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Labor market pooling and human capital investment decisions

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Abstract

Of the typically cited agglomeration advantages labor market pooling receives strong empirical support — yet remains under-explored theoretically. This paper presents a model of human capital formation in an imperfectly competitive, pooled local labor market with heterogeneous workers and firms. Firms produce for a competitive output market with differing technologies, thus requiring diverse skills. In anticipation of firm behavior, workers choose between specializing into specific skills and accumulating general human capital. While labor market pooling provides static efficiency gains, our approach also suggests that there are long-term effects: under a diversified industrial structure, industry-specific shocks lead to a labor market pooling advantage which raises the incentive for workers to acquire both general and specific human capital. This will not only strengthen a region's capability to adapt to change but will also contribute to higher growth.

JEL classification: I20, J24, J41

Keywords: human capital, heterogeneity, unemployment, labor market pooling

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1 Introduction

Agglomerations grow faster and their labor productivity is higher compared to less agglomerated areas. Looking at US states, Ciccone and Hall (1996) report a 6 percent increase in labor productivity upon a doubling of employment density. Higher productivity is also evident in higher nominal wages: Glaeser and Mare (2001) confirm the existence of an urban wage premium of about 33 percent across US metropolitan areas; accounting for personal and job characteristics as well as for unobserved ability still leaves a substantial portion of the wage premium. Moreover, apart from a wage level effect, there is also evidence of wage growth effect contributing to higher urban wages. This suggests that apart from higher productivity due to, for example, demand or information externalities, there is another mechanism at work which causes benefits to rise over time. While modern growth theories tend to explain these results by human capital spillovers, Quigley (1998) emphasizes city size effects and the "independent role of diversity in enhancing economic efficiency". Among the economies of agglomeration, he claims, are reduced transaction costs as heterogeneous firms and workers will find themselves better matched in larger markets, and cost savings from the stabilization of employment when the law of large numbers is applied to imperfectly correlated employment fluctuations.

In a summary of the empirical evidence on the sources of agglomeration economies, Hanson (2001) concludes that there seem to be two robust results emerging from the empirical literature: firstly, that individual wages are increasing in the number of educated workers, and secondly, that long-run industry growth is higher in locations with a wider range of industrial activities.

While this evidence is consistent with the idea of knowledge spillovers - highly skilled individuals interacting and learning from each other - it also supports a labor pooling interpretation: In what follows we employ a model of a pooled labor market in order to explain the productivity and wage premium typically found in agglomerations. Pooling creates an advantage for both workers and firms as it improves matching and facilitates adjustment to shocks, thus raising the return to human capital investment. Since human capital is endogenous, this opens up the possibility of growth effects. The model thus takes into account the density of an agglomeration, its industry composition, and its human capital.

1.1 Evidence

Empirical studies on the existence and extent of agglomeration economies, particularly those that estimates production functions, augmented by some proxy for agglomeration, are surveyed in Rosenthal and Strange (2001, 2004). These studies typically focus on the industrial scope of agglomeration economies and are overall "somewhat more favorable to the existence of localization economies than to urbanization economies" (Rosenthal and Strange, 2001). The other strand of the literature has focused on the relationship between employment growth (e.g. Henderson, Kuncoro, and Turner (1995) Rosenthal and Strange (2003)) or wages (e.g. Glaeser and Mare, 2001) and agglomeration. They all confirm the existence of positive agglomeration economies.

As regards the sources of agglomeration economies, most of the studies investigate the role of knowledge spillovers and the evidence seems to suggest that knowledge spillovers do exist but vanish quickly with distance (Audretsch and Feldman (2004), Jaffe, Trajtenberg, and Henderson (1993)). The strongest evidence is available for labor market pooling. Dumais, Ellison, and Glaeser (2002) study the sources of agglomeration economies at different levels of geographical aggregation. They find knowledge spillovers to occur only at the metropolitan level, input sharing to occur at the metropolitan and more so at the state level, and evidence in favor of labor market pooling at all levels of aggregation. Wheeler (2006) and Yankow (2006) confirm the existence of a wage growth effect following job changes, which supports a matching or coordination efficiency explanation of agglomeration economies.

Costa and Kahn (2000) compare location decisions of highly educated and less educated workers, and find that highly educated couples prefer cities for, among other reasons, it is easier to find the appropriate employment for both.

In a recent paper, Strange, Hejazi, and Tang (2006) use Canadian Survey data on firms' strategic perceptions in order to test a model where agglomeration follows from uncertainty. While firms with uncertainty regarding technological change and innovativeness are found in larger cities, firms with a strong skill-orientation prefer to locate in industry clusters.

