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Minimum Wages, Wage Dispersion and Unemployment

A Review on New Search Models

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Abstract

This paper analyses theoretical effects of minimum wages on employment and the wage distribution under a frictional setting. I review new developments in search theory and discuss the influence of minimum wages on wages and employment under each setting. Thereby, a major theoretical focus of the paper is the integration of heterogeneity on both sides of the market in equilibrium search models. In the homogeneous case minimum wages do not affect employment, while in the heterogenous case theoretical results are mixed. There is no unique connection between unemployment and minimum wages, and the effect can be positive, zero or negative. However, the most advanced models, integrating heterogeneity on both sides of the market, seem to support the hypothesis that an increase in the minimum wage generally leads to an increase in unemployment as well.

JEL classification: E24, J21, J31, J64

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1 Wage dispersion and unemployment: Alternative views

Many economists argue that institutions as minimum wages compress the wage structure and thereby contribute to high unemployment, especially for low-skilled individuals. This hypothesis is common in the economic literature in the context of the different experiences in the US and Continental Europe regarding wage dispersion and unemployment [see e.g. Katz and Autor, 1999, Blau and Kahn, 2002, Blanchard, 2006]. Krugman [1994, p.62] states: "...that growing U.S. inequality and growing European unemployment are different sides of the same coin". Many observers argue that skill-based technical progress, reorganization processes or globalization have decreased the demand for low-skilled work in industrialized countries, thereby lowering the market wage. However, in Continental Europe institutional reasons have prevented wages from falling (enough), causing a reaction via the amount of labor employed and thereby increasing unemployment of the low-skilled.[1] This view is not uncontested for several reasons. Firstly, changes in employment rates in Europe were quite similar across skill groups and changes in the employment rates of the low-skilled were quite similar in Europe and the US [Acemoglu and Pischke, 1999]. Another reason is that institutional differences can explain differences in unemployment and inequality levels, but not in changes. Explaining changes in these variables requires changes in institutions. The decline of the minimum wage in the U.S., however, is often argued to be such an institutional change. In neo-classical labor market models, it is assumed that the pivotal determinant of wages is marginal productivity. If people differ in their marginal productivity, in equilibrium, they obtain different wages. Thus, the wage distribution is determined by the distribution of marginal productivities. Under these assumptions binding minimum wages generally lower the wage dispersion of those workers employed, but at the same time tend to reduce employment. This is the case, because then some individuals are too unproductive to be still employed at the higher minimum wage. Thus, a minimum wage causes structural unemployment.

The situation is different under a frictional setting. The reason is that frictions are a source of monopsony power for employers and that wages are below marginal productivity [Manning, 2003b]. Clearly, there is potential for redistribution of rents without altering employment. Do minimum wages purely redistribute rents from the firms to the workers or do they cause structural unemployment as well? I show that the answer to this question is ambiguous and that the discussed model variants yield different results. I obtain mostly zero employment effects. However, there are cases, where the minimum wage generates even positive employment effects, because it does not alter the incentive of the firm to employ individuals but it does increase the incentive for individuals to work.

Search frictions in the present case arise because of incomplete information where the process of generating information is time-consuming. Under this setting, identical workers can earn different wages and the sources of wage dispersion are search du-

[1] Institutional factors that can imply wage compression are minimum wages, strong unions, benefit payments and the like [see, e.g. Weiss and Garloff, 2005].

ration and luck. A central result of these theories challenges the neo-classical framework: rising wage dispersion is associated with rising unemployment. Low wage dispersion is associated with low unemployment. This contradicts the basic idea on the relationship of wage dispersion and unemployment presented above.[2]

In what follows, I present different search models of increasing complexity (see table 1) and examine the effect of minimum wages on the realized wage distribution and employment. Since the labor market performance of individuals and the performance of firms are extremely diverse, heterogeneity is seen to be an important feature of labor markets and thus a focus lies on the integration of heterogeneity in search models. Subsections 2.1 and 2.2 establish the theoretical basis on which most models are built upon. In subsection 2.1, I present basics from partial search theory with exogenous wage dispersion and derive the reservation wage property. In subsection 2.2, I establish the baseline model, a model with an endogenous wage distribution and homogeneous individuals and firms. In order to discuss more realistic settings, I discuss model extensions that allow heterogeneity on one or the other side of the market and which serve to check the sensitivity of the results of the baseline model.

2 Frictional labor markets

2.1 Exogenous wage dispersion: basic results

As a reference scenario and to understand later derivations, I briefly introduce the partial search model. Starting point from search theory is that working places are heterogenous in some relevant aspect. Job searching individuals have an information problem. They must acquire the relevant information about the jobs. This time-consuming process of obtaining information in the simple baseline search model is modeled by an exogenous hazard rate at which individuals receive job offers and thus information about certain job characteristics they are interested in. In the simplest version, job seekers maximize the present value of their lifetime income. So, the wage is the only relevant characteristic of the job. The optimization problem of the job seeker thus consists in getting as high as possible a wage, without searching too long. Under the Poisson assumption (for the probability of obtaining a job offer) search is sequential and rational behavior is given by a critical wage level, where wage offers above are accepted and wage offers below are rejected (see below). The critical wage is called reservation wage and defines the expected search duration which itself determines the expected wage level. Wages that differ across individuals can be explained by the luck of a high wage offer and by different reservation wages.

The following illustration is inspired by Cahuc and Zylberberg [2004, chapter 3.1] and Franz [2006, chapter 6.2].

[2] This idea is taken as a test between the frameworks presented in Weiss and Garloff [2005].

Table 1: Heterogeneity in search models

	Frictions	Endogenous Wage Dispersion	Heterogeneity of employees	Heterogeneity of firms
Neo-classical Model Framework	-	X	X	-
Partial search theory	X	-	X	X
Burdett and Mortensen [1998]	X	X	-	-
Burdett and Mortensen [1998] with continuous search costs, Van den Berg and Ridder [1997]	X	X	X	-
Bontemps, Robin, and Van Den Berg [2000], Acemoglu und Shimer [2000]	X	X	-	X
Postel-Vinay and Robin [2002b], Holzner and Launov [2005]	X	X	X	X

Assumptions

The assumptions under which the reservation wage property and the reservation wage can be deduced are concluded in what follows.

- (B0) Environment: The model is dynamic, time will be treated as continuous and the environment is stationary.
- (B1) Employees: Individuals exclusively either work or search, which precludes both on-the-job search and the existence of inactive individuals. There is no choice in the number of hours worked or searched. Individuals are risk-neutral, have rational expectations and maximize expected present value of their life time income over an infinite time horizon. Job seekers obtain $z = b - a$ per time unit, where b are unemployment benefits and a search costs. Employees obtain a wage w per time unit. The value of unemployment is called W_U (the expected

income), while $W_L(w)$ is the value of employment at the wage w.[3] The stationary wage offer distribution $H(w)$ is known to job seekers, while the offered wage of a specific firm is generally not known.[4]

- (B2) Search: Search is sequential, which means that if an individual has received an offer, he decides whether to accept or not and then in the case of rejection, i.e. if the wage offer is below the reservation wage w_R, continues search.[5] This is an optimal stopping problem, since job offers that have been rejected once cannot be accepted later on [Dixit, 1990]. The future is discounted at interest rate r.
- (B3) Transition rates: At an exogenous, constant rate λ an individual samples independent wage offers from $H(w)$. Individuals leave unemployment at a rate that is the product of the job offer rate and their acceptance probability. Employees loose their job at the exogenous, constant rate δ (the job destruction rate). The number of sampled job offers and the number of terminated jobs are poisson-distributed.

The basic model

The value of employment $W_L(w)$ at wage w can be derived as follows. In a small time interval[6] dt a worker obtains the wage wdt. With probability δdt the worker looses her job in this time interval. If losing the job the worker is left with value W_U. With the complementary probability $(1 - \delta dt)$ the worker remains employed. Under stationarity the value of employment is constant over time and therefore the worker is left with $W_L(w)$. In case of linear discounting, the Bellmann-equation is:

$$\begin{aligned} W_L(w) &= \frac{1}{1+rdt}\{wdt + \delta dt W_U + (1-\delta dt)W_L(w)\} \\ rW_L(w) &= w + \delta(W_U - W_L(w)). \end{aligned} \tag{1}$$

The second line can be found multiplying by $(1 + rdt)$, subtracting $W_L(w)$ and dividing by dt. The value of employment at the reservation wage must equal the value of unemployment. Rewriting (1) as $W_L(w) - W_U = \frac{w - rW_U}{r+\delta}$, taking into account that $\frac{\partial W_L(w)}{\partial w} = \frac{1}{r+\delta} > 0$ and that W_U is independent from the wage previously paid, then w_R is a unique solution to $W_L(w_R) = W_U$ and is given from $w_R = rW_U$ as the reservation wage. The offered wage must be at least as high as what the worker would have gotten if she had remained unemployed. This is demonstrated in figure 1.

[3] $W_L(w)$ and W_U are called value equations or Bellmann-equations. These terms have been coined in the theory of dynamic programming. For mathematical details see, eg, Dixit [1990], chapter 11.

[4] The wage offer distribution is the distribution of wages when randomly drawing a firm, whereas the wage distribution refers to the distribution of wages when randomly drawing a worker.

[5] I assume sequential search, since it has been shown that sequential search is superior to fixed sample search [McCall, 1965].

[6] The small time interval must be chosen such that the Poisson probability that two events occur in that time interval is zero.

Figure 1: Calculation of the reservation wage

The value of unemployment can be calculated as follows. Job seekers obtain $z = b - a$ per time unit. At poisson rate λ job seekers obtain job offers w as independent draws from $H(w)$. If an individual gets a job offer, the expected value is given by W_λ which consists in two components. The first component is the share of job offers that the job seeker rejects since the wage is below the reservation wage $H(w_R)$ multiplied by the corresponding value of rejection W_U. The second part is the complementary probability, multiplied by the average value of an accepted job offer:

$$W_\lambda = H(w_R)W_U + (1 - H(w_R))E_w[W_L(w)|w > w_R] = \int_0^{w_R} W_U dH(w) + \int_{w_R}^{\infty} W_L(w)dH(w). \tag{2}$$

With probability $(1 - \lambda dt)$ the job seeker does not obtain a job offer in dt. The value in this case remains $W_U = W_{\bar{\lambda}} = \int_0^\infty W_U dH(w)$.[7] Thus:

$$\begin{aligned} W_U &= \frac{1}{1 + rdt}(zdt + \lambda dt W_\lambda + (1 - \lambda dt)W_U) \\ rW_U &= z + \lambda \int_{w_R}^{\infty} (W_L(w) - W_U)dH(w). \end{aligned} \tag{3}$$

The second line follows by solving the equation for rW_U and resuming the remainder under an integral.

Recognizing that $W_L(w) - W_U = \frac{w - rW_U}{r + \delta}$ and $w_R = rW_U$ the reservation wage is implicitly defined as:

$$w_R = rW_U = z + \frac{\lambda}{r + \delta} \int_{w_R}^{\infty} (w - w_R)dH(w) = z + \frac{\lambda}{r + \delta}[(1 - H(w_R))(E(w|w > w_R) - w_R)]. \tag{4}$$

Subtracting z on both sides of the equation, the left hand side is the instantaneous cost of rejecting a wage offer w_R. At the reservation wage the cost of waiting must equal the expected gains from waiting. It is given by the probability that a job seeker obtains a wage offer that is acceptable $\lambda(1 - H(w_R))$, multiplied with the discounted conditional expectation of the wage given it exceeds the reservation wage [Devine and Kiefer, 1991, p.16/23].

