important than matching efficiency and the two are negatively correlated. Labour market tightness is mostly driven by stochastic trends, which leads to permanent rotations of the job creation curve, i. e. movements along the Beveridge curve.

## Zusammenfassung

Zwischen 1979 und 2009 bewegte sich der deutsche Arbeitsmarkt entlang einer Beveridgekurve mit veränderlichem Anstieg. Zudem verschob sich die Kurve mehrfach nach außen und einmal nach innen. Die Lageparameter – Matchingeffizienz und Separationsrate – sowie die konstituierenden Zeitreihen Arbeitslosigkeit und Vakanzen werden in diesem Papier simultan in ihre permanenten und transitorischen Komponenten zerlegt. Dafür nutzen wir ein korreliertes Unobserved-Components-Modell und berücksichtigen Kointegration und Identifikationsprobleme. Die Ergebnisse belegen eine andauernde Überlagerung struktureller und transitorischer Schocks sowohl auf Verschiebungen der Kurve als auch auf Bewegungen entlang der Kurve. Dabei spielt die Separationsrate eine größere Rolle für Verschiebungen als die Matchingeffizienz; beide Größen korrelieren negativ. Die Arbeitsmarktanspannung, das Verhältnis von offenen Stellen zu Arbeitslosen, wird stark vom stochastischen Trend getrieben. Das bedeutet, dass auch Drehungen der Job Creation Curve und damit Bewegungen entlang der Beveridgekurve permanent sein können.

**JEL classification:** C32, E24, E32, J2, J69

**Keywords:** Beveridge curve, worker flow rates, tightness, unobserved components

# 1 Introduction

Economists used to describe the German labour market by rigidity, sclerosis and hysteresis as labour market flows were much lower than in the United States and unemployment became persistent over time (Blanchard/Summers 1986, Nickell 1997). Consequently, the German Beveridge curve (BC) – the downward sloping relationship between vacancies and unemployment – shifted outwards. By 2003, however, things started to change. A few years after severe labour market reforms had come into force labour market flows rose and the BC shifted inwards for the first time in decades. The Great Recession 2008/09 caused a limited and seemingly ordinary movement along the curve. This is in striking contrast to the development of the U.S. labour market, for instance, where unemployment rose sharply, long-term unemployment doubled, and the BC shifted outwards (Lubik 2011, Daly et al. 2012, Sala et al. 2012).

The Beveridge curve was introduced by Abraham/Katz (1986) and Blanchard/ Diamond (1989) as a helpful tool to investigate structural and cyclical effects on vacancies and unemployment, hence the functioning of the labour market. Since then, and with additional relevance during the Great Recession, the BC has been decomposed to investigate the sources of its shifts and movements along the curve and, thereby, the sources of unemployment dynamics (e.g. Barnichon/Figura 2012, Bouvet 2012, Daly et al. 2012). Our paper enriches this literature by disentangling permanent and transitory components of the constituents of the BC in an unobserved components analysis.

Following common wisdom shifts of the curve are mostly ascribed to changes in labour market trends caused by changes in institutions, technology, the sectoral composition of the economy, or demography. These shifts of the BC imply a change in the permanent component of unemployment, often named "structural". On the other hand, cyclical variation as a consequence of fluctuating productivity rotates the job creation curve (JCC) such that the intersection with the Beveridge curve – the exact position of the labour market in the V-U-space – changes. Consequently, movements along the BC were considered to be cyclical. With unemployment reacting sluggishly to vacancy changes, the typical BC loops may occur.

From our understanding, some findings so far raise doubt on this clear-cut pattern of shifts and movements along the curve. Especially in Germany, the stability of the curve has been questioned as shifts occurred cyclically (Börsch-Supan 1990, Kosfeld et al. 2008). Theoretical as well as empirical studies showed that matching efficiency or the separation rate – shifting parameters of the BC – might be cyclical due to variations in unemployment heterogeneity or endogenous separations (e.g. Davis et al. 2010, Fujita/Ramey 2009). The same arguments result into the JCC[1] rotating

[1] For an exact derivation see the overview of wage curve and job creation curve in Pissarides (2000), pp. 10-23, or Cahuc/Zylberberg (2004, chapter 9).

either permanently or transitorily which in turn creates permanent or transitory movements along the curve. A similar effect would be achieved by productivity fluctuations in case these are due to permanent shocks, as found e. g. by Sinclair (2010) or Weber (2011). Consequently, it is not straightforward to interpret shifts as “structural” and movements along the curve as “cyclical”.

Therefore, our paper proposes a correlated unobserved components model to disentangle trend and cycle as well as their interaction within the BC framework. So we intend to figure out when a shift was structural indeed and when a movement along the curve was cyclical indeed. Clearly, founded knowledge about the sources of economic and labour market development being of either permanent or transitory nature is a major support for the choice of appropriate political measures. Moreover, this analysis visualizes the overlay of structural and cyclical processes (Blanchard 1997). The correlations among trends and between trends and cycles give insight into the complex interactions on the labour market: Structural change may induce compensatory or amplifying effects on matching efficiency and separation rate. It may also cause trend changes that are temporarily offset by cyclical reactions.

Further related questions can be addressed: Did rather structural reasons or business cycle fluctuations underlay the data generating processes of unemployment and vacancies? Do separations take place rather because demand for goods declines in recessions or because production processes change due to technological progress and structural change? Is matching efficiency cyclical or rather driven by institutional change as given by the labour market reforms? What is the relative importance of matching, separations, or job posting for the development of the BC?

In an unobserved components model, the permanent or long-run component of a time series is captured by a stochastic trend. With the theoretical Beveridge curve being a steady state relation of a set of variables, not all of the single trends can be independent. Cointegration is modeled as we restrict one trend to be a compound of the others. Thus, we add to the existing literature by applying the concept of steady state to the unobserved long-run states, not to *actual* unemployment, which may not be well approximated by equilibrium unemployment in Germany (Elsby/Hobin/Sahin 2009, Nordmeier 2012). In addition to the trends, the cycles refer to the transitory components of the series.

In economics, disentangling trend and cycle of GDP turns out to be the most popular application of unobserved components models. Methodological augmentations have been proposed with regard to correlations between the unobserved components´ shocks (Morley et al. 2003), identification of the source of these correlations (Weber 2011), and inclusion of asymmetries by regime switching (Kim/Nelson 1999). Applications to the labour market are rare. They reassess the relation between labour market flows and productivity (King 2005), natural unemployment (King/Morley 2007) or Okun´s law (Sinclair 2009). Our paper adds to this strand of literature.

The remainder of the paper is organized as follows: In the next section, we summarize the theoretical and empirical literature on why BC components, especially matching efficiency and separation rate, should vary permanently or/and transitorily. Section 3 describes our model. Afterwards, we briefly address identification and estimation strategies and the data. In section 6, we interpret the results on the unobserved components and deliver an extended matching function. Finally, we summarize the paper and draw conclusions.

## 2 Related literature: Trend and cycle of Beveridge curve components

The BC is theoretically derived in the labour market model popularized by Mortensen/Pissarides (1994) and surveyed in Pissarides (2000). The law of motion of unemployment and the matching function lead to the BC as a downward sloping steady-state relation between vacancies and unemployment. Two of the variables forming the intercept are matching efficiency, i. e. the efficiency component of the job finding rate that includes search intensity as well as public and private job placement, and the separation rate (see model in section 3). In our analysis, the hazard rates give the probabilities of either finding a job out of unemployment or losing or quitting one's job and transitioning into unemployment. Thus, one can regard them as chances and risks in the labour market.

Shifts of the Beveridge curve – i. e. changes of matching efficiency or separation rate – may be either permanent or transitory. Permanent effects arise from changes of the trends which result, for example, from changes of the institutions on the labour market such as employment protection legislation or the generosity of unemployment insurance (Blanchard/Diamond 1989, Blanchard/Wolfers 2000, Boeri 2011). After the German labour market had been characterized by high regulation and sclerosis for years, the most severe labour market reforms in the history as Federal Republic were implemented between 2003 and 2005. They aimed at increasing search incentives, stimulating labour demand by deregulation of labour market segments, and improving the functioning of the market. Their effect on matching efficiency was positively evaluated (Fahr/Sunde 2009, Klinger/Rothe 2012, Hertweck/Sigrist 2012). Permanent effects on the separation rate have not yet been investigated directly. As a first hint, Rebien/Kettner (2011) state that after the reforms employees have become more willing to make concessions regarding their working conditions. Such a change in behaviour may have caused the separation rate to decrease permanently. At the same time, the share of monthly reversed labour market transitions may have increased (Nordmeier 2012) which reconciles higher flexibility with shrinking separation rate in the data.