1.2 The Theory

While scale economies in production (and the the sharing of specialized inputs) have received considerable attention, models of labor market pooling have so far remained rather less well explored as an ingredient into economies of agglomeration. An early formalization is presented in Krugman (1991), who sees the advantage of a pooled labor market in the reduction of risk: Workers benefit from lower income risk, firms benefit from the availability of workers if they wish to respond to a positive demand shock. A different benefit of pooling, namely improved matching, is emphasized in Helsley and Strange (1990), who incorporate labor market heterogeneity into a general equilibrium model of a system of cities. With cities' population growth being determined endogenously over migration, the labor market can be shown to generate agglomeration economies as both workers and firms expect to be better matched in larger cities. A similar idea is presented in Kim (1990): As cities grow, the average skill distance between workers and firms decreases so that with lower costs of mismatch workers face a stronger incentive to invest in human capital.

A dynamic effect of pooling is formalized by Kim and Mohtadi (1992): economic growth is determined from ever increasing specialization if constant population growth is assumed. With more workers and firms in the market the matching between them is improved, hence further specialization among workers is encouraged which generates ongoing growth. A similar mechanism for increased investments into skills is proposed in Wheeler (2001): with qualitatively heterogeneous skills and skill requirements, a larger market will enable greater sorting, so that urban markets will exhibit higher productivity, higher wages, and more wage inequality between skill levels. An explicit growth mechanism, however, is absent.

A model that combines uncertainty and the matching problem is presented in Strange et al. (2006): a firm with a particular but uncertain resource need will find it easier to respond to the uncertainty if there is a wider range of resource suppliers in the local market. Conversely, resource suppliers will prefer to locate where more downstream firms are active. Agglomeration increases the expected match quality in the face of uncertainty.

Rotemberg and Saloner (2000) demonstrate, that agglomerations, by pooling their labor input, help to alleviate the so-called hold-up problem. In particular,

competition among firms for trained individuals ensures workers with the appropriate return on their human capital investment. The hold-up problem arises when training occurs before a workers-firm pair is matched and in the absence of any ex-ante contract.

Other models of labour market pooling have analyzed the location decision of firms, see for example Matouschek and Robert-Nicoud (2005) and Combes and Duranton (2006). The latter emphasize the close relationship between pooling and knowledge spillovers as joint determinants of the location decision. Agglomeration is the result of a trade-off between the benefits and cost of pooling: the former arise from the knowledge that a poached worker brings to her new employer, the latter consist of higher wages that a firm has to pay in order to retain and attract workers. As the product market comes closer to perfect competition, firms eventually choose to locate in separate labor markets as the costs of higher wages outweigh the benefits of spillovers. Hence this paper demonstrates how the intensity of competition on product markets can be linked to outcomes/competition on the labor market. Human capital is heterogeneous, but there are no human capital investment decisions. Picard and Toulemonde (2004) model an imperfect pooled labor market with endogenous human capital formation. When deciding on their human capital investment workers do not know whether their abilities will be in demand by any of the firms. With more firms locating in the local market, the range of abilities in demand is extended and so the probability of finding a perfect match increases. One of the implications of the model, the authors claim, is that more general education, by expanding the range of abilities of a worker, will improve the matching (reduce imperfections) and will thus lead towards spatial dispersion of industry.

1.3 Our contribution

In the remainder of this paper, we present a microeconomic model of human capital formation in an imperfectly competitive and pooled local labor market, with heterogeneous workers and firms, when product markets are characterized by price uncertainty. Firms produce for a competitive output market and with differing technologies, thus requiring diverse skills. In anticipation of firm behavior, workers choose between specializing into a certain type of skills, and accumulating general skills. We thus look at the endogenous determination of the level of

both horizontally differentiated and general human capital.

Our model builds upon the idea that human capital has both a general and a specific component, as expressed in Kim (1989).[1] In a series of papers, Kim (1989, 1990, 1991) studies the impact of local labor market size on wages and human capital formation. Adapting the Salop model of product differentiation to the labor market, the skill space is represented by the circumference of a circle. In Kim (1989), wages rise due to better matching, and workers invest more in intensive human capital as the market is enlarged.

The distinction between general and specific human capital allows us to capture two important aspects of labor market pooling: increased match quality and risk reduction. While investments into specific skills improve a worker's productivity if matched with the appropriate technology, general skills increase a worker's flexibility, ie. her ability to retrain and adjust to a change in skill requirements.

Empirical support for this argument is provided by Glaeser and Saiz (2003): having first identified education as an important ingredient into agglomeration economies, they find evidence for a causation running from skills to growth, the mechanism being increased productivity (at the metropolitan area level). However, the finding that skills matter most in declining cities supports the so called Reinvention City view, according to which skills are important for a city's adjustment to negative shocks. Human capital therefore increases the flexibility of the local work force and thus helps a region and its firms to better adjust to shocks in demand or technology. The adaptive nature of skills is also emphasized in Iyigun and Owen (2006): within the framework of a growth model, they claim that in a changing environment, characterized by frequent technological change, education will produce adaptive skills which increase the future adaptability of the workforce. This, in turn, increases the resources devoted to R&D and therefore the rate of technical change.