Optimal behavior of the individuals is completely described by equation (4). A job seeker with net income z, who is confronted with a job offer rate λ, with a job destruction rate δ, with an interest rate r and a wage offer distribution $H(w)$ will accept any job offer that exceeds w_R and reject otherwise.

7 This is implied by the assumption that all model parameters are constant across time. Thus the value equations satisfy this property as well.

On-the-job search

Now, assume that employed job seekers obtain job offers at the exogenous job offer rate λ_L as well and that they do not incur search costs ($a_L = 0$).[8] The assumptions (B0), (B1'), (B2) and (B3) are assumed to hold where:

- (B1'): as (B1), but: Employees search on the job, receive independent job offers at constant exogenous rate λ_L from the wage offer distribution $H(w)$ and do not have any search cost.

One term must be added to the return to employment equation (1), which reflects the expected gain from a job change. An employee accepts all job offers that exceed its own wage $\bar{w}$ [Mortensen and Neumann, 1988]. It follows:

$$rW_L(\bar{w}) = \bar{w} + \delta(W_U - W_L(\bar{w})) + \lambda_L \int_{\bar{w}}^{\infty} (W_L(w) - W_L(\bar{w}))dH(w). \tag{5}$$

The return to unemployment is still given by equation (3). Evaluating equation (5) at w_R, equalizing $rW_L(w_R)$ with (3) and solving for w_R yields:

$$w_R = z + (\lambda - \lambda_L) \int_{w_R}^{\infty} (W_L(w) - W_U)dH(w). \tag{6}$$

Equation (6) can be rewritten in terms of the parameters of the model (see appendix 5.1):

$$w_R = z + (\lambda - \lambda_L) \int_{w_R}^{w^o} \frac{1 - H(w)}{r + \delta + \lambda_L(1 - H(w))} dw. \tag{7}$$

Intuitively, the possibility to search on the job lowers the reservation wage, since this opens the possibility to accept a low paid job and to search further on the job. If the chance to get a good job is independent on the job seekers status ($\lambda_L = \lambda$), unemployed job seekers will accept every job offer that exceeds net unemployment benefits z. With the characterization of the reservation wage for unemployed job seekers and the critical wage for employed job seekers, optimality in the behavior of the individuals is guaranteed. Unemployed job seekers accept every wage offer that exceeds w_R and continue searching otherwise, while employed job seekers accept every job offer that exceeds their current wage and continue working in their current job (and searching for a better paid job) otherwise.

The reservation wage increases with unemployment benefits b, and with the probability to obtain a job offer when unemployed λ. It decreases with search cost a, the probability to obtain a job offer when employed λ_L, the interest rate r and with the job destruction rate δ. Given the impact of the parameters on the reservation wage, the impact of the parameters on the average unemployment duration can be calculated, which increases itself with the reservation wage.

[8] The subscript L always confers to the employed individuals.

2.2 Endogenous wage dispersion - the baseline model

It is not straightforward to establish dispersion of job characteristics across identical firms as an equilibrium outcome. A necessary condition for the existence of equilibrium wage dispersion is a positive connection between wage level and output [Burdett and Judd, 1983]. If firms have monopsony power, firms make positive profits per employee. If firms can attract additional workers by setting high wages, there exists a trade-off between profits per worker and the number of workers in a firm. This means that there is indeed a positive connection between wages and production. The model of Burdett and Mortensen [1998] is based on this idea.

On-the-job search guarantees that part of the job seekers, namely the employed, can compare wages. They compare their own wage with the wage of an alternative job offer. This insures the positive relationship between wages and output. High wage firms attract many new workers from competing firms, while loosing only little. As a result they have a high employment, making low profits per employee. On the contrary low wage firms have a low employment level, thereby making high profits per worker.

The assumptions of the model are given by (B0), (B1"), (B2), (B3), (B4), (B5) and (B6), where:

- (B1"), as (B1'), but all N individuals produce an identical amount y of the consumption good per time unit, which can be interpreted as labor productivity, where $y > w_R$ holds and z is identical across all unemployed individuals.
- (B4) firms: An infinite amount of risk-neutral, c.p. identical firms on an interval $[0, 1]$ maximizes profits. There is nor market entrance or exit.
- (B5) wage formation: Firms determine wages ex ante from their profit maximization calculus.[9] This wage is payed forever, provided that the match does not end.
- (B6) economy: Small open economy with two goods and an exogenous interest rate r. The consumption good C is produced without capital (or with identical capital endowment in firms without depreciation), marginal productivity of labor is constant. The price of the consumption good is the numéraire, while w is the price of labor.

Starting with the reservation wage for the unemployed, it is given by equation (7), assuming that $r = 0$. Employed job seekers accept every wage offer that exceeds their current wage. Equilibrium unemployment follows from stationarity and equating in- and outflows to and from the pool of unemployed. In a small time interval dt $\lambda dt(1 - H(w_R))U$ unemployed individuals find a job, while $\delta dt(N - U)$ employees loose their

[9] Since firms determine wages ex-ante, it is possible that there is a meeting between agents where cooperation is profitable but where the match is not formed. Such a situation is called non-transferable utility.

job. Dividing by dt and building the limit for $dt \to 0$, I obtain $\dot{U} = \delta(N - U) - \lambda(1 - H(w_R))U$. Since firms that offer wages below the reservation wage make zero profits and since, as shown below, in equilibrium all firms make positive profits $H(w_R) = 0$ holds. Therefore equilibrium unemployment is

$$U = \frac{\delta N}{\delta + \lambda(1 - H(w_R))} = \frac{\delta N}{\delta + \lambda}. \tag{8}$$

Let $g(w)$ denote the density of the distribution of paid wages and $h(w)$ denote the density of the distribution of wage offers. The equilibrium employment of *one* firm that offers wage w is given by $l(w) = (N - U)g(w)/h(w)$. Then, $L(w) = (N - U)G(w) = \int_0^w l(\zeta)dH(\zeta)$ is the amount of employees, that is employed at a wage below w and $G(w)$ is the distribution of paid wages. Concentrating on employment changes in firms in an interval dt that offer (and pay) wages above w, they have inflows from the pool of unemployed and from firms that pay wages below w, while they loose employees only through exogenous job destruction. Employees that make a job-to-job transition remain in this wage class. Unemployed job seekers obtain a wage offer with probability λdt which exceeds with probability $(1 - H(w))$ the wage w. Individuals that are employed at a wage below w obtain with probability $\lambda_L dt$ a wage offer that exceeds w with probability $(1 - H(w))$. From this, total inflows are $\lambda dt U(1 - H(w)) + \lambda_L dt L(w)(1 - H(w)) = dt(\lambda U + \lambda_L L(w))(1 - H(w))$. With probability δdt exogenous shocks destroy existing jobs. The amount of jobs in firms that pay wages above w is given by $N - U - L(w)$. In equilibrium, employment in firms that pay wages above w is assumed constant (stationarity).

Solving for $G(w) = \frac{L(w)}{N-U}$ yields:

$$G(w) = \frac{\lambda U}{(N - U)} \frac{H(w)}{(\lambda_L(1 - H(w)) + \delta)} = \frac{\delta H(w)}{(\lambda_L(1 - H(w)) + \delta)}. \tag{9}$$

The second equality follows from using equation (8). It shows the relationship between the distribution of wage offers $H(w)$ across firms and the distribution of paid wages in a cross section of workers $G(w)$.[10] Since $L'(w) = l(w)h(w)$, equilibrium employment in firms that pay wage w, is given by:

$$l(w) = \frac{\lambda U(\lambda_L + \delta)}{(\lambda_L(1 - H(w)) + \delta)^2} = \frac{\lambda \delta N(\lambda_L + \delta)}{(\lambda_L(1 - H(w)) + \delta)^2(\delta + \lambda)} = \delta(N-U)\frac{(\lambda_L + \delta)}{(\lambda_L(1 - H(w)) + \delta)^2}. \tag{10}$$

Because of a higher inflow rate and a smaller outflow rate, employment grows with the wage: $l'(w) > 0$. In equilibrium, every firm pays a wage from the support of the equilibrium wage offer distribution and makes expected profits of $\Pi(w) = (y-w)l(w) = \frac{\lambda \delta N(y-w_R)}{(\delta+\lambda_L)(\delta+\lambda)}$ (see appendix 5.2), which are strictly positive.[11] The wage offer density $h(w)$ is defined on the support $[w_R, w^o]$, where $w^o = y-(y-w_R)\left(\frac{\delta}{\delta+\lambda_L}\right)^2$(see appendix

[10] These are different, since workers climb the job (wage) ladder through job-to-job transitions in the course of their career.

[11] Positive profits arise because search frictions are a source of monopsony power.

5.2).

Taking into account that $\Pi(w) = \Pi(w') = (y-w)l(w)\ \forall (w,w') \in [w_R, w^o]$[12] and equating the profit at the reservation wage with an arbitrary wage from the support of the wage offer distribution, one obtains $(y-w)\frac{\lambda U(\lambda_L+\delta)}{(\lambda_L(1-H(w))+\delta)^2} = (y-w_R)\frac{\lambda U(\lambda_L+\delta)}{(\lambda_L+\delta)^2}$. Solving for $H(w)$ yields the equilibrium wage offer distribution.

$$H(w) = \left\{ \begin{array}{cc} 0 & \text{for } w < w_R \\ \frac{\lambda_L+\delta}{\lambda_L}\left(1-\sqrt{\frac{y-w}{y-w_R}}\right) & \text{for } w_R \leq w < w^o \\ 1 & \text{for } w \geq w^o \end{array} \right\} \tag{11}$$

The distribution H does not contain mass points or wholes [Ridder and Van den Berg, 1997, p.101]. The corresponding density can be calculated as $h(w) = H'(w) = \frac{\lambda_L+\delta}{2\lambda_L}\sqrt{\frac{1}{(y-w_R)(y-w)}}$ and is increasing in w. Plugging $H(w)$ in equation (7) yields an expression for the reservation wage for this wage distribution: $w_R = \frac{(\delta+\lambda_L)^2 z + (\lambda-\lambda_L)\lambda_L y}{(\delta+\lambda_L)^2+(\lambda-\lambda_L)\lambda_L}$. Thus, the reservation wage is a weighted mean of the net unemployment benefits z and the labor productivity y.[13] If individuals cannot change jobs ($\lambda_L = 0$) the reservation wage equals z, $w^o = w_R$ and one obtains the monopsony solution [Diamond, 1971]. If on the other hand the job offer rate on the job becomes big ($\lambda_L \to \infty$), then the frictions vanish and the wage offer distribution collapses to a mass point at y. The same is true if there is no job destruction $\delta = 0$ or no search friction for the unemployed $\lambda \to \infty$ (that is no unemployment). For all intermediary cases ($0 < \lambda_L < \infty$, $U > 0$) the wage offer distribution has a positive variance. The monopsony power of the firms depends on the degree of the friction. The higher the friction, i.e. the lower λ_L and the higher δ, the higher is the monopsony power of the firms and both moments of the wage distribution depend on this monopsony power [Van den Berg and Ridder, 1993].