Beyond institutional change, permanent effects may also arise from sectoral change or – as Blanchard/Diamond (1989) put it – the intensity of reallocation in the economy (see also King/Morley 2007 or Davis/Haltiwanger 2001) and from technological progress. Finally, the structure of unemployment (a high share of long-term unemployed, for example) and the structure of the labour force (labour participation of women, young people and the elderly) may influence the position of the BC (Börsch-Supan 1991, Barnichon/Figura 2012, Bouvet 2012). In our approach, while changes of the labour force are not explicitly modelled, they are covered as soon as the composition effects are captured by matching efficiency and separation rate.

On the other hand, matching efficiency and separation rate were found to vary over the cycle, carrying cyclical variation over to the BC intercept. In their empirical study, Kosfeld et al. (2008) compute a time series of BC intercepts from a sequence of cross section regressions. The intercept series is then regressed on business cycle indicators that yield high significance. Going beyond that stage, our approach will provide information of whether this cyclicality is brought in by matching efficiency or separation rate.

The cyclicality of the BC was theoretically underpinned by Pissarides (1985) and Börsch-Supan (1991) who model the probability of match formation after employer and potential employee got into first contact dependent on productivity and reservation wage. Further considerations have been provided regarding the cyclicality of matching efficiency. Davis et al. (2010) state that recruiting intensity per vacancy varies countercyclically over time. In weak labour markets, employers find it easier to fill vacancies and, therefore, decrease advertising or search intensity, screen applicants less quickly, raise hiring standards and so on. A comparable cyclicality of search intensity of the unemployed was shown by Davis (2011). Both papers implement this recruiting intensity as an additional variable – an extraction from the intercept – in their matching functions, so one may equivalently think of matching efficiency varying with the cycle.

In addition, the heterogeneity hypothesis raised by Darby et al. (1985) and newly picked up by Barnichon/Figura (2011) states that matching efficiency varies with the business cycle as early in recessions the pool of unemployed workers includes more workers with higher job finding probability. In greater detail, Barnichon/Figura (2011) estimate matching efficiency as the Solow residual of the matching function and find it lagging behind the business cycle. Sedláček (2012) also relies on the heterogeneity hypothesis but finds a procyclical matching efficiency. He uses an unobserved components model to identify the time-varying efficiency parameter.

Furthermore, cyclical variation in the Mortensen/Pissarides labour market model has induced a large amount of studies with regard to the Shimer Puzzle – Shimer 2005, Hagedorn/Manovskii 2008, and many others seeking to extend that frame in order to draw realistic volatilities of the unemployment rate or labour market tightness from the calibrated model. This literature addresses the microfoundation of the labour market model and is not in our focus. An important issue in the context of our paper arises from this debate, though: the countercyclicality of the separation rate that was shown to be substantial and influential for unemployment dynamics (e. g. Fujita/Ramey 2009, Elsby et al. 2009, and for Germany Nordmeier 2012, Hertweck/Sigrist 2012).

In summary, the German BC may have shifted due to either permanent or transitory reasons.

Movements along the curve are defined by rotations of the job creation curve, an upward sloping relationship between vacancies and unemployment. Its slope – indicated by labour market tightness – depends in the Mortensen/Pissarides model on, for example, bargaining power, replacement rate but also on productivity, matching efficiency, and separation rate (Cahuc/Zylberberg 2004). Consequently, their permanent and transitory components are carried over to the JCC such that it rotates permanently *and* transitorily, leading to permanent *and* transitory movements along the Beveridge curve. Another realistic example for Germany is permanent wage moderation arising from a shrinking degree of unionization or less generous unemployment insurance benefits that worsen employees outside options. Such a scenario would lead to a permanent upward movement along the curve. A similar effect would be achieved if technological progress raised labour productivity permanently (and wages did not rise equivalently). Empirical job creation curves, however, have hardly been provided in the literature as the direct translation of the theoretical concept into an empirical model is not numerically solvable. Daly et al. (2012) estimate a "long-run shape" based on a regression of the vacancy rate on the natural rate of unemployment by the U.S. Congressional Budget Office.

## 3 The model

In this section we develop our labour market model. We follow the search and matching literature regarding theoretical key features (Mortensen/Pissarides 1994, Pissarides 2000, Petrongolo/Pissarides 2001 on the matching function).

Initially, we focus on the long-run equilibrium relations on the labour market. We explicitly anchor the steady state to the (unobserved) permanent components because it is *their* structural interrelations that are uncovered by the economic equilibrium. Thus, *they* form the cointegrating relation. As a consequence, our modeling approach solves the problem that actual unemployment may be insufficiently approximated by equilibrium unemployment in Germany (Elsby/Hobin/Sahin 2009, Nordmeier 2012).

Steady state unemployment is achieved if the transitions into and out of unemployment equate sustainably.[2] It is connected to the long-run flow equilibrium, i. e. the equation of equilibrium matches and equilibrium separations (all variables in logarithms):

(1) $M = S$

Econometrically, expression (1) implies cointegration between $S$ and $M$ if the observed series are I(1).

[2] For reduction of complexity we do not consider transitions into and out of the labour force. People in the labour force are either employed or unemployed.

Matches are formed by unemployed persons from *U* who leave unemployment with a certain probability, the job finding rate *f* which at least partly mirrors labour market institutions. Similarly, separations can be referred to as employees from *E* losing or quitting their job with a certain probability, the separation rate *s*. Changes in the hazard rates reflect the economic behaviour of agents, e. g. firms´ decisions on how many people to employ or dismiss. By contrast, the flow variables are subject to mere level effects due to changes in the stocks of unemployment and employment. We therefore rewrite the flow equilibrium in terms of the long-run components of the log-linearized hazard rates and respective stock variables.

(2) $f + U = s + E$

In this respect, the long-run component of unemployment *U* can be seen as some measure of structural unemployment, in theory often connected to the concept of the NAIRU. Equation (2) is the first step for the derivation of the BC. The second step is provided by the matching function that explains job finding probability depending on unemployed *U* and vacancies *V*. As usual, we specify a log-linear Cobb-Douglas-type matching function.

(3) $f = m + (\alpha - 1)U + \beta V$

*m* denotes matching efficiency while *α* and *β* are elasticities of matches with respect to unemployment and vacancies, respectively.

Integrating (3) into (2) and rearranging gives the Beveridge curve as steady state combinations of vacancies and unemployment.

(4) $V = \frac{1}{\beta}(s + E - m) - \frac{\alpha}{\beta}U$

Shifts of the BC are caused by changes to the intercept. Therefore, inward shifts occur if c. p. the separation rate or employment shrink and if matching efficiency or elasticity of matches with respect to vacancies rise. The slope of the BC is determined by the two match elasticities.

So far, the derivation of the BC relied on the long-run components of the constituting variables. In reality, however, those long-run components are not observable. Instead, empirical BCs consist of time series that include long-run (persistent) and short-run (transitory) components. To disentangle these components we develop a correlated unobserved components model. Thereby, each variable is decomposed into a stochastic trend $\tau$ that captures permanent effects and a stationary autoregression that captures transitory effects (the cycle *c*). The focus of our paper is on matching efficiency and separation rate as they mirror behaviour and institutions. We aim at specifying the time-varying properties of these parameters. As matching efficiency itself is not observable, we include the unobserved components specification into an empirical matching function.

$$
\begin{aligned}
&f_t = m_t + (\alpha - 1)U_{t-1} + \beta V_{t-1} + w_t \\
(5)\quad &m_t = \tau_t^m + c_t^m \\
&s_t = \tau_t^s + c_t^s
\end{aligned}
$$

As the steady state Beveridge relation (4) summarizes five nonstationary variables into one equilibrium relation, it implies cointegration: At most four stochastic trends can be independent. As a consequence, we use the Beveridge relation to specify the permanent component of unemployment as a composite trend of employment, separation rate, matching efficiency, and vacancies (and besides control for a deterministic trend and an intercept).[3]

$$
(6)\quad U_t = \frac{1}{\alpha}\left(-\tau_t^m + \tau_t^s + \tau_t^E - \beta\tau_t^V\right) + c_t^U
$$

Beyond separation rate and matching efficiency, trend-cycle decompositions of employment and vacancies complete the model.

$$
(7)\quad \begin{aligned}
E_t &= \tau_t^E + c_t^E \\
V_t &= \tau_t^V + c_t^V
\end{aligned}
$$

The inclusion of vacancies as an endogenous variable closes the model and plays the role of determining a job creation curve, i. e. a specification for tightness and the equilibrium vacancy-unemployment-relation. A direct translation of the standard theoretical approach would overload an empirical model, which would not be numerically solvable. Therefore, we use a general model version based on unobserved components. On this basis, the system of equations provides enough information to deduce unobserved components of tightness $\theta$.

$$
\begin{aligned}
&\theta_t = V_t - U_t \\
(8)\quad &\tau_t^\theta = \left(1 - \frac{\beta}{\alpha}\right)\tau_t^V - \frac{1}{\alpha}\left(-\tau_t^m + \tau_t^s + \tau_t^E\right) \\
&c_t^\theta = c_t^V - c_t^U
\end{aligned}
$$

The structural model further contains the specification of the unobserved components. Each trend component follows a random walk with drift.

$$
(9)\quad \tau_t^i = \mu^i + \tau_{t-1}^i + \eta_t^i \qquad \text{for } i = m, s, E, V
$$

Each cycle component is modeled as a stationary AR(*p*) process.

$$
(10)\quad c_t^i = \sum_{j=1}^{p} \phi_j^i c_{t-j}^i + \varepsilon_t^i \qquad \text{for } i = m, s, E, V, U
$$

[3] Choosing one of the other variables would just imply a linear transformation not altering the model.