Our approach is related to Jellal, Thisse, and Zenou (2005) who investigate the effects of product market fluctuations on the labor market when firms and workers both are heterogeneous. Full employment and unemployment equilibria are derived, with unemployment being the result of volatile prices and of mismatch. Our paper differs in two important aspects. Firstly, human capital in our

[1] Kim refers to specific human capital as intensive, and to general human capital as extensive.

model is endogenous. Secondly, in our set-up price shocks are revealed and firms adjust wages and employment, while workers form expectations of their wage.

Our paper adds to the existing literature in that it synthesizes the endogenous formation of heterogeneous skills, and labor market pooling. A labor market pooling advantage arises a the result of a portfolio effect and a matching effect. The model thus provides a link between product and labor markets.

As regards the portfolio effect, a pooled labor market protects both firms and workers against demand uncertainty. Assuming that demand shocks will be industry-specific, the portfolio benefit will be higher the lower the correlation of demand shocks. This will be the case if the industry structure in a region is rather diverse in the sense that its industries produce for diverse output markets.

In contrast, the matching effect of pooling applies to a context where workers' skills are specialized, and firms require specialized skills. This argument rests on the assumption that skills and technology are complementary. The average skill distance between workers and firms determines the degree of mismatch in the local labor market, and this is reflected in the matching effect.

Consequently, the pooling advantage is largest, if a diversified industrial structure is combined with a spectrum of specialized but similar skills. The model is thus consistent with the two "stylized facts" reported by Hanson (2001), i.e. the decisive role of human capital and industry structure in the economic performance of agglomerations.

We will proceed as follows: the next section introduces the model. Section 3 presents the derivation of labor market equilibria and analyzes workers' optimal investments into human capital. In section 4, we discuss a particular specification of our model where additional restrictions generate unemployment. Before concluding, we discuss the empirical implications of our analysis and discuss potential links between local human capital and growth arising from our model in section 5. All proofs are relegated to the appendix.

2 The Model

We consider a local labour market composed of a mass of workers normalized to unity and two firms competing for these workers. Firms produce for a market that is subject to random price fluctuations. Before entering the labour market,

workers decide about their human capital investment in anticipation of future labour market outcomes. Both workers and firms are risk-neutral and maximize expected income and profits, respectively.

2.1 Workers

A worker's human capital is characterized by a specific, worker-innate skill, $s \geq 0$, as well as a general ($x > 0$) and a specific ($y > 0$) component.[2] In her investment decision, a worker only decides about the latter two components, while her skill is predetermined and private information. Specific human capital directly increases a worker's productivity in the production process. For simplicity, let y equal the amount of output produced by the worker. In contrast, general human capital helps the worker to adapt her skills to the specific needs of the employer. If her skill s differs from the employer's skill requirement s^* a worker has to bear adjustment costs c^a which general skills help to reduce. Specifically, we assume the following specification of adjustment costs: $c^a(s, s^*) = |s - s^*|/x$. Human capital formation is costly: the cost function is assumed to be additively separable, increasing and convex in both types of human capital. For simplicity we use the specific functional form $c^h(x, y) = \alpha(x^2 + y^2)/2$, where $\alpha > 0$.[3]

Workers are confronted with two kinds of uncertainty: When making their investment decision, workers do not know, firstly, how far in terms of skills they are apart from their prospective employers. We assume that skills s are distributed uniformly over $[0, S]$ with density S. Only after human capital decisions have been made will the ordering of skills relative to prospective employers be revealed to workers. This uncertainty captures the effect of technological developments which affect production structures.[4] The distance between the average

[2]Note that the meaning of "general" and "specific" is somewhat different from Becker (1964) and related work. In particular, a worker might possess both more specific and more general human capital than another worker.

[3]We will further restrict the parameter α where necessary to ensure existence of optimal solutions.

[4]For example, a student in IT might specialize in a specific programming language, not knowing whether this or some other language will be demanded by his future employer. Ex ante, various programming languages might be equally important. Similarly, law students have to specialize at some point of their studies, without knowing for sure what they are going to need most in their future job.

worker and the firms can be interpreted as the degree of mismatch in the local labour market. The second uncertainty is with respect to firms' labour demand, as product market conditions keep changing (see below). Expectations over mismatch and demand will influence workers' investment decisions which in turn will affect labour market outcomes.

2.2 Product Market

When selling their products to the markets, the two firms face stochastic price realizations. We assume that the output price for firm $i \in \{1, 2\}$, $\tilde{p}_i > 0$ fluctuates around the expected value $E[\tilde{p}_i] = p$ with variance σ_p^2, the distribution of both prices being identical. Although the two firms produce for different markets, we allow for some dependence in demand which is captured by the correlation coefficient $\rho \in [-1, 1]$ of the two prices. Price realizations are observable to all players. Prices determine productivity and firms compete on the labour market.