Remarkably, the model generates an equilibrium wage dispersion across identical individuals. Further, it provides arguments for the empirical facts that big firms pay higher wages than small firms and that senior workers gain on average more than their junior counterparts. Problematic is however that the density of the distribution of paid wages $G'(w) = \frac{\delta(\lambda_L+\delta)h(w)}{[\delta+\lambda_L(1-H(w))]^2}$ is upward sloping, which is difficult to reconcile with observed wage distributions. In addition empirical studies point to the fact that this explanation of wage dispersion, i.e. monopsony power of firms through frictions in connection with on the job search is able to explain only a small part of the variance of an empirical wage distribution [Bontemps et al., 2000, p.348f.].

A binding minimum wage does not affect unemployment as long as the marginal productivity exceeds the minimum wage.[14] Employers respond to an increase of a binding minimum wage by raising their wage offers. This reduces the profit of the firms, but

[12] This is the case, since otherwise the result cannot be an equilibrium. If profits were different for different wages, then low-profit firms would have an incentive to change their paid wage, since by assumption all firms are identical.

[13] If the discount rate r is positive, the reservation wage formula can be generalized to $w_R = \frac{(\delta+\lambda_L)^2 z + \delta(\lambda+\lambda_L)y\tau}{(\delta+\lambda_L)^2+\delta(\lambda+\lambda_L)\tau}$, where $\tau = \frac{\lambda_L}{\delta} - 2\frac{r}{\delta} + 2\frac{r(\delta+r)}{\lambda_L\delta}\ln\left(1+\frac{\lambda_L}{\delta+r}\right)$ [Bontemps et al., 2000, p.314].

[14] Obviously, the same is true in the neo-classical model of the labor market. But if a minimum wage

not labor demand since the profit is positive for each individual employed. Even more, voluntary unemployment does not exist in this model. This result follows from the homogeneity assumption: each individual is a good allocation for each vacancy and vice versa. This result holds as long as wages are below marginal productivity, a typical result in search equilibrium models.

There is empirical evidence that indeed the unemployed accept every job offer, which is a central result of the model above. Many studies estimate an acceptance probability of almost one: "Il apparaît que la première offre d'emploi reçue est pratiquement toujours acceptée" Cahuc and Zylberberg [2001, p.77]; [see also Van den Berg, 1999, p.F290]. But this means that the mechanism that explains voluntary unemployment in the partial search model is not central for understanding unemployment. From this point of view unemployment is involuntary.

An increase in the binding minimum wage alters the complete wage distribution. The expectation of the wage distribution increases while the variance decreases. Unemployment does not change. Clearly, a major drawback of this model is the homogeneity assumption, since it is likely that homogeneity is a critical assumption for the no employment effects result. To see why, note that when reservation wages are heterogenous it is not more clear that firms offer wages that are above the reservation wage of all individuals. On the other hand, note that Jolivet et al. [2006, p.1] conclude from an empirical implementation of the homogeneous search model in a cross country comparison that the "(...) model fits the data surprisingly well".

3 Heterogeneity

In reaction to the drawback of the homogeneity assumption, several extensions have been discussed in the literature that introduce heterogeneity on either side of the market. For individuals heterogeneity can take the form of different reservation wages caused by differences in search costs or different productivities. For firms the effect of different productivities has been examined.

3.1 Different search costs

This subsection presents a model extension that allows for different search costs across unemployed job seekers [Burdett and Mortensen, 1998] since it is likely that this extension has a major impact on the above results for the minimum wage. If unemployed job seekers have different search costs a, then z varies conditional on b.[15]

is binding, there are always people whose marginal productivity is below this minimum wage, since everybody is paid its marginal productivity. So, the central point is that, under the search-theoretic perspective, people are not paid their marginal productivity, and therefore a binding minimum wage does not necessarily mean higher unemployment.

[15] Different search costs might arise when the access to labor markets is more different for one group than for the other or when individuals have different preferences for leisure.

First, notice that compared to the basic model different search costs have no effect on labor demand. Because of identical productivity every (or no) match is profitable. But there is an effect on offered and paid wages and therefore an effect on the behavior of the job seekers. Intuitively, this is the case, since for firms in this case it might be profitable to offer wages below the reservation wage of a part of the unemployed. Provided that the offered wage is still above the reservation wage of some individuals, a firm has still positive inflows. Then, for unemployed not every contact with a firm means a profitable match. This is true, although the match is potentially profitable.

The resulting equilibrium wage distribution combines the effect of the informational imperfection with the effect of the heterogeneity in search costs. On the one hand, the wage distribution compensates the job seekers for its different search costs. On the other hand, firms differ in size and they have to pay different wages to ensure their size.

The model

Assumptions (B0), (B1"'), (B2), (B3), (B4'), (B5) and (B6) hold, where:

- (B1"'), as (B1"), but N individuals differ in their search costs. Their net search costs z follow a continuous distribution on $[\underline{z}, \bar{z}]$, where $R(z)$ is the share of individuals whose search costs are below z. $r(z)$ is the corresponding density. In addition $\lambda_L = \lambda$ holds.
- (B4'), as (B4), but firms know the distribution $R(z)$, but cannot observe z individually.

Different reservation wage policies across individuals are a consequence of the assumption of different net search costs. Note that (B1"') implies that $w_R = z$. Then, equating in-and outflows to and from unemployment yields the equilibrium unemployment rate for any z-type. It is given by $U_z = \frac{\delta N r(z)}{\delta + \lambda[1-H(z)]}$.

The optimal behavior of the firm can be deduced as follows. The amount of unemployed that accepts a wage offer w is given by:

$$S(w) = \int_{\underline{z}}^{w} \left(\frac{\delta N r(z)}{\delta + \lambda[1 - H(z)]} \right) dz = \int_{\underline{z}}^{w} \left(\frac{\delta N}{\delta + \lambda[1 - H(z)]} \right) dR(z). \tag{12}$$

As before, let $G(w)$ be the distribution of paid wages and $L(w) = (N - S(\bar{z}))G(w)$ the amount of employees that are employed at a wage below w.[16] Under stationarity, employment must remain constant in firms that pay wages below w. Inflows to this

[16] This is true, since $S(\infty) = S(\bar{z})$ is the amount of unemployed that would accept a wage offer of ∞. Since all unemployed individuals would accept such a wage offer, $S(\bar{z})$ is the amount of unemployed over all z-types.

group of firms stem only from unemployment. $dS(z)$ unemployed z-individuals obtain an acceptable job offer below w with probability $H(w) - H(z)$. Thus, expected inflows are $\lambda \int_{\underline{z}}^{w} (H(w) - H(x))dS(x)$. Outflows are composed of job destruction δ and of offers from better paying competitors $\lambda(1 - H(w))$ to employees in this wage group $(N - S(\bar{z}))G(w)$. Taken together:

$$(\delta + \lambda[1 - H(w)])(N - S(\bar{z}))G(w) = \lambda \int_{\underline{z}}^{w} (H(w) - H(x))dS(x).$$

Solving for $L(w)$ and differentiating with respect to w, after some simplifications[17] the equilibrium employment in a firm, offering the wage w is given by:

$$l(w) = \frac{(N - S(\bar{z}))G'(w)}{h(w)} = \frac{\lambda\delta N R(w)}{(\delta + \lambda[1 - H(w)])^2}.$$

The wage offer distribution can be derived from the equality of profits $\Pi = (y - w)l(w)$ on the support of $H(w)$ in equilibrium. Let $\underline{w}$ be the lower bound of the support, then $(y - \underline{w})\frac{\lambda\delta N R(\underline{w})}{(\delta+\lambda)^2} = (y - w)\frac{\lambda\delta N R(w)}{(\delta+\lambda[1-H(w)])^2}$ holds and it follows:

$$H(w) = \frac{\delta + \lambda}{\lambda}\left[1 - \sqrt{\frac{(y - w)R(w)}{(y - \underline{w})R(\underline{w})}}\right], \text{ für } w \in [\underline{w}, w^o]. \tag{13}$$

w is the biggest solution to $w = \arg\max_w[(y - w)R(w)]$ and w^o the biggest value that satisfies $\frac{(y-\bar{w})R(\bar{w})}{(y-\underline{w})R(\underline{w})} = \frac{\delta^2}{(\delta+\lambda)^2}$ [Burdett and Mortensen, 1998, p.266].

Unemployed individuals in this extension of the basic model do not accept every wage offer they obtain, and their expected unemployment duration depends on their reservation wage. The resulting equilibrium unemployment $S(\bar{z}|H)$ is higher than the unemployment that would result if all firms would pay a wage equal to marginal productivity $S(\bar{z}|w = y)$.[18] In addition equilibrium unemployment depends positively on $\frac{\delta}{\lambda}$, an indicator of the amount of frictions in the labor market. Assuming that $\bar{z} < y$ and $H(\bar{z}) > 0$, every match is potentially profitable, but not every match is formed when employers and employees meet. This is the case since firms commit ex-ante to pay some wage of the wage offer distribution and since it is optimal for a part of the firms to offer wages below $\bar{z}$.

Introducing a binding minimum wage has several effects. It changes both the lower bound of the wage distribution and the upper bound. However, since the minimum wage shifts the wage offer distribution to the right and since the reservation wage does not depend on the wage offer distribution for $\lambda_L = \lambda$, the average unemployment

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17 $L(w)$ is given by $L(w) = G(w)(N - S(\bar{z})) = \frac{\lambda \int_{\underline{z}}^{w} (H(w)-H(x))dS(x)}{(\delta+\lambda(1-H(w)))}$.

Using (12), $dS(z) = \left(\frac{\delta N}{\delta+\lambda(1-H(z))}\right) dR(z)$ and $L'(w) = l(w)h(w) = (N - S(\bar{z}))G'(w)$ the above $l(w)$ follows [Burdett and Mortensen, 1998, p.265].

18 That is always true as long as $\underline{w} < \bar{z}$.

durations will *decrease*, since on average they obtain more acceptable wage offers; that is unemployment decreases when the minimum wage increases. This is the case since labor demand does not react, whereas job seekers accept job offers more often on average. Still, one critique to this model is that it implies a counterfactual wage density [Van den Berg, 1999, p.F299]. In addition, heterogeneity in workers or firms productivities is likely to have quite different effects on employment effects of minimum wages.

3.2 Heterogeneity across firms

A priori heterogeneity and endogenous wage dispersion

There exist several model extensions with exogenous or even endogenous, heterogeneous productivity in the search framework. In a competitive setting with constant returns to scale this situation could not persist, since more productive firms would pay higher wages and employees would move immediately to the better paying firm. In a market with frictions however, this is not the case.

The following derivations are based upon Bontemps et al. [2000]. I will assume that the assumptions (B0), (B1""), (B2), (B3), (B4"), (B5) and (B6') hold, where:

- (B1""), as (B1"), but N identical individuals with productivity $\tilde{y}$ produce unequal amounts y of the good C. $y = \tilde{y}t(k)$ is assumed to hold, where $t(\cdot)$ is a positive function of k and displays the following properties: $t'(k) > 0$, $t''(k) < 0$. k can be interpreted as capital intensity in a firm.
- (B4"), as (B4), but there is an amount of M firms, whose capital intensity is distributed according to $\Gamma(k) = \Gamma(y)$. The constant, exogenous random variable K is drawn before production starts and has a finite expectation. There is a unique realization of Y that corresponds to each realization of K. Realizations of Y are continuously distributed on the support $[\underline{y}, \bar{y}]$. It is assumed that $\underline{y}$ exceeds the common reservation wage of the employees or the binding minimum wage if it exists.
- (B6'), as (B6), but the consumption good C is produced with labor and capital, and there are no depreciations.