All roots of the lag polynomials $\Phi^i(L)=1-\phi_1^i L-...-\phi_p^i L^p$ in modulus lie outside the unit circle. Given this mean reverting property, they explain transitory deviations from the trend.

Besides the matching shock the model includes 4 trend shocks $\eta_t^i$ and 5 cycle shocks $\varepsilon_t^i$. Unlike in conventional unobserved components studies, these are allowed to correlate with each other according to the covariance matrix in appendix 8.2 (compare Morley et al. 2003). This provides us with 9 correlations between measurement and transition shocks and 36 further correlations among the transition shocks. These correlations uncover how intensely the developments on the labour market overlie and interfere with each other. This includes structural and cyclical effects as well as shifts of and movements along the curve. Thereby, cyclical shocks may affect the trend and vice versa.

A few example hypotheses may underline the importance of allowing for correlated shocks: (1) The correlation between the trends of matching efficiency and separation rate is supposed to be negative. As these parameters enter the intercept of the Beveridge curve with different signs, a negative correlation of their trend components implies that permanent shocks such as structural reforms will shift the BC through both matching efficiency and separation rate. The effects do not compensate such that in an extreme example the BC would not shift at all. (2) Their transitory components are also expected to be negatively correlated as the previous literature showed that matching efficiency is rather procyclical whereas separation rate is countercyclical. (3) An overlay of structural and cyclical effects would be indicated by correlations between trend and cycle components. For example, higher trend matching efficiency may have produced better matches such that a lower ratio of jobs becomes unproductive in recessions. In that case, it would be negatively correlated with cyclical separation rate. Other such correlations might occur because of adjustment lags that lead to temporary reactions (of a cycle) to permanent shocks, as explained below. (4) A positive correlation between trend matching efficiency and trend tightness could imply that a structural improvement of the functioning of the labour market induces an increase in labour demand shown as rising vacancies such that tightness increases. Then, the BC would shift inwards and up at the same time.

A common result in the unobserved components literature is a negative correlation between trend and cycle of one and the same variable. Even though trend and cycle are simultaneously determined it is intuitive to rationalise this negative correlation from the permanent component´s perspective: If trend rises, the observed series takes a while to adjust to the new equilibrium path. The sluggish reaction induces a lag until full adjustment, i. e. a negative cycle in the meantime. One example can be found in real business cycle theory (Kydland/Prescott 1982), where permanent production shocks also operate as drivers of business cycles.

## 4 Identification and estimation

The structural form of our correlated unobserved components model contains elasticities of matches, drift terms, autoregressive coefficients, variances and covariances of the innovations as unknowns. Identification of such models, especially of the correlations, was treated for the univariate case in Morley et al. (2003) and for the multivariate case by Sinclair (2009). Identification requires the reduced form of the model to provide enough – estimable – information to deduce the structural unknowns. The reduced form of a correlated unobserved components model is a VARIMA(p,1,p) (see appendix 8.1 for the exact derivation). It directly contains the autoregressive coefficients in the AR part. The drifts can be extracted from the reduced-form intercept. All other parameters are merged into its MA part by means of Granger´s Lemma. Hence, the system of equations stemming from the nonzero autocovariances of the MA part must be rich enough to derive this information. Thereby, the number of nonzero autocovariances is given by the lag order of the MA part which in turn depends on the lag order of the unobserved autoregressive cycles (see appendix 8.1).

Beyond the AR coefficients and drift terms, the number of unknowns in the structural form with r=4 trends and k=5 cycles amounts to 59: 4 match elasticities (see section 6.3) + (r+k+1) variances + $(r^2+k^2+rk+r+k)/2$ covariances. Comparing this number of unknowns to the pieces of information given by the autocovariances leads to the conclusion that in our setup 2 nonzero autocovariance matrices – thus a lag length of 2 – are necessary.

The lag length was chosen by empirical investigation. We conducted residual analyses on univariate auxiliary regressions. The null of no residual autocorrelation in LM tests could not be rejected for lag lengths of at least 3. Moreover, information criteria confirmed a reasonable fit to the data, even though the choice was not always uniform. In any case, with an empirical lag length of 3 and a lag length of 2 necessary for identification, the model is identified. This choice balances the need for parsimony in a complex model and sufficiently rich dynamics of the given variables.

For estimation purposes, the structural model is cast in state-space representation (see appendix 8.2). Maximum likelihood is applied to estimate the parameters of the matching function and all variances and covariances. Thereby, the likelihood function is constructed using the prediction error decomposition from the Kalman filter.

## 5 Data

To calculate the hazard rates, we use a 2 percent random sample of the Integrated Employment Biographies (IEB, Version 9.0), which is provided by the Institute for Employment Research (IAB) and allows to aggregate individual labour market states and transitions in between. The IEB covers all individuals in Germany who either have been employed subject to social security, have received unemployment benefits, have participated in programs of active labour market policies (from 2000 on), or have officially been registered as job-seekers at the German Federal Employment Agency (from 2000 on); for data reports on earlier versions see Jacobebbinghaus/ Seth 2007, Oberschachtsiek et al. 2009.

For every person in our dataset aged between 15 and 65 years we define the main employment status at the 10$^{th}$ of each month from January 1979 to December 2009. If the employment status changes from one month to the next, we count this transition as an exit from one status and an entry into another status. To model such changes, a non-intersecting data set is required for each person. In the case of parallel spells, only the most important state is examined. The dominant status is selected using a priority list. Our ranking criteria are appointed by logical reasons combined with the priority for higher data quality. As a result, states associated with employment generally dominate unemployment and non-employment. However, marginal employment ranks behind unemployment since it may only be used to add income to unemployment benefit within the legal restrictions. This rule ensures that unemployment spells are not interrupted by just marginal employment. States related to the second labour market (job creation schemes in the public or quasi-public sector) and further training or qualification have higher priority than unemployment spells. Furthermore, short gaps between spells have to be filled off hand. We interpolate up to 14 days if the status before and after a gap was identical or if a gap up to 14 days precedes or follows an unemployment spell.

The job finding rate is calculated as ratio of transitions from unemployment into employment (subject to social security, second labour market – e.g. job creation schemes, marginal employment) and unemployment in the preceding month. Similarly, the separation rate gives the relation of the reverse transitions to employment in the preceding month.

Data on the stock variables in our model are provided by the official statistics of the German Federal Statistical Office (employment subject to unemployment insurance) and the Federal Employment Agency (registered unemployment and vacancies).[4]

We adjust all series for seasonality by X12-ARIMA. In the few cases when the seasonal pattern was not appropriately captured by the standard procedure, we estimate these seasonal outlier effects in auxiliary regressions and adjust for them. Similarly, the structural breaks due to German reunification were eliminated. Augmented Dickey Fuller tests with structural breaks (level shifts due to reunification) as well as KPSS tests on the reunification adjusted series were conducted to check (non)stationarity (Table 1). KPSS rejects the null of stationarity for all series, and

---

4 In contrast to many other countries there are official monthly time series for the stock of voluntarily reported vacancies in Germany. This is the best approximation we can use. The series of total vacancies from the representative German job vacancy survey (Kettner et al. 2011) is too short and of too low frequency. Corrections such as the relation of inflows of registered vacancies to all hires (Franz 2006, p. 106) do not consider structural or business cycle specialties of the vacancy reporting rate.

ADF does not reject the null of nonstationarity but for vacancies. We will allow for stochastic trends in all variables. Vacancies might be a borderline case, but here the trend variance could still be estimated at zero in our unobserved components framework.