The introduction of stochastic prices allows us to analyze the interrelation of (product) demand uncertainty, labour demand, labour market equilibria and human capital investment decisions. In the model, the correlation coefficient indicates the degree of industrial specialization within the local economy: the industrial structure of a local market can be considered more specialized (diversified) the higher (lower) the correlation of firms' product market prices. High local specialization via the agglomeration of firms of identical or very similar industries makes the market vulnerable to industry-specific shocks, as these affect the whole market.

Apart from the obvious demand shocks and industrial specialization interpretation, our analysis can also be applied to markets where firms differ in their innovation rates. If process innovation is an important feature of local industrial structure, then a firm can turn a successful innovation into some productive advantage. Higher productivity temporarily increases the firm's labour demand and market power, until rival innovations, knowledge spillovers, or the expiry of related patents will level out productivity differences between competitors. In a low-technology industrial environment with little innovative activity, the probability that external factors affect firms' productivity jointly is much higher. In this sense, higher (lower) correlation of prices may also be interpreted as a local environment with lower (higher) innovation.

2.3 Labour Market

We consider two firms which differ in their production technology in the sense that they require workers of different skills. We assume that the two firms' technologies are characterized by maximum differentiation. We order worker skills according to their closeness to firm 1's required skill: $s = 0$ denotes the skill which matches firm 1's (firm 2's) demand perfectly (least), whereas $s = S$ is the worst (perfect) skill-match for firm 1 (firm 2). Hence, by definition of skill space and ordering, the two firms are located at the endpoints of a line S which represents the skill space. The length of that line can then be interpreted as the degree of worker heterogeneity. A smaller S means that firms become more similar in terms of their technologies and skill requirements, and that the average distance (and hence mismatch) between a worker and the nearest firm decreases. The fixed mass of workers is then distributed over a smaller interval with higher density of the uniform distribution.

The two firms are unable to observe a worker's innate skill. Given that workers do not know their future skill position relative to the firms' optimal skill demand, they all form the same expectations and hence develop the same human capital structures. Firms cannot use the human capital structure as a signal of skill.[5] This reduces firms' ability in the labour market to offering a unique wage to all workers and let them incur the (unobservable) adjustment costs. Workers will then sort themselves to the firm which offers the highest wage net of adjustment cost. After retraining, all workers in a firm will be identical. Our model of wage competition is thus equivalent to price competition within the well-known Hotelling product market structure.[6] Similarly to the reservation prices in the product market version, we assume that workers require a non-negative wage.

Our assumption about firms' locations at the end of the line $[0; S]$ and the a-priori non-observability of their relative positions along S for workers is less critical than it seems. One could imagine the set of skills to be a circle of circum-

[5]Similarly, workers are unable to reveal their skills otherwise, as they will all be tempted to announce the skill which promises them the highest wage, as long as the skill is non-verifiable.

[6]See Hotelling (1929), Anderson, De Palma, and Thisse (1992) or Bester, De Palma, Leininger, Thomas, and Von Thadden (1996) for examples, as well as the application of similar structures to local labour markets in Kim (1989, 1990), Thisse and Zenou (2000), Jellal et al. (2005) or Hamilton, Thisse, and Zenou (2000).

ference $2S$ where the two firms' location choice follows workers' human capital decisions. In the spirit of the analysis of Kats (1995), one can show that there exist sufficient conditions for an equilibrium with maximum differentiation between the two firms.[7] Since we are looking at an application of this type of model, we refrain from explicitly deriving equilibria in locations and instead take locations as given. However, in order to ensure our market equilibrium comprises both firms, we impose the following parameter restriction:

$$\max[\tilde{p}_i - \tilde{p}_j] < \frac{3S}{xy} \quad \text{for } i, j \in \{1, 2\} \text{ and } i \neq j \tag{1}$$

Technically, this ensures a labour market equilibrium where both firms employ a positive mass of workers even in the case where the price difference reaches its highest value ($\max[\tilde{p}_i - \tilde{p}_j]$).[8]

Another parameter restriction is given by the participation constraint of workers, as we will first consider only full employment equilibria. As long as

$$\min \tilde{p}_i \geq \frac{3S}{2xy} \quad \text{for } i \in \{1, 2\} \tag{2}$$

is fulfilled, equilibria will see all workers employed even when both firms realize their lowest possible prices. Figure 1 summarizes the timing of our model. In

Figure 1: Time structure

the following section, we will now solve the model, starting with the derivation of the general labour market equilibrium.

[7] We are aware of the analysis of Bester et al. (1996) and the implicit coordination assumption of previous studies' location equilibria they identify. However, by definition, there are no end points in the circular structure which firms could assign positive probability mass in location decisions. Hence, the probability of the two firms locating at identical positions is zero.

[8] At the same time, this condition also ensures that no firm will find it profitable to drive its competitor out of the market.