Equilibrium unemployment is given by equation (8) $U = \frac{\delta N}{\delta + \lambda}$. The reservation wage of the U unemployed is given by equation (7) $w_R = z + (\lambda - \lambda_L) \int_{w_R}^{w^o} \frac{1-H(w)}{r+\delta+\lambda_L(1-H(w))} dw$, where w^o is the upper born of the wage distributions.

Describing the dynamics of employment in firms offering a wage above w ($w_R < w < w^o$), imposing stationarity and using equation (8) helps us deducing equilibrium employment $l(w)$ in firms offering w.

$$\frac{L(w)}{N-U} = G(w) = \frac{U}{(N-U)} \frac{\lambda H(w)}{[\lambda_L(1-H(w)) + \delta]} = \frac{\delta H(w)}{(\lambda_L(1-H(w)) + \delta)}, \tag{14}$$

as in the homogeneous model (equation 9). By the same arguments as above, the reservation wage (or minimum wage) is the lower bound of the wage offer distribution (see section 2.2). And thus:

$$l(w) = \delta(N-U)\frac{\delta+\lambda_L}{[\delta+\lambda_L(1-H(w))]^2}. \tag{15}$$

Note that $l(w)$ must divide by M to obtain the average amount of employment in one firm offering w: $\breve{l}(w) = \frac{l(w)}{M}$.

Firms maximize expected profits:

$$\Pi(w|y) = (y-w)\breve{l}(w) = \delta(y-w)\frac{(N-U)}{M}\frac{(\delta+\lambda_L)}{[\delta+\lambda_L(1-H(w))]^2}. \tag{16}$$

Again, the firm faces the trade-off between the profit per employee and the equilibrium amount of employees. In general, the wage a firm pays can depend on its productivity y. However, facing the results of section 2.2, note that it is possible that firms of an identical y-type pay different wages if different wages yield identical profits. If this is the case, then a firm of type y chooses a wage randomly according to $H(w|y)$. Let

$$K_y = \arg\max_w \{\Pi(w|y)|\max(w_R, w_{\min}) < w < y\}$$

be the entity of profit maximizing wages from which the y-firm draws one. Then, in the case of continuous productivity dispersion it can be shown that $K_y = K(y)$ is unique [Bontemps et al., 2000, p.315/350]. For each firm of a given y-type there is only one optimal wage, which is increasing in y. This simplifies the analysis since then the probability $H(w)$ that a firm pays a wage lower than $w = K(y)$ is determined by the probability $\Gamma(y)$ that the firm has a productivity below y. Since $K(y) = w$ and since $K'(y) > 0$ the inverse $y = K^{-1}(w)$ can be calculated. The share of firms that offer wages below w equals the share of firms, whose productivity is below $y = K^{-1}(w)$, or: $H(w) = \Gamma(K^{-1}(w))$.

The first order condition for the profit equation can be derived by differentiating (16) with respect to w:

$$\begin{aligned} \frac{\breve{l}'(w)}{\breve{l}(w)} &= \frac{l'(w)}{l(w)} = \frac{1}{y-w}, \text{ and} \\ -\delta - \lambda_L(1-H(w)) + 2(y-w)\lambda_L h(w) &= 0. \end{aligned} \tag{17}$$

The first line follows from using the first equality and the second line from using the second equality in equation (16). The second line determines the optimal wage for each firm $w = K(y|H(\cdot))$ implicitly for wages above the reservation wage.

For each firm equilibrium profits $\Pi(\cdot)$ and employment $\breve{l}(\cdot)$ can be obtained as a function of y. Using this, an explicit expression for $K(y)$ can be obtained. y-firms make profits of $\Pi(y) = (y - K(y))\breve{l}(K(y))$. Differentiating with respect to y yields $\Pi'(y) = (1-K'(y))\breve{l}(K(y)) + (y-K(y))\breve{l}'(K(y))K'(y)$. Using optimality and the envelope theorem, the following result is obtained:

$$\Pi'(y) = \breve{l}(K(y)). \tag{18}$$

Thus, profits of the firms are increasing with productivity. Integrating equation (18) yields an explicit expression for $\Pi(y)$ (see appendix 5.3). Using this and a suitable form of the equilibrium employment equation, I obtain the following expression for $K(y)$:

$$K(y) = y - [\delta + \lambda_L(1 - \Gamma(y))]^2 \int_{\underline{w}}^{y} \frac{1}{[\delta + \lambda_L(1 - \Gamma(\varrho))]^2} d\varrho, \tag{19}$$

where $\underline{w} = \max(w_R, w_{\min})$.

The wage offer distribution follows from $H(w) = \Gamma(K^{-1}(w))$. But, in general there is no closed form expression $K^{-1}(w)$.

The equilibrium wage offer distribution $H(w) = \Gamma(K^{-1}(w))$ uniquely determines the distribution of paid wages $G(w)$ in equation (14). It also determines the equilibrium profit of firms $\Pi(w|y)$ in (16) depending on y. Profit is maximized if firms choose the wage according to equation (19) and is given by equation (38). Equilibrium unemployment is given by equation (8).

Wage dispersion in this model arises as a result of the interaction of both search frictions and productivity dispersion. Productivity dispersion itself is not sufficient for wage dispersion. It has been shown above, however, that informational frictions alone with on-the-job search are a sufficient condition for wage dispersion. However, first, the integration of different productivities across firms is an important ingredient, empirically. Second, it has been mentioned before that the homogeneous model implies counterfactual wage distributions and it is only able to explain part of the variance of wages between individuals. In this context, the resulting wage distributions depend on the productivity distribution. If for example a Pareto-distribution for the productivity is assumed, a realistic shape for the wage distribution can be obtained [Bontemps et al., 2000]. Indeed, the model is able to generate wage distributions that are in accordance with the data. This is astonishing, especially since the assumption of homogeneous workers has been retained.

Since $K'(y) > 0$, an increasing variance of the distribution of the productivities, increases the variance of the wage distribution, whereas equilibrium unemployment remains unchanged. A minimum wage affects both the lower and the upper bound of the wage distribution. As in the homogeneous model, equilibrium unemployment is in general not affected while the monopsony power of the firms is affected. Rents can be redistributed from firms to workers. Although, in general, an increasing minimum wage drives unproductive firms out of the market, labor demand does not react, because the missing demand of the low productive firms is fully compensated by their more productive counterparts. However, the assumption that there is always a continuum of firms that demands labor is critical in that context. The fact that firms are driven out of the market stipulates another point, namely that a minimum wage has dynamic effects on the composition of the firms. This aspect is however beyond the scope of this paper.

3.3 Heterogeneity on both sides of the market

If trying to explain the variance of paid wages between observationally equivalent workers, basically two components are required: first, there are firm effects on the wage and, second, there is an effect of the degree of frictions on wages. In addition, wages vary considerably between workers with different observed characteristics, controlling for firm characteristics and search frictions. Summing up, three factors are needed to explain empirical wage distributions: heterogeneous firms, heterogeneous workers and search frictions [see Bontemps et al., 2000, Abowd et al., 1999]. So far, the presented models explain wage variation by search frictions [Burdett and Mortensen, 1998], by search frictions and heterogeneity of the employees [Burdett and Mortensen, 1998], by search frictions and exogenous technology differences [Bontemps et al., 2000]. The model presented in this section integrates the three important factors for the explanation of an empirical wage distribution. The model is due to Postel-Vinay and Robin [2002a].

Assumptions

Heterogenous productivity of the individuals is integrated in the model in the following way. Consider the labor market for a specific homogeneous professional group, where all homogenous job seekers are substitutable to a certain degree. However, individuals differ in their productivity as measured by an index ε. Individuals determine their productivity by drawing a value from the continuous productivity distribution $\Omega(\varepsilon)$ on an interval $[\varepsilon_{\min}, \varepsilon_{\max}]$ with density $\omega(\varepsilon)$. It is assumed that unemployed job seekers of type ε obtain a net unemployment income of $z(\varepsilon) = \varepsilon b$. w brings the individual the utility $\Xi(w)$ and individuals maximize the present value of their expected utility over an infinite time horizon. Leisure does not enter the utility function of the individuals.

Each firm produces with technology y which is distributed according to a cdf $\Gamma(y)$ with density $\gamma(y)$ on a bounded support $[\underline{y}, \bar{y}]$ and which is determined by an ex-ante random draw. A firm maximizes the present value of its expected profits over an infinite time horizon. It is assumed that the "home productivity" b exceeds $\underline{y}$. Marginal productivity of an efficiency unit of labor is constant given the y-type of the firm. That is, an individual of type ε and a firm of type y produce together an output $y\varepsilon$, a production function with homogeneity degree of 2.

The sequential process of contacts between employers and employees is as follows. Unemployed job seekers contact firms at rate λ, while employed job seekers contact firms at λ_L. It is assumed that each firm of type y makes wage offers to individuals with a specific probability that is identical over all ε-types. The contact probability for a type-y firm follows a distribution function $\Psi(y)$ with density $\psi(y)$. Postel-Vinay and Robin [2002a] argue that the relative frequency of contacts for a y-firm $\psi(y)/\gamma(y)$ is determined by the search intensity of this firm. There is, however, no microfoundation for the ratio of $\psi(y)$ and $\gamma(y)$.

Upon a meeting, both sides have complete information about all relevant characteristics of the other side. Therefore, the wage offer of the firm can condition on the type ε of the job seeker. In addition, if an employee gets an outside job offer from a competing firm, the employing firm can make a binding counteroffer. Implications of this assumption are detailed in Postel-Vinay and Robin [2002b]. In a model with endogenous technology dispersion, they show that job-to-job transitions depend basically on the productivities of the competing firms. Before discussing their derivations, assumptions (B0), (B1""'), (B2), (B3'), (B4"'), (B5') and (B6") summarize the foundation of the model:

- (B1""'), as (B1'), but N individuals maximize their utility function $\Xi(w)$ and enter and leave the labor market at rate n. Newcomer enter the labor market as unemployed job seekers. Individuals differ in their productivity ε , according to a distribution function $\Omega(\varepsilon)$ on $[\varepsilon_{\min}, \varepsilon_{\max}]$. ε-type unemployed obtain $z = \varepsilon b$. $W_U(\cdot)$ ($W_L(\cdot)$) are the values of unemployment and employment, respectively. When a job seeker and a firm meet, the probability that the productivity of the firm is below y is $\Psi(y)$.
- (B3'), as (B3), but matches dissolve at rate $\delta + n$, where δ is the job destruction rate and n is the rate at which individuals leave (and enter) the market.
- (B4"'), as (B4), but firms differ in their productivity y, distributed according to $\Gamma(y)$ on $[\underline{y}, \bar{y}]$, with $\underline{y} > b$. Upon a meeting the firm observes both the type ε and the productivity y of the firm that employs the individual at present.
- (B5'), as (B5) but firms condition their wage offer $w(\varepsilon, y, \cdot)$ that is nonnegotiable on the type ε of the individual and on the productivity y of the firm that employs the individual so far.
- (B6"): Specific labor market, where r is the discount rate in this market and where a homogeneous product is produced from heterogeneous agents. A type y firm and a type ε individual produce together the output $y\varepsilon$ of the homogeneous good. The price of the produced good is the numéraire.