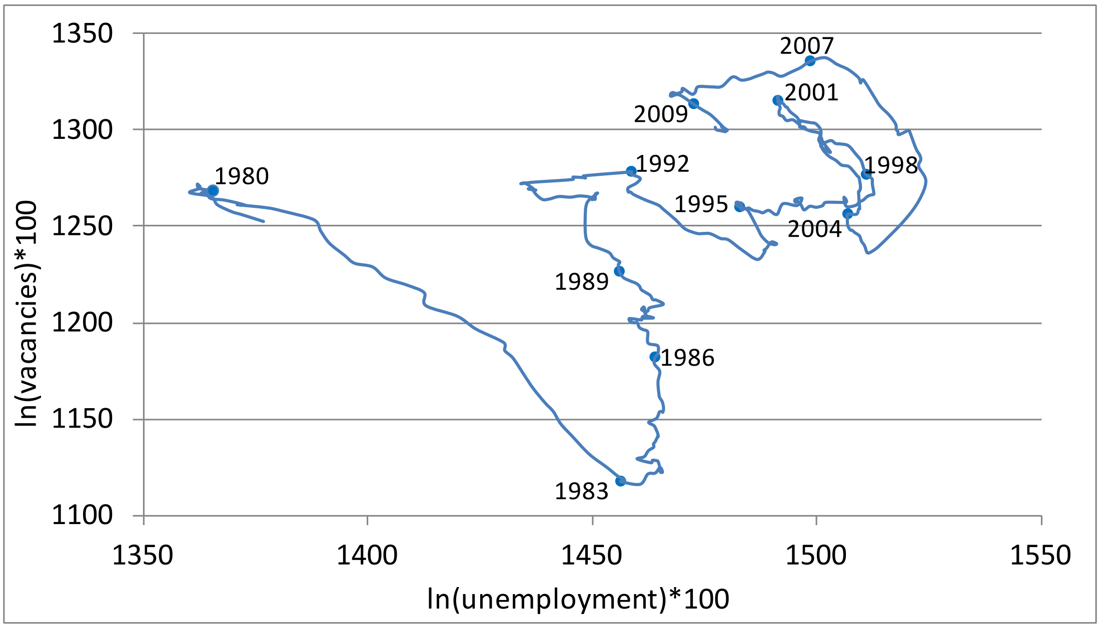
**Table 1**
**Unit root tests**

| series | test | value of test statistic | 5 percent critical value | lags | break date | deterministics |
|---|---|---|---|---|---|---|
| job finding rate | ADF<br>KPSS | -2.60<br>6.86 | -2.88<br>0.46 | 2 | 1991m3<br>none | constant |
| separation rate | ADF<br>KPSS | -2.55<br>4.66 | -2.88<br>0.46 | 3 | 1991m2<br>none | constant |
| employment | ADF<br>KPSS | -1.42<br>0.88 | -3.03<br>0.15 | 5 | 1992m1<br>none | constant, trend |
| vacancies | ADF<br>KPSS | -3.82<br>0.28 | -3.03<br>0.15 | 6 | 1991m6<br>none | constant, trend |
| unemployment | ADF<br>KPSS | -1.59<br>1.21 | -3.03<br>0.15 | 1 | 1991m6<br>none | constant, trend |

Note: ADF critical values according to Lanne et al. (2002), KPSS critical values according to Kwiatkowski et al. (1992), KPSS test on reunification adjusted series.

The Beveridge curve is presented in Figure 1. In the early 1980s, the German labour market experienced a quick and sharp decline in vacancies and increase in unemployment which makes the impression of a movement along a BC to its lower right edge. Afterwards, the BC steepened considerably. Thus, already before the German reunification, it became harder to exploit an increasing number of vacancies to reduce unemployment. Reunification itself caused a substantial right shift as a high number of people became unemployed in the course of the transition of Eastern Germany towards a market economy. (Statistical effects are already computationally eliminated.) The outward shift cannot only be attributed to the direct effects of the transition as it kept on until 2005. Moreover, this shift can be similarly observed in all federal states of Germany (Bouvet 2012) – it is not a purely Eastern German phenomenon but must be rationalized by structural, e. g. institutional reasons that prompt unemployment to persist.

**Figure 1**
**The German Beveridge curve, 1979 to 2009**




Source: Federal Employment Agency, data manipulation see text. Bullets refer to January of indicated year.

The upswing around the millennium induced a rather expectable movement of the Beveridge curve.

Starting in late 2006, the BC shifted inwards for the first time in decades. By that time, Germany had experienced a year-long phase of moderate wage increases. In addition, a whole bunch of labour market reforms had come into force (for an overview see Klinger/Rothe 2012). They included features as, for example, tightened unemployment benefit, deregulation of employment protection and temporary agency work, increase in part-time and fixed-term contracts. Those features are considered to improve the BC position (Bouvet 2012).

The inward shift came to an end in late 2008 when the Great Recession hit the German labour market. Its response to a drop of GDP by more than 5 percent was notedly moderate. Labour demand did not drop as much as was expected from the comparison to previous recessions. This calls for another or a new structural effect that could have overlaid the crisis period and that does not show up in a shift of the BC but in a just modest movement along the curve.

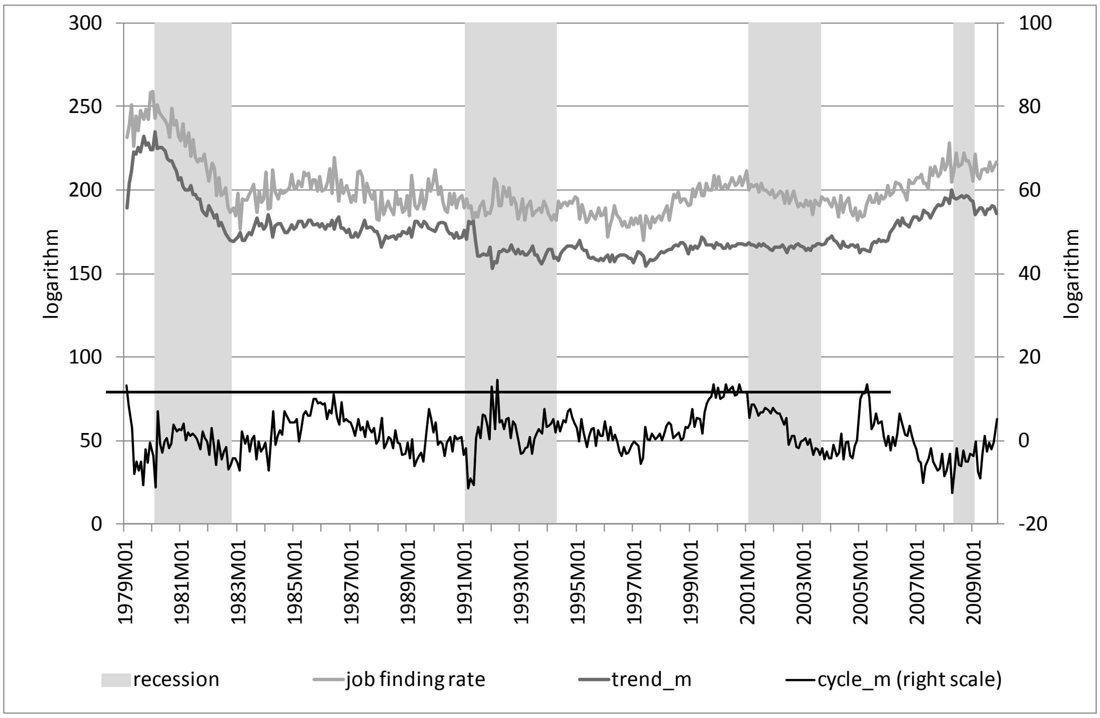
The results of the unobserved components analysis will shed light on the driving forces of the dynamics outlined above.

## 6 Results

### 6.1 States, innovations, and Beveridge curve dynamics

The estimated unobserved components of matching efficiency, separation rate, and tightness are shown in Figures 2 to 4.[5] They share a common feature that is typical of correlated unobserved components models or Beveridge-Nelson-type decompositions (Morley et al. 2003): highly volatile trends. They reflect the multitude of shocks that cause persistent effects on the labour market and that can also compensate each other. Summarizing the co- or countermovement of the trends and cycles in the graphs, we can conclude on permanent or transitory dynamics of the German Beveridge curve.

**Figure 2**
**Job finding rate and matching efficiency: observations, trend and cycle**




Source: Institute for Employment Research and own estimation. Business cycle dating by ECRI.