3 Full Employment Analysis

3.1 Labour Market Equilibrium

An equilibrium in the local labour market consists of a pair of wages, w_1 and w_2, and the allocation of workers to firms (as long as net wages are non-negative). With full employment in equilibrium, the marginal worker of skill $\hat{s}$ who is indifferent between working for firm 1 and firm 2 can be derived via $w_1 - \frac{\hat{s}}{x} = w_2 - \frac{S-\hat{s}}{x}$:

$$\hat{s}(w_1, w_2) = \frac{S}{2} + \frac{x}{2}(w_1 - w_2) \tag{3}$$

Workers in $[0, \hat{s}]$ then constitute firm 1's labour supply, those in $[\hat{s}, S]$ choose to work for firm 2. Then, profits of the two firms can be given depending on prices and wages:

$$\pi_1(w_1, w_2) = \frac{\hat{s}(w_1, w_2)}{S}(\tilde{p}_1 y - w_1) \quad \text{and} \tag{4}$$

$$\pi_2(w_1, w_2) = \frac{S - \hat{s}(w_1, w_2)}{S}(\tilde{p}_2 y - w_2) \tag{5}$$

Partial differentiation with respect to own wages results in the individual reaction function of firm i

$$w_i(w_j) = \frac{\tilde{p}_i y + w_j}{2} - \frac{S}{2x} \tag{6}$$

which then leads to the equilibrium (*Bertrand*) wage of firm i

$$w_i^B = \frac{2\tilde{p}_i + \tilde{p}_j}{3} y - \frac{S}{x} \tag{7}$$

and equilibrium marginal worker

$$\hat{s}^B = \frac{xy}{6}(\tilde{p}_1 - \tilde{p}_2) + \frac{S}{2} \tag{8}$$

Two conditions have to be fulfilled for this equilibrium to exist for all price realizations: First, the marginal worker has to be within $[0, S]$; this condition is ensured by restriction (1). Second, the marginal worker has to receive a non-negative net wage even when price realizations for both firms are lowest; restriction (2) ensures this is the case.

3.2 Labour Market Pooling

So far, the equilibrium derivation is similar to the (full employment) analysis in earlier models like Kim (1989, 1990), Thisse and Zenou (2000), Hamilton et al. (2000) or Jellal et al. (2005). We depart from their set-up by allowing workers' productivity to differ between firms, hence equilibrium wages and employment can be asymmetric. We now proceed to show how our model thus captures advantages from labour market pooling for both firms and workers.

Consider workers first: Wages are uncertain in two respects. One is a worker's position in skill space relative to the two firms; the other is her productivity with a firm as prices have not been realized. Hence, the expected net wage of a worker is:

$$E[w^T] = E_{\tilde{p}}\left[\frac{\hat{s}}{S}(w_1 - E_{s\leq\hat{s}}[c_1^a(s)]) + \frac{S-\hat{s}}{S}(w_2 - E_{s>\hat{s}}[c_2^a(s)])\right] \tag{9}$$

The expectations are taken with respect to firstly prices. The first term within brackets captures the expected net wage of a worker at firm 1, weighted with the probability of employment there. As the worker's position within $[0, \hat{s}]$ is uncertain, expectations with respect to s, too, have to be taken (while the value of $\hat{s}$ still depends on price realizations). The second term captures the same probability weighted payoff to the worker in case she is employed at firm 2.

By symmetry, expected profits of the two firms are identical and only depend on the realization of prices. They can thus be derived for example via firm 1 profits

$$E[\pi_1] = E_{\tilde{p}}\left[\frac{\hat{s}}{S}(\tilde{p}_1 y - w_1)\right] \tag{10}$$

Inserting equilibrium wages and employment from (7) and (8) into (9) and (10) yields our first set of results.

Proposition 1 *In the labour market equilibrium,*

1. *the expected payoffs of workers and firms and the expected total surplus are, respectively:*

$$E[w^T] = py - \frac{5S}{4x} + \frac{xy^2}{18S}\sigma_p^2(1-\rho) \tag{11}$$

$$E[\pi_i] = \frac{S}{2x} + \frac{xy^2}{9S}\sigma_p^2(1-\rho) \tag{12}$$

$$E[TS] = py - \frac{S}{4x} + \frac{5xy^2}{18S}\sigma_p^2(1-\rho) \tag{13}$$

2. *all expected payoffs decrease in price correlation, ρ, and (weakly) increase in price uncertainty, σ_p^2 (portfolio effect);*

3. *expected wages and total surplus decrease in skill differentiation, S (matching effect).*

In equilibrium, the labor market features two pooling effects, a portfolio and a matching effect. The latter has already been present in other models of local labor markets such as Thisse and Zenou (2000) or Hamilton et al. (2000). A mismatch arises from worker-firm pairs with differing skill-supply and skill requirement and is captured in the adjustment costs.[9] In a more agglomerated area, we would expect to find a greater number of firms and a finer division of labor among them. In our setting, this is captured by firms being closer in terms of their skill requirements (S is lower). This implies that the costs of worker mismatching decrease, thus raising overall productivity in the market. However, a finer division of labor also increases the competitive pressure on the labor market, as firms compete for less heterogenous workers. Thus, workers reap the benefits in the form of higher wages, while firms' profits may decrease.[10]