The model

Let $W_U(\varepsilon, b, \varepsilon b) = W_U(\varepsilon)$ be the value of unemployment of an individual of type ε and let $W_L(\varepsilon, y, w)$ be the value of employment of the same ε-type, depending on the productivity of the employing firm and the paid wage. The value equation depends not only on the wage that the firm pays but also on the productivity of this firm. This is the case, since the productivity of the firm determines the career opportunities (in the sense of potential wage gains) in this firm. If an employed individual of type ε

obtains a wage offer from a competing firm, the upper bound of the wage increase for the individual is determined by the productivity of the employing firm. To see this, notice that the maximal counteroffer that the employing firm can make is bound by its productivity. However, because of perfect information, the competing firm will choose its wage offer such that the individual is indifferent between changing the firm and not. Job seekers obtain always exactly their reservation wage upon engagement. For unemployed job seekers, it can be calculated from $W_L(\varepsilon, y, w_R) = W_U(\varepsilon)$, where $w_R = w_R(\varepsilon, y, z) = w_R(\varepsilon, y)$. The reservation wage of the unemployed job seekers, like the reservation wage of employed job seekers depends on the career opportunities in the firm, that makes the offer.

If a y'-firm with $y' > y$ makes an offer, it chooses the wage such that the individual is indifferent between the value of the highest wage $w = \varepsilon y$ he can get in his firm without career opportunities[19], and the value of the wage with the positive career opportunities if changing to the y'-firm. Let $w_w(\varepsilon, y, y')$ be the wage that makes the ε–individual indifferent between the firms y, y'. Then: $W_L(\varepsilon, y, \varepsilon y) = W_L(\varepsilon, y', w_w)$. If the competing firm has a lower productivity than the employing firm $y' < y$, than the firm is ready to pay at most $\varepsilon y'$. The counteroffer that is able to inhibit the individual from changing the firm is because of the better career opportunities smaller than $\varepsilon y'$ and given by $w_w(\varepsilon, y', y)$, where $W_L(\varepsilon, y, w_w) = W_L(\varepsilon, y', \varepsilon y')$ holds.

The value of unemployment can be derived as usual from the no-arbitrage condition, where the instantaneous income has to be replaced by the instantaneous utility $\Xi(\varepsilon b)$. The value of a job offer is given by the value of unemployment since firms offer exactly the reservation wage to the individual. Future utility flows are discounted by the discount rate r plus the instantaneous mortality rate n. It follows $W_U(\varepsilon) = \frac{1}{1+(r+n)dt}\{\Xi(\varepsilon b)dt + \lambda dt W_U(\varepsilon) + (1 - \lambda dt)W_U(\varepsilon)\}$, or:

$$W_U(\varepsilon) = \frac{\Xi(\varepsilon b)}{r+n}.$$

The value of employment contains several components. If an ε-individual that is employed at wage w in a y-firm obtains an offer from a competing firm, three possibilities arise. First, if the productivity y' from the competing firm is so small that the employing firm could poach the ε-employee from the y'-firm for a wage $w_w(\varepsilon, y', y) < w$, nothing changes. Let the critical productivity of a competing firm for which $w_w(\varepsilon, \breve{y}, y) = w$ holds, be $\breve{y}(\varepsilon, y, w)$. Than the probability that the offer does not change anything is given by $\Psi(\breve{y})$. The second possibility is that $\breve{y} < y' < y$. That is, the competing firm cannot win the Bertrand-competition, but is able to offer the employee a higher value than it has in the current firm with his current wage. That is, the employee gets the wage increase $w_w(\varepsilon, y', y) - w$ and his new value of work is given by $W_L(\varepsilon, y', \varepsilon y')$. This happens with probability $\Psi(y) - \Psi(\breve{y})$. The value equation must account for the expected value of labor over the productivities of the competing firms in this case. Finally, if the productivity of the competing firm y' is higher as the productivity of the

[19] Without career opportunities means that the employing firm pays marginal productivity and can therefore offer no higher wage.

employing firm y it wins the Bertrand-competition, the employee changes the firm and its wage changes by the amount $w_w(\varepsilon, y, y') - w$.[20] With the corresponding probability $(1 - \Psi(y))$ his new value of work is then $W_L(\varepsilon, y', w_w(\cdot)) = W_L(\varepsilon, y, \varepsilon y)$. The value equation summarizes these possibilities:

$$\begin{aligned} [r + \delta + n + \lambda_L(1 - \Psi(\breve{y}(\cdot)))]W_L(\varepsilon, y, w) &= \Xi(w) + \delta W_U(\varepsilon) + \\ \lambda_L[\Psi(y) - \Psi(\breve{y}(\cdot))]E_\Psi[W_L(\varepsilon, x, \varepsilon x)|\breve{y} &< x < y] + \lambda_L(1 - \Psi(y))W_L(\varepsilon, y, \varepsilon y). \end{aligned} \tag{20}$$

Evaluating this formula at $w = \varepsilon y$, I find that:

$$W_L(\varepsilon, y, \varepsilon y) = \frac{\Xi(\varepsilon y) + \delta W_U(\varepsilon)}{r + \delta + n}. \tag{21}$$

This is true, since $\breve{y}(\varepsilon, y, \varepsilon y) = y$. Using (21) for the conditional expectation in (20) together with the conditional distribution $\Psi(x|\breve{y} < x < y) = \frac{\Psi(x)}{\Psi(y) - \Psi(\breve{y})}$, using in addition integration by parts, the fact that $W_L(\varepsilon, y, w) = W_L(\varepsilon, \breve{y}, \varepsilon\breve{y})$ and the relationship $\lambda_L\left(\frac{\Xi(\varepsilon y) - \Xi(\varepsilon\breve{y})}{r + \delta + n}\right) = \frac{\lambda_L \varepsilon}{r + \delta + n}\int_{\breve{y}}^{y} \Xi'(\varepsilon x)dx$, a new expression for the value of work is obtained.

$$(r + \delta + n)W_L(\varepsilon, y, w) = \Xi(w) + \delta W_U(\varepsilon) + \frac{\lambda_L \varepsilon}{r + \delta + n}\int_{\breve{y}}^{y}(1 - \Psi(x))\Xi'(\varepsilon x)dx \tag{22}$$

The return to working at wage w can be decomposed in the instantaneous utility of the wage minus the loss in case where the job gets lost $-(\delta(W_L(\cdot) - W_U(\cdot)) + nW_L(\cdot))$ plus the instantaneous probability to get an offer of a competing firm times the expected discounted utility gain in this case.[21] Using that by definition $W_L(\varepsilon, y, w) = W_L(\varepsilon, \breve{y}, \varepsilon\breve{y})$ and equation (21) on the left hand side of equation (22), $\Xi(w) = \Xi(\varepsilon\breve{y}) - \frac{\lambda_L \varepsilon}{r + \delta + n}\int_{\breve{y}}^{y}(1 - \Psi(x))\Xi'(\varepsilon x)dx$ is obtained. If a firm with $y' > y$ makes an offer to an employee, then the employee changes the firm and obtains the wage $w_w(\varepsilon, y, y')$. Plugging this in the last formula and using that $\breve{y}(\varepsilon, y', w_w(\cdot)) = y$, one obtains an implicit characterization of the wage an individual obtains when changing job.

$$\Xi(w_w(\varepsilon, y, y')) = \Xi(\varepsilon y) - \frac{\lambda_L \varepsilon}{r + \delta + n}\int_{y}^{y'}(1 - \Psi(x))\Xi'(\varepsilon x)dx \tag{23}$$

Analogously, the reservation wage of an unemployed job seeker when obtaining an offer from a y'-firm ($y' > b$) is given implicitly by using $w_R(\varepsilon, b, y') = w_w(\varepsilon, b, y')$ and

[20] The wage change can be both a wage increase and a wage cut. This depends on the wage the employee has earned in the old firm and on the productivities of both firms. If for example the employee earns already marginal productivity in his firm, the wage change is always a wage cut.

[21] The expected utility gain from a job offer can be calculated as the integral over the probability that the offer stems from a firm, whose productivity is above x, where $x \in [\breve{y}, y]$, times the marginal utility of the highest wage $x\varepsilon$ this firm can afford to pay. Putting $\frac{\varepsilon}{r+\delta+n}$ under the integral, the integral is the expected discounted utility gain from a wage offer of a firm with productivity $y' \in [\breve{y}, y]$. This is true since all firms with productivity above $x \in [\breve{y}, y]$ can afford to pay at least wage $x\varepsilon$ and therefore insures at least marginal utility $\Xi'(\varepsilon x)$ for the individual and since for values $x > y$ the value of employment remains unchanged.

$\check{y}(\varepsilon, y', w_R(\cdot)) = b$.

$$\Xi(w_R(\varepsilon, b, y')) = \Xi(\varepsilon b) - \frac{\lambda_L \varepsilon}{r+\delta+n}\int_b^{y'}(1-\Psi(x))\Xi'(\varepsilon x)dx \tag{24}$$

Both wages are reservation wages in the sense that they correspond to the minimum wage offer that a type y'-firm must make a ε-individual to induce it to work in this firm. In both cases this reservation wage depends on the current productivity, either of the employing firm or the home productivity. Since a firm with $y' > y$ offers career opportunities, the wage $w_w(\cdot)$ it pays is lower than the maximal wage a y-firm can afford. The discounted value of the career opportunities is given by the second addend in equation (23). Thus, the model generates voluntary job-to-job transitions under wage cuts. The analog holds for the reservation wage of the unemployed, it is lower than the value of their home production.

Paid wages are either the first wage $w_R(\varepsilon, b, y')$ or a wage that results from a Bertrand-competition between two firms y, y', that is $w_w(\varepsilon, y', y)$ mit $\check{y} < y' < y$ (or $w_w(\varepsilon, y, y')$, if $y' > y$). So there are always three components contained in the wage: individual productivity, firm productivity and luck. For a CRRA-utility function (as e.g. $\Xi(w) = \ln w$) the reservation wage from equation (23) can be decomposed additively in its three components, i.e. $\ln w_w(\varepsilon, y, y') = \ln\varepsilon + \ln w_w(1, y, y') = \ln\varepsilon + \ln y + \frac{\lambda_L}{r+\delta+n}\int_y^{y'}\frac{1}{x}(1-\Psi(x))dx$ (ibid., p.2305). From this the decomposition of the variance of paid wages can be derived from which the model has been motivated (see appendix 5.4).

$$var_w(\ln w) = var_\varepsilon(\ln\varepsilon) + var_y[E_{(y'|y)}(\ln w_w(1, y, y')|y)] + E_y[var_{(y'|y)}(\ln w_w(1, y, y')|y)]$$

The variance of wages can be decomposed in a component that is attributable to individual productivity differences (ε), a component that comes from different firm productivities (y) and in a component that comes from labor market frictions (see also appendix 5.4).

Before characterizing the equilibrium, some additional definitions must be made. Let $L(\varepsilon, y)$ be the share of individuals whose productivity is below ε and that are employed in firms with productivity below y. Then, $L_y(y) = \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} L(\varepsilon, y)d\varepsilon$ is the share of individuals that are employed in firms with productivity below y. Let $l(\varepsilon, y)$ and $l_y(y)$ be the corresponding densities. Further let $G(w|\varepsilon, y)$ be the conditional distribution of paid wages. Equilibrium unemployment follows from $\lambda u = (1-u)(\delta+n)$, or

$$u = \frac{\delta+n}{\lambda+\delta+n}. \tag{25}$$

This is the usual condition for unemployment, accounting however for the fact that there is turnover in the population.