Figure 2 displays the job finding rate as well as trend and cycle of matching efficiency; note that the trend constantly lies below the observed series due to the additional terms U and V on the right hand side of the matching function in (5). In the early 1980s, the job finding rate experienced a severe downturn caused by a drop in trend matching efficiency that has never fully recovered ever since. Probably, negative productivity shocks, e. g. following the second oil crisis, decreased transitions into work, and unfit institutions inhibited labour market functioning from improving again. After 1985, for example, several law changes made unemployment insurance

[5] Further results on variables not discussed in this section but included in the complete model specification are available from the authors on request.

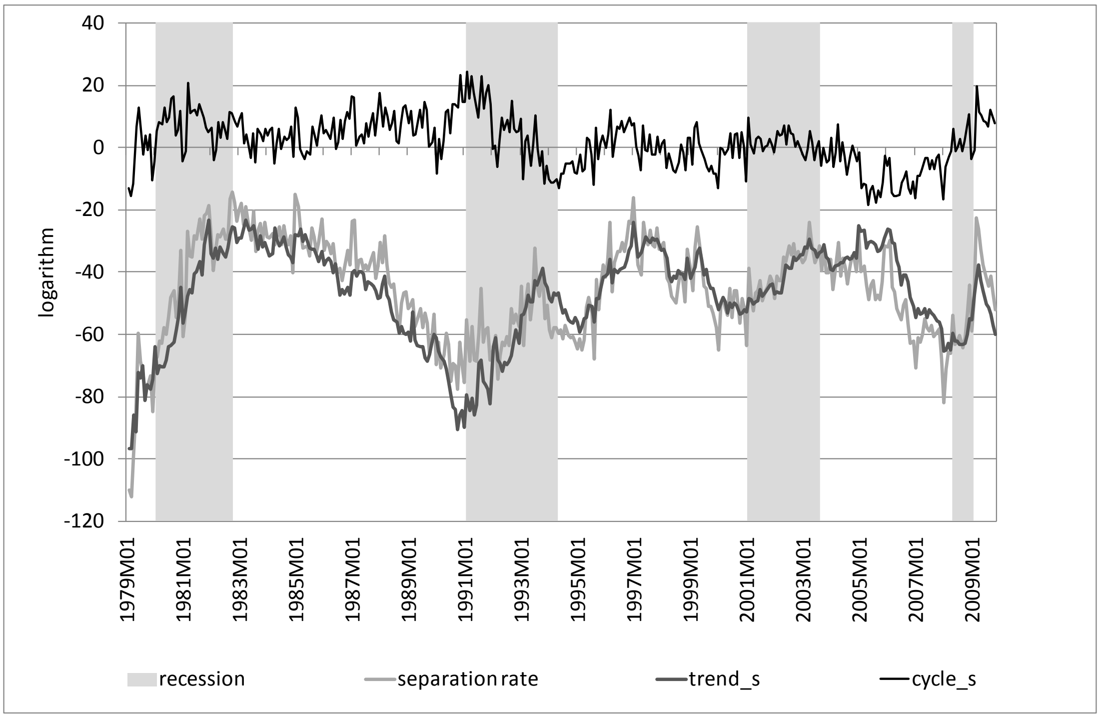
more generous for people aged 42 and older.[6] A further, much slighter decrease took place at the beginning of the 1990s. An increase of the job finding rate around millennium must be classified as only transitory as the trend of matching efficiency stayed flat. By contrast, following wage moderation and labour market reforms that worsened employees´ outside options, trend efficiency developed slightly better than job finding probability and from 2005 on it improved remarkably. A similar increase has not been observed during the past 30 years. This finding supports empirical studies that state a positive impact of the labour market reforms on matching efficiency (Fahr/Sunde 2009, Klinger/Rothe 2012, Sala et al. 2012). The Great Recession at the turn of the years 2008/2009 did not lead to a sharp cyclical reaction but seems to mark the fading out of previous structural effects on matching efficiency.

The development of the separation rate is substantially driven by the trend; increases during GDP recessions and decreases during GDP expansions were often caused by permanent shocks. After reunification, cycles have become more pronounced, especially increases in the cyclical separation rate at the beginning of the 1990s and during the Great Recession. In the years before the Great Recession, structural improvements on the German labour market became visible also in a decline of the trend separation rate which was even accompanied by decreasing cyclical separations. Compared to history, however, this decrease was not as outstanding as the increase of trend matching efficiency at the same time.

Matching efficiency and separation rate determine the intercept of the BC with different sign. Changes in these parameters lead to shifts of the curve if they do not compensate each other. From the graphical analysis of the unobserved states we can conclude that the BC shifted for both permanent and transitory reasons. In the early 1980s, the flat BC slope does not (solely) indicate a movement along the curve, but a distinct permanent shift due to trend matching efficiency shrinking and trend separation rate rising sharply. The only inward shift in the past 30 years was caused by exactly the reverse development. In between, however, the shifts to the right after reunification were merely caused by further increases in the structural separation rate whereas matching efficiency experienced only minor changes.

[6] On the micro level, however, studies on the effects of this prolongation of unemployment compensation entitlement on unemployment duration yield discordant results (Hunt 1995, Fitzenberger/Wilke 2010).

**Figure 3**
**Separation rate: observations, trend and cycle**




Source: Institute for Employment Research and own estimation. Business cycle dating by ECRI.

This visual impression is confirmed by a comparison of shock variances that reveal the relative importance of the two parameters in explaining historical shifts of the BC (Table 2). The variances of the shocks to trend and cycle separation rate are much larger than those of the matching efficiency shocks. Thus, most of the shifts are explained by separation rate rather than matching efficiency. This connects to the studies by Fujita/Ramey (2009) or Hertweck/Sigrist (2012) that stress the importance of the separation rate for unemployment dynamics. Furthermore, the cycle shock exhibits a similar variance as the trend shock for matching efficiency whereas for the separation rate, the cycle shock variance is three times as large as the trend shock variance. These results confirm previous studies on the instability of the German BC (Börsch-Supan 1990, Kosfeld et al. 2008).

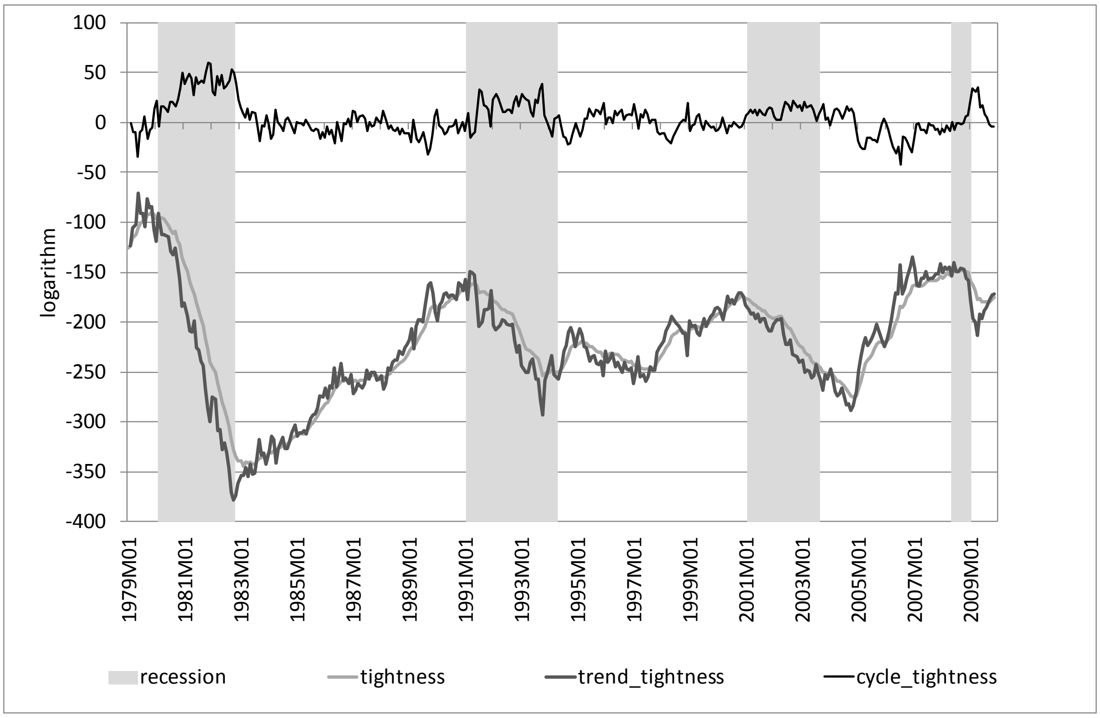
**Table 2**
**Trend and cycle shock variances of matching efficiency, separation rate, and tightness**

| | variance | | | |
|---|---|---|---|---|
| | trend | | cycle | |
| | | percent of total VAR(i) | | percent of total VAR(i) |
| matching efficiency | 5.67 | 57.9 | 4.04 | 41.3 |
| separation rate | 10.08 | 21.0 | 32.93 | 68.6 |
| tightness | 154.20 | 274.0 | 62.51 | 111.2 |

Note: variance proportions of tightness larger than 100 because of negative covariance between trend and cycle.