The portfolio effect of labor market pooling has also been acknowledged in the literature before. However, it has not been formalized in a local labor market model before. Interestingly, the effect arises even under our assumption of risk neutrality. Rather, the effect is generated by the combination of labor market flexibility and asymmetries in firms' productivity. The ability of firms to adjust employment and wages allows them to shift employment from the lower-productive firm to the one with higher productivity. This increased employment efficiency benefits firm. Part of this gain, however, is transferred to the workers due to the wage competition between firms. Hence, all market participants gain from (expected) employment adjustments due to productivity shocks. These adjustments only occur for less-than-perfect price correlations ($\rho < 1$) and increase with lower correlation and higher uncertainty (higher σ_p^2).

[9] These costs might take the form of lower initial wages for new employees.

[10] The overall effect of S on profits is ambiguous: Firms also benefit from a lower segment size as it increases the relative importance of asymmetric labor market outcomes in firm profits. For sufficiently low correlation and sufficiently high uncertainty, profits may increase when S falls.

3.3 Human Capital Formation

We now turn to analyzing workers' choice of human capital investment in the first stage of the model. The maximization of an individual worker is

$$\max_{x,y} E[w^T] - c^h \tag{14}$$

Assuming existence of interior solutions to this problem, we can state our next result.

Proposition 2 *Workers' optimal human capital investments, x^* and y^*, have the following properties:*

1. *extensive and intensive human capital are complements, as long as $\rho < 1$;*
2. *both types of human capital increase in the expected product price, p, as long as $\rho < 1$;*
3. *both types of human capital decrease in price correlation, ρ;*
4. *both types of human capital increase in price uncertainty, σ_p^2, as long as $\rho < 1$;*
5. *specific human capital decreases in skill differentiation, S.*

The first interesting result in proposition 2 is that the two types of human capital complement each other even tough the two types are independent in both the productivity structure and the cost function of human capital formation. This complementarity is due to the aforementioned portfolio effect: For perfectly correlated prices ($\rho = 1$), both types of human capital would be independent. As a consequence, a region and its workforce can therefore be both highly productive and flexible. The accumulation of specialized skills does not necessarily lead a region's work force being locked into specific jobs or firms.

The complementarity of human capital types induced by the portfolio effect also drives the subsequent results in proposition 2. Higher expected productivity (p) directly raises the value of specific human capital. Complementarity then raises the returns to both types of human capital even further, such that the overall effect on both types is positive. Price uncertainty and correlation directly

influence the degree of labor turnover in response to asymmetric prices. As this turnover is on average positive, both types of human capital are higher in a more diversified and uncertain market.

The effect of skill differentiation S on human capital levels is more complex, as the matching and portfolio effects interact. Without the portfolio effect ($\rho = 1$), an increase in worker heterogeneity would lead to a greater degree of mismatch in the labor market. As general human capital can alleviate this effect, workers would be expected to increase their general human capital. At the same time, the benefit of diversification and turnover decreases with greater skill differentiation. Hence, the benefit of both types of human capital captured in the portfolio effect decreases, inducing lower investments into human capital. While the latter effect is unambiguous with respect to specific human capital, the overall effect on general human capital is indeterminate.

In sum, our results suggest two channels for higher human capital in a local labor market. First, an industrial structure that provides greater diversification unambiguously increases the return to both types of human capital. Second, more agglomerated markets with higher division of labor among firms increases the returns to specific human capital. While cities may feature both types of channels, the latter may on average be more prevalent as cities greatly differ in the degree of (industrial) diversification. Hence, our model predicts more specialization of labor in cities relative to less agglomerated areas.

4 Scenario: The Case of Unemployment

So far we have assumed full employment. However, unemployment is a significant and persistent phenomenon in today's industrialized economies. This section shows how the possibility of unemployment affects local human capital formation.[11] Specifically, we will allow for some price realizations to be so low that unemployment arises. Hence, we now consider a market where (temporary) drops in productivity may be quite severe.

[11]For the set-up of a local labor market, Thisse and Zenou (2000) and Jellal et al. (2005) have shown how unemployment may arise from firms' market power and a low level of worker productivity relative to the degree of mismatch.