The conditional wage distribution in equilibrium is characterized by the following. There are $G(w|\varepsilon, y)l(\varepsilon, y)N(1-u)$ employees of type ε, who are employed in a type y-firm at wage below w. Employees of this category leave the class either because of job destruction δ, because of death n, or because they obtain an offer from a firm, whose productivity is above $\check{y}(\cdot)$. So, outflows amount to $(\delta+n+\lambda_L(1-\Psi(\check{y})))G(w|\varepsilon,y)l(\varepsilon,y)N(1-u)$. Inflows come from the pool of unemployed $\lambda\psi(y)\omega(\varepsilon)$, since firms offer the reservation wage, which is always acceptable to the individuals.[22] On the other hand, y-firms can poach ε-individuals to a wage below w from firms, whose productivity is below $\check{y}(\varepsilon,y,w)$. The expected flow is given by $\lambda_L N(1-u)\psi(y)\int_{\underline{y}}^{\check{y}(\varepsilon,y,w)} l(\varepsilon,x)dx$. In equilibrium, using equation (25) and canceling out $N(1-u)$, it follows:

$$(\delta+n+\lambda_L(1-\Psi(\check{y})))G(w|\varepsilon,y)l(\varepsilon,y) = \left[(\delta+n)\omega(\varepsilon)+\lambda_L\int_{\underline{y}}^{\check{y}(\varepsilon,y,w)} l(\varepsilon,x)dx\right]\psi(y). \tag{26}$$

Evaluating this equality at $w=\varepsilon y$, and using that $G(\varepsilon y|\varepsilon,y)=1$ and $\check{y}(\varepsilon,y,w)=y$, $(\delta+n+\lambda_L(1-\Psi(y)))l(\varepsilon,y)=\left[(\delta+n)\omega(\varepsilon)+\lambda_L\int_{\underline{y}}^{y} l(\varepsilon,x)dx\right]\psi(y)$ is obtained. The solution to this differential equation is (ibid., p.2341):

$$l(\varepsilon,y)=\frac{(1+\kappa_L)\psi(y)}{[1+\kappa_L(1-\Psi(y))]^2}\omega(\varepsilon)=l_y(y)\omega(\varepsilon), \text{ with } \kappa_L=\frac{\lambda_L}{\delta+n}. \tag{27}$$

Using the primitive $L(\varepsilon,y)=\frac{\Psi(y)}{1+\kappa_L(1-\Psi(y))}\Omega(\varepsilon)=L_y(y)\Omega(\varepsilon)$ and $l(\varepsilon,y)$ this is easily checked. Equation (27) basically says, that the employment of type ε individuals is independent of the type y of the firm, i.e., there is no sorting.

Using (27), equation (26) can be solved for $G(w|\varepsilon,y)$. The conditional distribution of wages for ε-individuals in y-firms is given by:

$$G(w|\varepsilon,y)=\left(\frac{1+\kappa_L(1-\Psi(y))}{1+\kappa_L(1-\Psi(\check{y}(\varepsilon,y,w)))}\right)^2. \tag{28}$$

The equilibrium size of a y-firm can be derived by the equilibrium conditions as $N\frac{l_y(y)}{\gamma(y)}=\frac{N(1+\kappa_L)}{[1+\kappa_L(1-\Psi(y))]^2}\frac{\psi(y)}{\gamma(y)}$. The first term implies that the size of a firm increases with the productivity of the firm, since upon meetings they are more often capable of attracting individuals from competing firms than low productivity firms. The second term reflects by assumption the search intensity of a firm, which can increase or decrease with firm productivity. So, firm size does not uniquely depend on productivity. Since the random variable ε does not depend on the random vector (y,y') and since this implies independence between ε and y, the conditional distribution of $(y'|y)$ can be derived. Let $y'<y$, then $G(w_w(\varepsilon,y,y')|\varepsilon,y)=G(\varepsilon y'|\varepsilon,y)=\frac{\Omega(\varepsilon)\breve{G}(y,y')}{\Omega(\varepsilon)L_y(y)}=\tilde{G}(y'|y)=\left(\frac{1+\kappa_L(1-\Psi(y))}{1+\kappa_L(1-\Psi(y'))}\right)^2$. This is true since $\check{y}(\varepsilon,y',w_w)=y'$.

Summarizing, it is to be noticed that the model is able to integrate the three factors that are empirically important for explaining wage dispersion into a theoretical framework. An attractive feature of the model is that it provides a rationale for voluntary job-to-job transitions under wage cuts, since this seems to be a phenomenon that is empirically important [for Germany, see Fitzenberger and Garloff, 2007, Pfeiffer, 2003]. On the other hand, (real) wage cuts for job stayers seem to be empirically important, too [Postel-Vinay and Robin, 2002a, Pfeiffer, 2003, p.2313f.,p.40ff.]. Of course, this cannot

[22] This is true because of the assumption $\underline{y}>b$.

be explained by the model. A further interesting result is that ε and y are distributed independently, which implies that there is no sorting within professional groups. This is implied by equation (27).[23] Although it is true that more productive firms prefer employing more productive individuals and although they can attract them if competing with low productive firms, firms can earn positive profits for each employee. Therefore in the model, they employ everybody and in equilibrium there is no sorting. The limitation to professional groups is important, since there is evidence for positive assortative matching in labor markets [Van den Berg and Van Vuuren, 2006].

Now, introducing a binding minimum wage has multiple effects. Assume that $w_{\min} > \underline{y}\varepsilon_{\min}$. There is a set of matches that are not profitable anymore and they are not performed. For workers with productivity $\varepsilon_{crit} < \frac{w_{\min}}{\underline{y}}$, i.e. workers that are affected by the minimum wage, the specific unemployment rate is higher. However, those workers that are still employed, earn higher wages. Let $y_{crit} = \frac{w_{\min}}{\varepsilon_{\min}}$ denote the productivity boundary, above which all matches are profitable. Then firms with $y < y_{crit}$ have less employees and make lower profits than in the absence of the minimum wage. For all other firms there is a number of matches that are in fact profitable, but which require the firms to pay the obligatory minimum wage instead of the reservation wage. This reduces profits of the firms but leads to higher average wages for the employees. Summarizing, in this model, the effects of a binding minimum wage on employment is negative, while rents are distributed from firms to workers.

3.4 Production functions and marginal productivity

So far, the models discussed allow for differences both on the side of the firms and on the side of the individuals. They maintain, however, the assumption that the productivity of an individual does not depend on the number of individuals employed. Yet, another possibility to introduce heterogeneity is the assumption of non-constant marginal productivities. Again, this difference is likely to be crucial for the effects of minimum wages on wages and employment. Spill-over effects between different skill-groups might become relevant. So far, there are only little attempts in the literature to incorporate this production function view in the search framework. One such attempt is Ridder and Van den Berg [1997], which draws on Manning [1993], Mortensen and Vishwanath [1993] and Mortensen and Vishwanath [1994]. This model assumes a non-linear production function with one production factor, only. Introducing several skill groups and linking them over a production function is analytically very demanding. There are not many models performing such a task. One such model with several production factors linked over a production function will be discussed below and is due to Holzner and Launov [2005].

I start by discussing the model where one production factor is allowed to have non-constant marginal productivity in a production function, depending on its use. For the

[23] Note, that since there is no assumption concerning the distribution of the individuals across firms, this is a result of the model and not an assumption. Other papers using similar frameworks conclude on the contrary that there is positive assortative matching. [see, e.g. Gautier et al., 2005]

presentation, I refer to Ridder and Van den Berg [1997]. Consider the framework as outlined in 2.2. As before, let $l(w)$ be the steady-state number of individuals employed in a firm that pay a wage w, and let $y(l(w))$ denote the output depending on the employment $l(w)$.[24] Assume that y is concave [$y'(l) > 0, y''(l) < 0$] and that $y(0) = 0$. Assume, in addition, that the job offer rate off-the-job and on-the-job are identical $\lambda = \lambda_L$.

The reservation wage is given by equation (7) and reduces to the monetary value of obtaining unemployment benefits net of search costs, because the option value of unemployment vanishes through the equality of job offer rates, i.e. $w_R = z$. Equilibrium unemployment is unchanged and follows from the steady-state condition for inflows into and outflows from unemployment $u = \frac{\delta}{\delta+\lambda}$ since firms do not offer wages below the reservation wage. The objective function of the firms[25] is given by

$$\pi(w) = y(l(w)) - wl(w). \tag{29}$$

Concavity of y implies that there is a size $l(w)$ where $y'(l(w)) \leq w$. This implies that, at the upper bound of the wage distribution, a mass point can be obtained. In this case it is present both in the wage offer distribution $H(w)$ and in the distribution of paid wages $G(w)$. It arises since at the employment level where the wage equals marginal productivity every additional worker (even when obtained at that wage) would contribute a negative amount to the objective function and thus there is no incentive to pay a higher wage.[26] Still, firms paying a marginal productivity wage make positive profits because of the concavity.

Depending on the parameters this model has several possible solutions. One solution is an equilibrium where all firms pay a common wage, which can be either equal to the reservation wage or equal to marginal productivity and guarantees employment $\frac{N\lambda}{\lambda+\delta}$.[27] The reservation wage solution is obtained if marginal productivity in the symmetric equilibrium is below or equal to z, i.e. $y'(\frac{N\lambda}{\lambda+\delta}) \leq z$. In this case, it does not pay to deviate: paying a lower wage guarantees zero employees, whereas paying a higher wage increases the number of employees thereby decreasing marginal productivity. This makes the contribution of a an additional worker negative. So, deviating does not pay and this is an equilibrium if profits are positive. Otherwise, there is no production. A dispersed equilibrium cannot exist in this case. To see this, recognize that for employment to be positive all wages must be above or equal to z. However, assume that z was the upper bound of the wage distribution (if it is higher, profits are even lower). Employment in the continuous part of the wage distribution is given by equation (10). Employment at the upper bound of the wage distribution is higher than in the equal

[24] Note, that here y is uniquely determined by the employment of the firm. Given employment y is no random variable.

[25] It is to be noted that the objective function given here is not equal to the expected profit because of the non-linearity of the production function. It can be justified, however, by a second order Taylor approximation [Holzner and Launov, 2005].

[26] The incentive to pay a higher wage is the reason why mass points cannot exist in the homogeneous model. This mechanism is destroyed by decreasing marginal productivity.

[27] The equal wage employment is given by the employment rate times the number of employees N divided by the number of firms 1.

wages equilibrium $l(w^o) = \frac{N\lambda}{\delta} > \frac{N\lambda}{\lambda+\delta}$. Thus marginal productivity is strictly lower than z. Firms paying wage z want to shrink. Thus, there is no such equilibrium.

Similarly, unique equilibria with dispersed wages or partly dispersed wages are obtained under the following parameter constellations. If the profit at z for a dispersed equilibrium is higher than the profit at a common wage equilibrium (which equals marginal productivity) $\bar{w} = y'(\frac{N\lambda}{\lambda+\delta}) > z$, then an equilibrium with dispersed wages is obtained, since deviating from the common wage to the reservation wage pays directly. Otherwise if

$$y\left(\frac{N\lambda}{\delta+\lambda}\right) - \bar{w}\frac{N\lambda}{\delta+\lambda} > y\left(\frac{N\delta\lambda}{(\delta+\lambda)^2}\right) - z\frac{N\delta\lambda}{(\delta+\lambda)^2} \tag{30}$$

the common wage equilibrium is obtained, since deviating does not pay (neither below, nor above).