At first glance, two descriptions seem to contradict our interpretation so far: First, despite the sharp changes in matching efficiency and separation rate, the development of the BC at the beginning of the 1980s may well be misinterpreted as movement along the curve. Second, the inward shift in the aftermath of labour market reforms and wage moderation appeared relatively late compared to the major changes of the shifting parameters. The apparent conflict dissolves when we consider the overlay of shifts on the one hand and movements along the curve caused by – permanent or transitory – rotations of the job creation curve on the other. To get an idea of these rotations, we calculate unobserved components of labour market tightness from the components of vacancies and unemployment as well as their interactions with the other variables, all estimated in the model. As Figure 4 shows, much of the variation in the data is ascribed to the trend. It exhibits a variance almost three times as high as the cycle variance (see Table 2). Consequently, rotations of the JCC occur mostly for permanent reasons. This is in line with business cycles driven by permanent productivity shocks (Morley et al. 2003, Weber 2011) and with structural processes like sectoral or technological change going along with business cycle fluctuations (e. g. Caballero/Hammour 1994), i. e. reacting differently in recessions and expansions. As is typical of unobserved components models, the series of tightness adjusts sluggishly to trend shocks. This pattern creates a cycle with opposite sign during the time of adjustment (Morley et al. 2003) and implies a negative correlation between trend and cycle of tightness.

**Figure 4**
**Tightness: observations, trend and cycle**




Source: Federal Employment Agency and own estimation. Business cycle dating by ECRI.

Taking the development of trend matching efficiency, trend separation rate and trend tightness together, it becomes obvious that the flat downward slope of the BC in the early 1980s actually reflects permanent shifts to the right and down at the same time. Shrinking matching efficiency and rising separation rate led to the outward shift that was amended by a permanent movement down the curve as tightness shrank simultaneously. Similarly, the steep increase in tightness after 2005 retarded the appearance of the BC inward shift until tightness had reached its new persistent plateau. Tightness rose so sharply because vacancies reacted more quickly to the change in economic conditions whereas it took unemployment some time to adjust to the new institutional framework on the labour market. Besides this, deregulation in some segments led to an increase in demand for highly flexible labour such as short-term, fixed-term, part-time or temporary agency work. An increase in these kinds of jobs may inflate vacancies by a high turnover but not equivalently reduce unemployment. The slow adjustment of unemployment is confirmed by a negative correlation between its trend and cycle.

The trend restriction in equation (6) that translates cointegration into our model provides us with an analytical tool to calculate the relative importance of changes in trend matching efficiency or trend separation rate for changes in “structural” unemployment: As an example comparison, we calculate average monthly changes for economic expansion years 1999/2000 and 2006-2008 when the remarkable BC inward shift took place. In the first phase, log trend unemployment shrank by 0.7 per month on average, in the second phase the monthly reduction was -1.2. The role of trend matching efficiency changes in these reductions increased from 31.8 in the first phase to 56.3 percent in the second phase while the importance of trend separation rate changes decreased from 94.1 to 58.8 percent. Employment, vacancies and the deterministic trend generally played a minor role which was larger in the first upswing, however.

## 6.2 Correlations and labour market implications

The correlated unobserved components model allows shedding at least some light on the complex interactions on the labour market. Because of sluggish adjustment of time series to permanent shocks, most unobserved components studies estimate a negative correlation between trend and cycle of one and the same variable. In our application, this is only true for tightness whereas matching efficiency and separation rate show up with hardly any correlation between their trend and cycle shocks (Table 3). This accounts either for a very quick adjustment of the shifting parameters to changes in their trends or – which is more plausible with regard to the multiple sources of correlations in our multivariate model – for compensatory underlying effects.

Matching efficiency and separation rate are negatively correlated in trends (-0.4) as well as in cycles (-1.0). With regard to the permanent components this correlation reveals that institutional and structural change or permanent productivity shocks work through both channels in a similar way: the BC relocates through both shifting parameters into the same direction; the effects usually do not compensate each other (as would have been the case with positive correlation). Moreover, the two parameters follow an opposite cyclical pattern.

**Table 3**
**Correlations between the shocks of the main unobserved states**

| | trend_m | trend_s | trend_θ | cycle_m | cycle_s | cycle_θ |
|---|---|---|---|---|---|---|
| trend_m | 1 | -0.42 | 0.55 | 0.01 | -0.12 | -0.42 |
| trend_s | | 1 | -0.53 | 0.52 | 0.14 | 0.62 |
| trend_θ | | | 1 | 0.40 | -0.11 | -0.82 |
| cycle_m | | | | 1 | -1.00 | -0.68 |
| cycle_s | | | | | 1 | 0.18 |
| cycle_θ | | | | | | 1 |

Source: own calculation.

The cross correlation between the trend of matching efficiency and cyclical separation rate is economically irrelevant. The other way round, trend separation rate and cyclical matching efficiency are correlated at 0.5, suggesting a time-consuming structural adjustment process on the labour market that is transitorily compensated or at least hidden by cyclical patterns. Two examples may underpin this deliberation: First, a higher separation rate for structural reasons, e. g. when structural destruction in some sectors outperforms structural growth in others, may lead to lower trend matching efficiency because sector-specific human capital of those people who lost their job has become obsolete. Cyclical matching efficiency may rise temporarily, however, as such processes do not involve all companies of the concerned sectors at once but sequentially. Meanwhile, people who just became unemployed may still find jobs. A second example is the reaction to a permanent productivity shock caused by technological progress. As interpreted above, such a shock may lead to increasing trend matching efficiency and shrinking trend separation rate as jobs stay more productive. The positive correlation with cyclical matching efficiency suggests a transitory reduction in the efficiency parameter. As new technological standards need time to spread over the economy and the labour market needs time to supply suitably educated workers, this transitory decline in matching efficiency appears plausible.

The correlations between the shifting parameters and tightness emphasize the overlay of shifts and movements along the curve, which was carved out in the previous subsection. The correlations between trend tightness and trend matching efficiency at 0.6 as well as trend separation rate at -0.5 imply permanent outward shifts being accompanied by permanent movements down the curve or permanent inward shifts being accompanied by permanent movements up the curve. The first, negative implication characterizes the BC development in the first 25 years of our observation period. Without that overlay, the curve in Figure 1 would have shifted even more steeply into the upper right corner. The latter, positive implication became visible after 2006. Economically, this can be explained by a common dependence of separations, vacancies, and unemployment on permanent productivity shocks. Moreover, labour market institutions that change trend matching efficiency correspondingly change tightness as they influence vacancies via the search costs and benefits of the companies and unemployment via its duration.

## 6.3 The matching function

The unobserved components approach enables us to disentangle permanent and transitory effects in the matching function. Not only can we consider time varying matching efficiency but also different elasticities of matches with respect to trend versus cyclical unemployment and vacancies. Thus, we generalize the matching function with regard to the unobserved components.

(11) $f_t = \tau_t^m + c_t^m + (\alpha - 1)\tau_{t-1}^U + (\alpha^c - 1)c_{t-1}^U + \beta \tau_{t-1}^V + \beta^c c_{t-1}^V + w_t$

Parameter estimates are given in Table 4.

The generalization does not reveal different conclusions regarding unemployment. The elasticities of matches with respect to both trend and cyclical unemployment are estimated at nearly 1 which is high compared to the summaries by Petrongolo/ Pissarides (2001) or Broersma/van Ours (1999). However, since we allowed for time-varying matching efficiency in our function, results are not directly comparable. Economically, an elasticity substantially smaller than 1 implies a disproportionately smaller reduction in matches when unemployment shrinks. This may not be plausible as a reduction in unemployment typically keeps merely bad risks within the pool. Matching function estimates for the group of the long-term unemployed by Klinger/Rothe (2012) came up with an extremely robust elasticity between 0.9 and 1, too.