4.1 Model Adjustments

In order to simplify the analysis, we now assume a specific shock structure: A firm faces either of two price realizations, $\tilde{p}_i \in \{p-\varepsilon; p+\varepsilon\}$, $\varepsilon \in [0; p)$, which occur with equal probability (hence, $\sigma_p^2 = \varepsilon^2$). Consequently, there are only four possible combinations of the two firms' prices. Given our symmetry assumptions, we can define the probabilities for these realizations as follows:

$$\begin{aligned} \text{Prob}[\tilde{p}_1 = \tilde{p}_2 = p+\varepsilon] = \ & \tfrac{1+\rho}{4} \ = \text{Prob}[\tilde{p}_1 = \tilde{p}_2 = p-\varepsilon] \\ \text{Prob}[\tilde{p}_1 = p+\varepsilon, \tilde{p}_2 = p-\varepsilon] = \ & \tfrac{1-\rho}{4} \ = \text{Prob}[\tilde{p}_1 = p-\varepsilon, \tilde{p}_2 = p+\varepsilon] \end{aligned} \tag{15}$$

As before, ρ captures the correlation between the two prices and thus determines the probability of symmetric versus asymmetric prices.

We introduce unemployment by assuming that for a low price realizations, the skill distance between those workers (in the middle of line S, the distance to the nearest firm is sufficiently large to let their productivity fall below zero (their reservation wage):

$$(p-\varepsilon)y - \frac{S}{x} < 0 \tag{16}$$

This restriction ensures that for low price realizations at both firms, wages will be depressed to the extent that some workers in the middle of the skill space $[0, S]$ would earn a negative net wage. At the same time, we continue to assume that both firms are active and that there is full employment in the other three price realizations (symmetrically high prices and asymmetric prices). The former is still satisfied under restriction (1), the latter requires the following restriction in place of (2):

$$p \geq \frac{3S}{2xy} \tag{17}$$

Overall, these parameter restrictions constrain both the extent of price fluctuation and the level of expected prices from above and below:[12]

$$\varepsilon \in \left(p - \tfrac{S}{xy}, \tfrac{3S}{2xy}\right) \quad \text{and} \tag{18}$$

$$p \in \left[\tfrac{3S}{2xy}, \tfrac{5S}{2xy}\right) \tag{19}$$

[12]These restrictions are necessary to exclude a case of intermediate net productivity that leads to a rather peculiar equilibrium, as already noted by Salop (1979). See also Thisse and Zenou (2000) or Jellal et al. (2005).

With the above adjustments in place, we now reconsider the labor market equilibria.

4.2 Labour Market Equilibria

First, notice that by restrictions (1) and (17) our analysis from the previous section holds as long as at least one firm's price realization is high. Hence, for three of our four possible cases, the results for equilibrium wages and employment remain as given by (7) and (8), respectively. Hence, it suffices to focus on the case of symmetrically low price realizations. In this case, the two firms act as monopsonists in their part of the labor market as workers' alternative is the (zero) reservation wage.

Consider firm 1's wage setting decision (firm 2's decision is identical). The marginal worker accepting a wage offer w_1 is indifferent between the net wage and the reservation wage: $w_1 - \frac{\hat{s}_1}{x} = 0$. Firm 1 thus faces the labor supply function

$$\hat{s}_1(w_1) = w_1 x \tag{20}$$

Firm 1's profits are then $\pi_1(w_1) = \frac{\hat{s}(w_1)}{S}((p-\varepsilon)y - w_1)$ and maximizing these yields the *monopsony* wage offer

$$w^M = \frac{y}{2}(p-\varepsilon) \tag{21}$$

and the marginal worker at

$$\hat{s}_1^M = \frac{y}{2}x(p-\varepsilon) \tag{22}$$

Restriction (16) ensures that some workers in the middle of the labor market segment do not accept the wage offer. This ensures that the two firms can act monopsonistically and that workers with $s \in [\hat{s}_1^M, S - \hat{s}_1^M]$ remain unemployed. Finally, the expected wage of a worker (with respect to his skill position s) in the case of symmetrically low price realizations equals

$$E[w^M] = \frac{1}{4S}(p-\varepsilon)^2 y^2 x \tag{23}$$

Before reconsidering the human capital decisions of workers, it is worth pointing out two aspects of the scenario considered here: (1) the combination of low productivity realization and outside options (for example due to the introduction

of unemployment benefits) is the key determinant of unemployment in this version of the model; (2) the existence of an outside option has ambiguous effects for the average worker as it provides a safety net at the cost of granting firms monopsony power. It is only for a severe drop in productivity that the overall effect on expected wages is positive in comparison with a market where no outside option exists.[13]

4.3 Human Capital Formation Reconsidered

To analyze human capital formation of the workers, we need to re-calculate the expected wage of a worker, taking now into consideration the four possible cases. This yields

$$\begin{aligned} E[w^T] &= \frac{1+\rho}{4}\left((p+\varepsilon)y - \frac{5S}{4x} + \frac{1}{4S}(p-\varepsilon)^2 y^2 x\right) \\ &\quad + \frac{1-\rho}{2}\left(py - \frac{5S}{4x} + \frac{1}{9S}\varepsilon^2 y^2 x\right) \end{aligned} \tag{24}$$

which is the probability-weighted sum of the expected wage for symmetric price realizations plus the expected wage for asymmetric realizations. While the expected wage features some of the terms familiar from the expected wage in the full employment setting, unemployment now introduces an additional complementarity between the two types of human capital: even for perfectly aligned prices ($\rho = 1$), the two types reinforce their positive effect on wages via the reduction of unemployment.