Now, consider the dispersed equilibrium. Still, it is possible that there is a both a dispersed part and a mass point having a probability mass $\gamma = Prob(w = \bar{w}) < 1$. Steady-state employment in these firms can be calculated by equating in- and outflows.[28]

$$\lambda(N - \gamma l(\bar{w})) = \delta l(\bar{w}) \tag{31}$$

This yields $l(\bar{w}) = \frac{N\lambda}{\delta+\lambda\gamma}$ as employment in one firm that offers the wage at the mass point. Clearly, the wage at the mass point must correspond to the marginal productivity given the employment, i.e. $y'(\frac{N\lambda}{\delta+\lambda\gamma}) = \bar{w}$.[29] Paying the mass point or being in the continuous part must yield identical profits and thus:

$$y\left(\frac{N\delta\lambda}{(\delta+\lambda)^2}\right) - z\frac{N\delta\lambda}{(\delta+\lambda)^2} = y\left(\frac{N\lambda}{\delta+\lambda\gamma}\right) - y'\left(\frac{N\lambda}{\delta+\lambda\gamma}\right)\frac{N\lambda}{\delta+\lambda\gamma}. \tag{32}$$

If there is a value between 0 and 1 for γ that solves this equation, then a mass point is obtained. Otherwise there is no mass point in the wage distribution.

For the general case (unspecified production), it is not possible to obtain a closed form solution for H and G. Note however, that the counterfactual shape of the wage density does not get lost. On the contrary, to compensate for the decreasing profit per worker because of a decreasing marginal productivity, employment must grow even faster with the wage than in the linear model.[30] Then, equation (10) implies that the density is even steeper.

Increasing minimum wages might change the equilibrium obtained and compress the

[28] The equation can be derived by recognizing, that $l(\bar{w})$ per assumption is the employment in one firm that offers $\bar{w}$ and that the measure of firms is 1.

[29] If the wage was higher, firms would want to employ less. If the wage was lower, the usual argument that increasing the wage by a small amount increases profits holds.

[30] With decreasing marginal productivity, there are two effects that drive the profit per worker down, when employment is growing: first, to attract more worker the wage must increase and even with linear production the profit per worker decreases. Second, with increasing employment the marginal productivity decreases, driving down the other component of the profit per worker.

wage distribution. The probability of obtaining a mass point at the *upper bound* of the wage distribution increases. In general, however this does not result in lower employment, since all equilibria but one yield the same employment. If minimum wages have employment effects however, that means shifting from an equilibrium with production to an equilibrium where there is no production at all. Clearly, this is not a very realistic possibility.

An extension of the baseline model to a production function with several skill groups i each of size q_i with $\sum q_i = N$ and heterogeneous production technologies as indexed by j is due to Holzner and Launov [2005]. Note, that this is the most general model discussed in this paper: it allows for differences between firms employing different technologies and for differences in workers that vary in productivity a priori and depending on their use. Let me introduce some notation, first. w_{ij} is the wage offer a firm of type j offers to an individual of type i. Let H_{ij} be the wage offer distribution for firms that produce with a technology j for skill group i. I.e., $H_{ij}(w_{ij})$ is the amount of type-j firms that offers a wage below w_{ij} for type-i individuals. H_i is the wage offer distribution for skill group i aggregated over all firm types, i.e. the distribution individuals care about, when looking for a job. Finally, H_j is the I-dimensional wage offer distribution of type-j firms to all skill groups. Similarly, $l_i(w_{ij})$ gives the employment of skill group i in a type j firm that offers w_{ij}, while $l(w_j)$ is the I-dimensional employment of all skill groups in this firm.

The reservation wage w_i^R for individuals of skill group i is given by equation (7) and is indexed by i. Skill-specific unemployment is given by equality of in- and outflows and determined by the skill-specific friction parameter λ_i.[31] In addition, the dynamics for each skill group is similar as in the standard model, meaning that equation (10) holds for each skill group, indexed by the index i, except for the following modification. In the denominator $(\delta+\lambda_L(1-H(w)))^2$ must be replaced by $(\delta+\lambda_L(1-H_i(w_i)))(\delta+\lambda_L(1-H_i(w_i^-)))$. The modification follows from the fact that H_i is allowed to contain mass points and thus $H_i(w_i) = H_i(w_i^-) + \gamma_i(w_i)$. If there are no mass points, the original employment equation is obtained. Recognize that λ_L, δ are assumed identical across skill groups.

Firms with production technology j maximize their expected profit by choosing the wage vector $w_j = (w_{1j}, w_{2j}, ..., w_{Ij})$, $j = 1, ..., J$ i.e.:

$$\pi_j = \max_{w_j} E[y_j(l(w_j)) - w_j' l(w_j)]. \tag{33}$$

Using a second order Taylor-Approximation, $E[y_j(l(w_j)) - w_j' l(w_j)]$ can be rewritten as $y_j(E(l(w_j))) - w_j' E(l(w_j))$. Note that, due to tractability reasons, it is assumed that firms do not react on short-run variations in employment, implying that firms specify their wage policy at the outset and do not change it. Holzner and Launov [2005] assume complementarity between the production factors (supermodularity) in the pro-

[31] Firms do not offer wages below the reservation wage of the individuals. This can be justified by assuming for the production function that each skill is essential in production and is implied by the supermodularity assumption made below.

duction function y_j. This guarantees, provided a continuous distribution, that (type j) firms cover exactly the same position in the wage offer distribution of each skill group.[32] As above, if there exists a mass point, it exists at the upper bound $\bar{w}_{ij}$ of the skill-specific wage distribution H_{ij} for firms with technology j. At this point, marginal productivity given the employment at that wage equals the wage $y'_j(l_{ij}(w_{ij})) = w_{ij}$.

It is assumed that firms profits differ according to the technology j employed. In this case, firms sort according to their profitabilities in the skill-specific wage offer distributions meaning that more profitable firms pay higher wages. Thus, the share of firms that offers wages below the upper bound $\bar{w_{ij}}$ of a skill-specific wage offer distribution of firms of type j equals the share of firms s_j with technology j and less profitable $H_{ij}(\bar{w}_{ij}) = s_j$. The resulting skill-specific wage offer distributions H_i have no holes (connected support) and the reservation wage (of skill group i) is the lower bound of the wage offer distribution (of skill group i) as in the standard model. Excluding mass points, Holzner and Launov [2005] are able to derive an analytical form for the wage offer distribution. They show that depending on the degree of homogeneity of the production function, the model is able to generate increasing or decreasing skill-specific wage offer densities h_{ij}.[33] That means that they do not require differences in technologies to generate a well-shaped wage density, as opposed to the models discussed so far. In addition, they show that for higher wages decreasing densities are more likely.

Introducing a binding minimum wage has the following effects. Assume that a binding minimum wage is introduced for one skill group only. This compresses the wage distribution for this skill group from the left. In addition, the *complete* skill-specific wage distribution H_i is shifted to the right. This follows from the fact that the upper bound of the skill specific wage offer distributions $\bar{w}_{ij}$ for each technology j depends positively on the lower bound. As long as the skill-specific wage offer distribution does not contain a mass point, the other wage offer distributions remain unchanged and firms still cover the same position in the wage distributions for each skill group. It is possible however, that the increase of the binding minimum wage leads to a mass point. Increasing the minimum wage above marginal productivity of the most unprofitable technology would make it optimal for the firms to employ less individuals. This is however not allowed for by the model. It is possible that the minimum wage increases to a level where firms with the most unprofitable technology make negative profits and thus are driven out of the market.

Independently of the precise effects on the wage distribution, employment effects of increasing a minimum wage are zero, since labor demand as represented by λ_i does not react, even when firms go bankrupt. The reason is that λ_i does not depend on the

[32] Intuitively this is the case, since under supermodularity in production a firm that has high employment in each skill group and a firm that has low employment in each skill group together produce more output than any two firms that produce with any other combination of these amounts. Notice that this characteristic carries over to the profits of the firms.

[33] Intuitively, with increasing returns to scale there is a factor which counteracts the effect of decreasing profits per employee from the standard model. To insure constant profits, this implies that employment must grow more slowly as compared to the linear production case. A decreasing wage offer density guarantees employment growth to be slowly.

measure of active firms nor does it depend on the profitability of the use of a specific skill group (as long as the productivity of this skill group is high enough to guarantee some employment). Thus, the minimum wage redistributes rents from firms to workers and might eliminate unprofitable technologies but does not increase unemployment. Introducing a binding minimum wage for all skill groups simultaneously is similar in its effects, with the modification that the point where a technology becomes unprofitable is attained faster.

Note that the existence of mass points in the wage distribution makes it reasonable to think about rationing of jobs [Ridder and Van den Berg, 1997], because there are cases where profits at the mass point are higher with lower employment. The model of Holzner and Launov [2005] does not account for this possibility which must be seen as a drawback.

4 Conclusion

The aim of this paper is twofold. On the one hand, it presents models of the labor market that give ample consideration to the frictional character of the labor market. A number of models that explicitly take account of the incompleteness of information and the process of acquiring information are introduced. In the basic model, an equilibrium wage distribution is derived for ex-ante homogeneous employees and employers. In extensions to the model, the impact of differences on the employer and employee sides on the resulting equilibrium wage distribution are examined. Because heterogeneity of the actors in the labor market is seen as an important factor, it is sensible to study models that explicitly model heterogeneity on both market sides and that are still analytically tractable. All these equilibrium search models are seen to be able to generate residual wage dispersion, which is an important component of wage dispersion as a whole [see, e.g. Juhn et al., 1993, Katz and Autor, 1999, Lemieux, 2006]. Thus, these models contribute to our understanding of wage inequality and its changes.

Recently, the possibility to implement minimum wages in Germany has been extended dramatically in Germany. In addition, the scientific debate about employment effects of minimum wages in Germany has obtained new impulses by the paper of König and Möller [2007].[34] So, the second aim of this paper was to discuss the impact of minimum wages on employment and realized wages under this frictional setting. It turns out that the compression hypothesis, i.e. the assumption that institutional wage compression leads to higher unemployment, is not supported by all models analyzed. I obtain all sorts of employment effects: positive, zero, and negative ones. For example, in the case of continuous search costs, a minimum wage can even lead to a reduction of unemployment. The mechanism is that individuals in this case find more wage offers acceptable. Where I obtain zero employment effects this stems from the

[34] So far, the conclusion in the international literature on employment effects of minimum wages is mixed depending on the country, the extent of the minimum wage, the age group and the industry for which it holds. But all in all, the literature seems to be a bit more in favor of negative employment effects [Neumark and Wascher, 2007].

fact that search frictions guarantee firms a monopsonistic position on the labor market. A minimum wage then restricts the monopsony power of firms and redistributes rents from firms to workers. Labor demand effects do not occur or only in the unlikely case where the minimum wage increases so strong that *all* matches (for a certain skill group) become unprofitable. Only one of the proposed models support the compression hypothesis: namely, the model with heterogeneity on both sides of the market. In this model, this is the case, since the minimum wage makes a part of the matches unprofitable and thus not every meeting does result in a match. A further possibility to incorporate labor demand effects in a reasonable way in a model with search frictions is to endogenize λ. This is done in Fitzenberger and Garloff [2007] and shown to imply negative employment effects of minimum wages. Summarizing, in the search context, no definite answer is available to the question of the influence of a binding minimum wage on unemployment. In the spirit of Koning et al. [1995], one could argue that as long as the minimum wage is not too high, there are no employment effects of minimum wages and these only redistribute rents. However, considering the labor market as an ensemble of segmented specific labor markets, as they do, suggests that a too high minimum wage could make a whole segment unprofitable. In this case there are pronounced employment effects. Note that when search frictions or more general monopsonistic structures are important, it can be desirable to introduce or increase a minimum wage in order to redistribute rents from firms to workers without incurring the cost of increasing unemployment [see, e.g. Manning, 2003a].