**Table 4**
**Estimation results for the matching function**

| | | coefficient | standard error | significance |
|---|---|---|---|---|
| elasticity of matches with respect to | | | | |
| trend unemployment | $\alpha$ | 0.998 | 0.087 | *** |
| cyclical unemployment | $\alpha^c$ | 1.042 | 0.187 | *** |
| trend vacancies | $\beta$ | 0.019 | 0.220 | |
| cyclical vacancies | $\beta^c$ | 0.271 | 0.137 | ** |

Significance levels: *** 1 percent, ** 5 percent.
Source: own calculation.

In contrast to unemployment, the impact of vacancies in the generalized matching function differs considerably between trend and cycle: trend vacancies being not significant, it is only the elasticity of matches with respect to cyclical vacancies that shows up with a significant coefficient of 0.3. [7] This result resembles estimates of

[7] In sum, our matching function then reveals slightly increasing returns to scale. Petrongolo/Pissarides (2001) also summarize studies that reject the null of constant returns. They argue as theoretical disadvantage, however, that increasing returns to scale are compatible with more than one equilibrium, one with high and one with low search activity.

aggregate stock-flow matching functions that were rationalized by a systematic element in search (Coles/Smith 1998, Gregg/Petrongolo 2005, Fahr/Sunde 2009): unemployed people already know the existing stock of vacancies but did not match. In the next round of applications, they will focus on newly incoming vacancies. Even though some people that just entered unemployment may match with old vacancies, this effect is not large enough to create a significant contribution of the stock of vacancies in the matching function.[8] As these vacancies do not merge into matches, they become persistent and form at least part of the permanent component. Instead, inflows into vacancies have a significant impact with an elasticity usually estimated between 0.3 and 0.4. Those new vacancies are filled more easily, raise matches and, consequently, disappear – they are transitory. Given that such a procedure does not only refer to inflows but also holds for a proportion of our stock variable, this proportion of very short-term vacancies forms our cyclical component.

# 7 Conclusions

The picture of the German Beveridge curve between 1979 and 2009 shows movements along the curve with changing slope as well as many outward and one inward shift. As a reduced-form relationship, it reveals no insights into the drivers of these dynamics. Structural content is added to these descriptions by means of an unobserved components analysis, further considering cointegration between labour market variables. It disentangles each of the BC constituents into a permanent and a transitory series and allows analysing the time-varying properties of matching efficiency, separation rate, and tightness. Thereby, we gather information on the determinants of shifts of the BC, movements along the curve, and their interaction.

The inspection revealed the following main results: First, the flat negative slope in the early 1980s is a result of shifts to the right and downwards at the same time. Trend matching efficiency and trend tightness shrank, trend separation rate rose. Between 1985 and 2005, matching efficiency played a minor role for BC dynamics. By contrast, the increasing separation rate led to an ongoing outward shift. It was not compensated by increasing matching efficiency. Unemployment, especially long-term unemployment, rose instead. After 2005, in the aftermath of severe labour market reforms and long-lasting wage moderation, rising matching efficiency and shrinking separation rate allowed for an inward shift of the BC. A sharp contemporary increase in tightness retarded the appearance of that inward shift. It became visible after tightness had reached a new plateau by 2007. Beyond the observation sample of this paper, tightness increased further and added another upward shift to the right shift of the curve between 2006 and 2008.

Second, the analysis revealed a large variance of the shifting parameters that is due to transitory shocks: about 40 percent of the total variance of matching efficiency

[8] Incidentally, this implies an elasticity of 1 for the stock of unemployment in constant returns to scale specifications as in Gregg/Petrongolo (2005).

and 70 percent of the total variance of separation rate. However, as we are able to disentangle trend and cycle, we can conclude that the major shifts, especially the improvement in the past years, were for permanent reasons.

Third, separation rate is more important in explaining BC shifts than matching efficiency. This is true for both, trend and cycle.

Fourth, changes in labour market tightness are also driven by permanent shocks, which is in line with productivity fluctuations driven by a stochastic trend found in previous unobserved components models and theoretically underpinned by the RBC literature. Other structural shocks may influence tightness, too. Especially its latest permanent increase was accompanied by higher flexibility of labour demand through, for example, temporary agency work. As a consequence, labour turnover may inflate vacancies but does not equivalently reduce unemployment.

In summary, the labour market is marked by an ongoing overlay of permanent and transitory movement that refers to both, shifts and movements along the Beveridge curve. Policy recommendations must take these movements and their permanent or transitory sources into account. Clearly, appropriate design and choice of policy measures depends on observed labour market development being of either structural or cyclical nature and on the concrete institutional or economic driving forces. Thus, structural decompositions of this kind are a promising direction of future research, providing valuable landmarks for appropriate policy design. One further step into this direction can be given by digging deeper into the core of the correlations between trends and cycles.

# 8 Appendix

## 8.1 Identification

Using matrix representation, the first part of the structural model given in equations (5) to (7) reads as follows. Thereby, $k$ is the number of endogenous variables and $r$ the number of independent stochastic trends. The vector of endogenous variables, $y$, includes job finding rate $f$, separation rate $s$, employment $E$, vacancies $V$, and unemployment $U$ (all in logs):

$$\underset{k\times 1}{\begin{pmatrix} y_{1t} \\ \vdots \\ y_{kt} \end{pmatrix}} = \underset{k\times r}{\begin{pmatrix} I_r & & & \\ & & & \\ -\frac{1}{\alpha} & \frac{1}{\alpha} & \frac{1}{\alpha} & -\frac{\beta}{\alpha} \end{pmatrix}} \underset{r\times 1}{\begin{pmatrix} \tau_{1t} \\ \vdots \\ \tau_{rt} \end{pmatrix}} + \underset{k\times 1}{\begin{pmatrix} c_{1t} \\ \vdots \\ c_{kt} \end{pmatrix}} + \underset{k\times r}{\begin{pmatrix} -\frac{\alpha-1}{\alpha} & \frac{\alpha-1}{\alpha} & \frac{\alpha-1}{\alpha} & \frac{\beta}{\alpha} \\ & & & \\ \underset{k-1\times r}{0} & & & \end{pmatrix}} \underset{r\times 1}{\begin{pmatrix} \tau_{1,t-1} \\ \vdots \\ \tau_{r,t-1} \end{pmatrix}} + \underset{k\times k}{\begin{pmatrix} 0 & 0 & 0 & \beta^c & \alpha^c \\ & & & & \\ \underset{k-1\times k}{0} & & & & \end{pmatrix}} \underset{k\times 1}{\begin{pmatrix} c_{1,t-1} \\ \vdots \\ c_{k,t-1} \end{pmatrix}} + \underset{k\times 1}{\begin{pmatrix} w_t \\ 0 \\ \vdots \end{pmatrix}}$$

$$\underset{k\times 1}{y_t} = \underset{k\times r}{T} \underset{r\times 1}{\tau_t} + \underset{k\times 1}{c_t} + \underset{k\times r}{A} \underset{r\times 1}{\tau_{t-1}} + \underset{k\times k}{A^c} \underset{k\times 1}{c_{t-1}} + \underset{k\times 1}{w_t}$$

To derive the reduced form we first take first differences that lead to stationary series. $L$ is the lag operator:

$$\Delta y_t = (T + AL)(I - L)\tau_t + (I + A^c L)(I - L)c_t + (I - L)w_t$$

Trend vector and cycle vector are given in the structural form as multivariate random walk and multivariate autoregression, respectively (see equations (9) and (10)):

$$\begin{pmatrix}\tau_{1t}\\ \vdots \\ \tau_{rt}\end{pmatrix} = \begin{pmatrix}\mu_1\\ \vdots \\ \mu_r\end{pmatrix} + \begin{pmatrix}\tau_{1,t-1}\\ \vdots \\ \tau_{r,t-1}\end{pmatrix} + \begin{pmatrix}\eta_{1t}\\ \vdots \\ \eta_{rt}\end{pmatrix} \quad \Rightarrow \quad (I-L)\underset{r\times 1}{\tau_t} = \underset{r\times 1}{\mu} + \underset{r\times 1}{\eta_t}$$

$$\underbrace{\begin{pmatrix}\Phi^1(L) & 0 & \cdots \\ 0 & \ddots & \\ \vdots & & \Phi^k(L)\end{pmatrix}}_{\Theta(L)} \begin{pmatrix}c_{1t}\\ \vdots \\ c_{kt}\end{pmatrix} = \begin{pmatrix}\varepsilon_{1t}\\ \vdots \\ \varepsilon_{kt}\end{pmatrix} \quad \Rightarrow \quad \underset{k\times 1}{c_t} = \underset{k\times k}{\Phi(L)^{-1}} \underset{k\times 1}{\varepsilon_t}$$

$$\text{with } \Phi^i(L) = 1 - \phi_1^i L - \ldots - \phi_p^i L^p \ , \ i = 1,\ldots,k$$

Inserting into the differenced equation and rearranging gives the reduced form:

$$\Phi(L)\Delta y_t = \Phi(1)(T + A)\mu + \Phi(L)(T + AL)\eta_t + (I + A^c L)(I - L)\varepsilon_t + \Phi(L)(I - L)w_t$$

A complete description of the data generating process includes the cointegration relation between the model variables; see Morley (2007) for a similar example. In our setup, cointegration is achieved by the long-run flow equilibrium in equation (2) by the fixed cointegrating vector [1 1 -1 -1]. As there are no free parameters to be identified from this equilibrium relation, we skip this part of the reduced form for brevity.

According to Granger´s Lemma (Granger/Morris 1976), the three right-hand-side MA expressions add up to a new MA process whose order is determined by the highest lag length of the original processes[9]: $\omega_t \sim MA(p+1)$. Hence, the reduced form is a VARIMA($p$,1,$p$+1) process. Thereby, the only row that lag length $p$+1 applies to is the matching function, all other elements of $\omega_t$ are *MA(p)*.

The cycle coefficients are directly identified from the AR coefficients of the VARIMA. The drift terms are determined from the reduced-form intercept once we know about the matrices *T* and *A* that include the structures of the restricted trend and the matching function.

Information on *T* and *A* as well as information on the variances and covariances of the structural shocks must be identified from the autocovariances of the newly formed MA process $\omega_t$:

$$\begin{aligned} &\Gamma(j) = E\left(\omega_t \omega_{t-j}'\right) && \text{for} \quad 0 \le j \le p+1 \\ &\Gamma(j) = 0 && \text{for} \quad j > p+1 \end{aligned} \qquad p \ge 2$$

The first autocovariance matrix $\Gamma(0)$ provides information on *k* variances and (*k*(*k*-1))/2 covariances. The following autocovariance matrices $\Gamma(1)$ to $\Gamma(p)$ each provide

9 The lag order of the new MA process would decline if the original processes shared common roots.

$k^2$ pieces of information as the elements below and above the main diagonal are no longer identical. The last non-zero autocovariance matrix $\Gamma(p+1)$, however, provides only $k$ pieces of information as the vector $\omega_{t-p-1}$ contains only one non-zero element. In sum, the autocovariances of the MA term of the reduced form deliver $\left(\frac{1}{2}+p\right)k^2+\frac{3}{2}k$ equations containing the parameters of

$T$, $A$, $A^c$, as well as $COV\begin{bmatrix} w_t \\ e_t \end{bmatrix}$.

The number of unknowns in the structural form beyond the AR coefficients and drift terms is given by: 4 parameters and a variance of the matching function, r trend shock variances, k cycle shock variances and the related covariances between all of them (see $COV\begin{bmatrix} w_t \\ e_t \end{bmatrix}$ in appendix 8.2). They add up to $\frac{1}{2}\left(r^2+k^2\right)+r\cdot k+\frac{3}{2}(r+k)+5$.

The necessary condition for identification requires the number of unknowns being not larger than the number of autocovariance equations. The comparison uncovers that this condition is fulfilled for lag lengths $p \geq 2$.

## 8.2 State space representation

In the state-space model, both trends and cycles are treated as unobserved states. The measurement equation connects them to the observed series (*ars* abbreviates the total number of autoregressive coefficients $k \cdot p$):

$$\underset{(k\times 1)}{y_t} = \underset{(k\times 2\cdot r+ars)}{G} \; \underset{(2\cdot r+ars\times 1)}{z_t} + \underset{(k\times 1)}{w_t}$$

with the vectors of endogenous variables $y_t = \begin{pmatrix} f_t & s_t & E_t & V_t & U_t \end{pmatrix}'$ and unobserved states

$$z_t = \begin{pmatrix} \tau_t^m & \tau_t^s & \tau_t^E & \tau_t^V & c_t^m & c_t^s & c_t^E & c_t^V & c_t^U & c_{t-1}^m & c_{t-1}^s & c_{t-1}^E & c_{t-1}^V & c_{t-1}^U & c_{t-2}^m & \cdots & c_{t-p+1}^U & \tau_{t-1}^m & \tau_{t-1}^s & \tau_{t-1}^E & \tau_{t-1}^V \end{pmatrix}'$$

The connection to the observables is given in the parameter matrix *G*.

$$G = \left(\begin{array}{cccc|ccccc|ccccc|c|cccc}
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta^c & (\alpha^c - 1) & \underset{k\times ars-2\cdot k}{0} & -\frac{\alpha-1}{\alpha} & \frac{\alpha-1}{\alpha} & \frac{\alpha-1}{\alpha} & \frac{\beta}{\alpha} \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & & & & \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & & \underset{k-1\times r}{0} & & \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & & & & & \\
-\frac{1}{\alpha} & \frac{1}{\alpha} & \frac{1}{\alpha} & -\frac{\beta}{\alpha} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & & & & &
\end{array}\right)$$

The residual vector contains only the innovation of the matching function: $w_t = \begin{pmatrix} w_t & 0 & 0 & 0 & 0 \end{pmatrix}'$

The transition equation describes the evolution of the unobserved states:

$$\underset{(2\cdot r+ars\times 1)}{z_t} = \underset{(2\cdot r+ars\times 1)}{d} + \underset{(2\cdot r+ars\times 2\cdot r+ars)}{H} \; \underset{(2\cdot r+ars\times 1)}{z_{t-1}} + \underset{(2\cdot r+ars\times 1)}{e_t}$$

The transition matrix *H* specifies the random walks of the trends and the autoregressions of the cycles. Further elements (zeros and ones) are needed to account for lagged states. Each matrix $\Phi_i$ contains the AR coefficients of the *i*-th lag on the main diagonal and zeros on the secondary diagonals.

$$H = \left(\begin{array}{c|ccc|c} I_r & & \underset{r\times ars}{0} & & \\ \hline \underset{ars\times r}{0} & \underset{k\times k}{\Phi_1} & \cdots & \underset{k\times k}{\Phi_p} & \underset{2r+ars\times r}{0} \\ \cline{2-4} & & I_{ars-k} & \underset{k\times k}{0} & \\ \hline I_r & & \underset{r\times ars}{0} & & \end{array}\right)$$

Drift terms are specified for the four trends: $d = \begin{pmatrix} \mu^m & \mu^s & \mu^E & \mu^V & \underset{r+ars\times 1}{0} \end{pmatrix}'$

The vector of transition shocks is given by

$$e_t = \begin{pmatrix} \eta_t^m & \eta_t^s & \eta_t^E & \eta_t^V & \varepsilon_t^m & \varepsilon_t^s & \varepsilon_t^E & \varepsilon_t^V & \varepsilon_t^U & \underset{r+ars-k\times 1}{0} \end{pmatrix}'$$

The covariance matrix (including measurement shocks) reads as

$$COV\begin{bmatrix} w_t \\ e_t \end{bmatrix} = \begin{pmatrix} E(w_t w_t') & E(w_t e_t') \\ E(w_t e_t')' & E(e_t e_t') \end{pmatrix}$$

As $w_t$ only contains the shock of the matching function but is zero else, $E(w_t w_t')$ is a quadratic matrix of dimension *k* with the first element representing the variance of that shock and all other elements being zero. The covariance matrix of the transition shocks, $E(e_t e_t')$, is quadratic of dimension *2r+ars* but only the upper left $r+k \times r+k$ submatrix contains non-zero elements referring to *r* trend shock variances, *k* cycle shocks variances and their respective covariances. Thus, this relevant part of $E(e_t e_t')$ is given by

$$\begin{pmatrix} \sigma^2_{\eta^m} & \cdots & \sigma_{\eta^m\eta^V} & & \cdots & \sigma_{\eta^m\varepsilon^U} \\ \vdots & \ddots & & & & \vdots \\ \sigma_{\eta^m\eta^V} & & \sigma^2_{\eta^V} & & & \\ & & & \sigma^2_{\varepsilon^m} & & \sigma_{\varepsilon^m\varepsilon^U} \\ \vdots & & & & \ddots & \vdots \\ \sigma_{\eta^m\varepsilon^U} & \cdots & & \sigma_{\varepsilon^m\varepsilon^U} & \cdots & \sigma^2_{\varepsilon^U} \end{pmatrix}$$

Accordingly, only $k \times (r+k)$ covariances between measurement and transition shocks in $E(w_t e_t')$ could be non-zero. With respect to the matching function innovation being the only non-zero element in $w_t$, however, the relevant shocks are all summarized in the first row of this $k \times (r+k)$ submatrix.

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