In sum, search approaches offer a good alternative and complement to neo-classical model frameworks. The frictional framework provides a basis for a better understanding of labor market mechanisms in a world of imperfect information. It adds to our understanding as far as labor market dynamics is concerned and as far as the determinants of residual wage dispersion are concerned. Thus, the model is a valuable alternative framework, for evaluating labor market policies, as for example minimum wages. It turns out from our analysis above that the impact of minimum wages is complex. Further theoretical and empirical work is necessary to decide on the issue of employment and wage effects of a minimum wage legislation.

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5 Appendix

5.1 Derivation of the reservation wage as a function of the parameters

Consider the derivative $\frac{\partial W_L(\bar{w})}{\partial \bar{w}}$ from (5). Rewriting (5) as $W_L(\bar{w}) = \frac{\bar{w}+\delta W_U+\lambda_L \int_{\bar{w}}^{\infty} W_L(w)dH(w)}{[r+\delta+\lambda_L \int_{\bar{w}}^{\infty} dH(w)]}$, the resulting derivative is given by $W_L'(\bar{w}) = \frac{\partial W_L(\bar{w})}{\partial \bar{w}} = \frac{1}{r+\delta+\lambda_L(1-H(\bar{w}))}$.[35] Denote with w^o the upper limit of $H(w)$, then integration by parts yields $\int_{w_R}^{w^o}(W_L(w)-W_U)dH(w) = [(W_L(w)-W_U)H(w)]_{w_R}^{w^o} - \int_{w_R}^{w^o} H(w)W_L'(w)dw$. This leads to:

$$
\begin{aligned}
w_R &= z+(\lambda-\lambda_L)\left[W_L(w^o)-W_U-\int_{w_R}^{w^o} H(w)W_L'(w)dw\right] \\
&= z+(\lambda-\lambda_L)\left[\int_{w_R}^{w^o}(1-H(w))W_L'(w)dw\right] \\
&= z+(\lambda-\lambda_L)\int_{w_R}^{w^o}\frac{1-H(w)}{r+\delta+\lambda_L(1-H(w))}dw.
\end{aligned}
$$

The second row follows by using $W_L(w^o)-W_U = \int_{w_R}^{w^o} W_L'(w)dw$ and the above expression for $W_L'(w)$.

5.2 Derivation of the equilibrium employment at wage w

Starting point for the derivation of (10), is the following equation which describes inflows and outflows to firms paying wages above w, $(\lambda U+\lambda_L L(w))(1-H(w)) = \delta(N-U-L(w))$.

Differentiating both sides of the equation with respect to the wage, substituting $h(w)$ for $H'(w)$, using that $L'(w) = l(w)h(w)$, and dividing by $h(w)$ yields:

$$[\delta+\lambda_L(1-H(w))]l(w) = \lambda U+\lambda_L L(w). \tag{34}$$

Firms maximize expected profits $\Pi(w) = (y-w)l(w)$. The first-order condition for a profit maximum yields the following differential equation:

$$\frac{l'(w)}{l(w)} = \frac{1}{y-w}. \tag{35}$$

This equation holds for all firms that pay wages above w_R. With the help of (35) $l(w)$ can be determined explicitly. Integrating both sides $\int \frac{l'(w)}{l(w)}dw = \int \frac{1}{y-w}dw$ or $\log l(w) + d_1 = -\log(y-w)+d_2$ is obtained, where d_1 and d_2 are integration constants. Letting $d = d_2 - d_1$ and exponentiating both sides yields:

$$l(w) = \frac{\exp(d)}{y-w} = \frac{D}{y-w}. \tag{36}$$

[35] The result is obtained by using the quotient rule and the fact that $\frac{\bar{w}+\delta W_U+\lambda_L \int_{\bar{w}}^{\infty} W_L(w)dH(w)}{[r+\delta+\lambda_L \int_{\bar{w}}^{\infty} dH(w)]} = W_L(\bar{w}) = \frac{A(\bar{w})}{B(\bar{w})}$. This yields $W_L'(\bar{w}) = \frac{A'B-AB'}{B^2} = \frac{1}{B}(A'-W_L(\bar{w})B') = \frac{1}{B}(1-\lambda_L W_L(\bar{w})h(\bar{w}) + \lambda_L W_L(\bar{w})h(\bar{w})) = \frac{1}{B}$.

The integration constant D can be derived through the constraint that (34) imposes on the above equation. Evaluating (34) and (36) at w_R and imposing equality, I obtain $l(w_R) = \frac{\lambda U}{\delta+\lambda_L} = \frac{D}{y-w_R}$ or $D = \frac{\lambda U}{\delta+\lambda_L}(y - w_R)$. Using D and U in (36) provides the solution to the differential equation: $l(w) = \frac{\lambda\delta N}{(\delta+\lambda_L)(\delta+\lambda)} \cdot \frac{y-w_R}{y-w}$.

The equilibrium profits of a firm that pays a wage from the support of the wage distribution is given by $\Pi(w) = (y - w)l(w) = \frac{\lambda\delta N(y-w_R)}{(\delta+\lambda_L)(\delta+\lambda)}$.

The upper limit of the support of wage distribution w^o can be calculated by inserting w^o in (34) and in $l(w) = \frac{\lambda\delta N}{(\delta+\lambda_L)(\delta+\lambda)} \cdot \frac{y-w_R}{y-w}$, and solving for $l(w^o)$, respectively. Noting that $L(w^o) = N - U$, I obtain

$$w^o = y - (y - w_R)\left(\frac{\delta}{\delta+\lambda_L}\right)^2. \tag{37}$$

as upper limit of the support of the wage distribution and the distribution of paid wages. Note, that the highest paid wage is below the marginal productivity of the employees.

5.3 Profits with continuous productivity dispersion

The solution of (18) $\Pi'(y) = \breve{l}(K(y))$ is obtained when integrating $\Pi(y) = \int_{\underline{y}}^{y} \Pi'(\varrho)d\varrho = A + \int_{\underline{y}}^{y} \breve{l}(K(\varrho))d\varrho$. A is the integration constant and follows from (16), when evaluated at ($\underline{y}$,$\underline{w}$), where $\underline{w} = \max\{w_R, w_{\min}\}$. Therewith, $A = \frac{\delta}{\delta+\lambda_L}\frac{N-U}{M}(\underline{y} - \underline{w})$. Furthermore, the share of firms that pays wages below $K(y)$ is equal to the share of firms whose productivity is below y: $H(K(y)) = \Gamma(y)$. Using (15), it is $\breve{l}(K(y)) = \delta\frac{(N-U)}{M}\frac{\delta+\lambda_L}{[\delta+\lambda_L(1-\Gamma(y))]^2}$ and thus the profit function becomes:

$$\begin{aligned} \Pi(y) &= \frac{\delta}{\delta+\lambda_L}\frac{N-U}{M}(\underline{y} - \underline{w}) + \int_{\underline{y}}^{y} \delta\frac{(N-U)}{M}\frac{\delta+\lambda_L}{[\delta+\lambda_L(1-\Gamma(\varrho)]^2}d\varrho \\ \Pi(y) &= \int_{\underline{w}}^{y} \delta\frac{(N-U)(\delta+\lambda_L)}{M}\frac{1}{[\delta+\lambda_L(1-\Gamma(\varrho))]^2}d\varrho. \end{aligned} \tag{38}$$

The second row follows from the fact that $\Gamma(y) = 0$ for $y \in [\underline{w}, \underline{y}]$ and thus the integral on the interval $[\underline{w}, \underline{y}]$ in the second row, equals the first summand in the first row. This equation yields the profit of a type y firm depending on the model parameters and on the distribution of firm productivities. Solving $\Pi(y) = (y - K(y))\breve{l}(K(y))$ with respect to $K(y) = w$ yields an expression for the wage as a function of the productivity y: $w = K(y) = y - \frac{\Pi(y)}{\breve{l}(K(y))}$. Using the corresponding expressions yields: $K(y) = y - [\delta + \lambda_L(1 - \Gamma(y))]^2 \int_{\underline{w}}^{y} \frac{1}{[\delta+\lambda_L(1-\Gamma(\varrho))]^2}d\varrho$.

5.4 Variance analysis

Start by assuming a utility function with constant relative risk aversion (CRRA) and consider a worker who is employed in a firm of type y, then: $\ln w_w(\varepsilon, y, y') = \ln\varepsilon + \ln w_w(1, y, y')$. The conditional (on y) expectation of the log-wage is given by Postel-

Vinay and Robin [2002a, p.2310]: $E_{(\varepsilon,y'|y)}(\ln w|y) = E_\varepsilon(\ln\varepsilon)+E_{(y'|y)}(\ln w_w(1,y,y')|y)$.[36]
Using independence of ε and (y,y'), the conditional (on y) variance is given by $var_{(\varepsilon,y'|y)}(\ln w|y) = var_\varepsilon(\ln\varepsilon) + var_{(y'|y)}(\ln w_w(1,y,y')|y)$. Applying this variance decomposition, the variance of wages can be decomposed in the variance of the conditional expectation of wages and in the expectation of the conditional variance:

$$
\begin{aligned}
var_w(\ln w) &= var_y[E_{(\varepsilon,y'|y)}(\ln w|y)] + E_y[var_{(\varepsilon,y'|y)}(\ln w|y)] \\
&= var_y[E_\varepsilon(\ln\varepsilon) + E_{(y'|y)}(\ln w_w(1,y,y')|y)] + E_y[var_\varepsilon(\ln\varepsilon) \\
&\quad + var_{(y'|y)}(\ln w_w(1,y,y')|y)].
\end{aligned}
$$

Thus, the following decomposition of the wage variance is obtained:

$$var_w(\ln w) = var_\varepsilon(\ln\varepsilon) + var_y[E_{(y'|y)}(\ln w_w(1,y,y')|y)] + E_y[var_{(y'|y)}(\ln w_w(1,y,y')|y)].$$

The first summand of this formula results from productivity differences among employees. The second summand reflects the effect of different firm productivities on the variance of paid wages. The expected wage changes along with y, the productivity of the firm. The variance of the conditional expectation reflects the variance of wages between firms of different productivities. Note, that the conditional expectation of the log-wage and thus the variance of the second summand depends on the joint distribution of y, y'. The third summand reflects wage fluctuations for firms and workers whose productivity is identical. Thus, the wage fluctuations among identical individuals in identical firms are contained in this part.[37] From the point of view of an individual it is explained by the luck of receiving a valuable job offer that implies pay raises. The extent of this variance is explained by frictions because frequent job offers lead to a faster adjustment of wages to the marginal productivity and thus lowers the variance.

[36] The indices indicate with respect to which variable the expectation is to be constructed.
[37] This is the part of the wage dispersion that is explained by the model of Burdett and Mortensen [1998].

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