As before, a worker solves the optimization problem

$$\max_{x,y} E[w^T] - c^h \tag{25}$$

in order to determine her human capital investments.[14]

Proposition 3 *In the model with unemployment, workers' optimal human capital investments, x^* and y^*, have the following properties:*

1. *extensive and intensive human capital are complements;*

[13] Note that the outside option is normalized at zero only for convenience.

[14] As before, the optimization problem of workers requires further parameter restrictions which we state in the appendix.

2. *both types of human capital increase in the expected product price, p;*

3. *both types of human capital decrease in price correlation, ρ;*

4. *specific human capital decreases in skill differentiation, S.*

These results are very similar to those of the basic model without unemployment. The main difference is that the effect of uncertainty (here captured by the extent of the price fluctuation ε) is now ambiguous. While uncertainty in the base model simply increased (beneficial) fluctuations on the labor market, it now also raises the level of unemployment in case of symmetrically negative shocks. Hence, uncertainty now also carries a cost as unemployment completely destroys the value of any human capital investment. This is an important feature of the extension as it stresses a potential negative effect of uncertainty not capture in the basic model.

Additionally, note that the results on human capital complementarity and the (related) effect of the product price in proposition 3 are strict. Unemployment thus provides an additional mechanism to link the different types of human capital. In sum, the introduction of unemployment into the base model rather strengthens our key results. However, it comes at the cost of narrow parameter restrictions and less analytical tractability.

5 Discussion

5.1 Results

The main results for a pooled labor market (in contrast to non-pooling) are:

1. In the short-run, pooling provides an advantage to both firms and workers as productivity, wages, profits and total surplus in the agglomeration are rising in the strength of the portfolio effect (with given skill space). Pooling raises productivity as it improves the allocation of labor such that specific human capital is employed more effectively (static efficiency gain); profits rise because at given productivity levels, the additional output at the more productive firm overcompensates for the output loss at the less productive firm; wages rise, as they are tied to profits.

2. With given properties of the shock distribution, as specific skills become more similar and the average degree of mismatch is reduced, net wages will rise in the face of lower adjustment costs; the impact of mismatch on firms' profits is ambiguous: on the one hand, competition for workers is intensified, forcing firms to pay higher gross wages; on the other hand, the pooling advantage which enters profits in much the same way as wages, is strengthened. Total surplus, however, is clearly rising. Hence, even if firms profits are falling, this negative impact is more than compensated for by the increase in wages.

3. In the long-run, as wages increase due to the pooling advantage this raises the return to both types of human capital, so incentives to invest in specific skills and in general skills improve. Therefore, we should find a more educated/skilled workforce in agglomeration that benefit from a pooling advantage. In particular, specific skills and general education should be increasing in density (S) and industry diversity (ρ).

4. Higher human capital levels raise productivity, wages, profits, and total surplus. In the case with unemployment, employment is stabilized, and average productivity increases even more because specific skills are employed in production more often. Wages rise for two reasons: firstly, productivity with all firms rises; secondly, adjustment costs are decreasing in general skills.

5. Specific and general human capital are complements: if a worker becomes more productive it pays to invest in general skills which help to adapt her specific skills to a new technology. Conversely, if flexibility is high, it pays to become more productive for flexibility to be rewarded.

These results imply that in a pooled local labor market, we would expect to see higher wages and higher productivity across those industries and occupations which are related.

Glaeser and Mare (2001) explain the urban wage premium by a level as well as a growth effect: while the former may result from information externalities, the latter is consistent with the idea of faster human capital accumulation. The evidence would also be consistent with our model, since the benefits of improved

matching and a portfolio effect combine to raise the return to human capital investment, hence the build-up of human capital in cities is faster. In Glaeser and Mare (2001) it is spillovers which reduce the cost of an additional unit of human capital, in our model it is the higher return of an additional unit.

However, the following two papers provide empirical results that are in contrast to the predictions of our model:

Bleakley and Lin (2006) test the hypothesis that thick markets improve the search and matching process (increasing returns to matching) and will therefore provide additional incentives for workers to invest in occupation and/or industry-specific skills. Workers will find own-sector employers more easily in denser markets, they argue, and with sector-specific human capital depreciating outside its sector, workers will be more productive in thicker markets and, in addition, will have a higher incentive to invest in such human capital. For that reason, they expect a lower rate of industrial (and of occupational) transitions in thicker markets a view that is supported by their empirical analysis. Even when displaced workers only are looked at, these findings pertain. However, potential experience seems to matter as density is found to have a positive impact on the occupational mobility of those with less than 10 years of experience. This is interpreted as evidence of low search cost early in one's career, which temporarily increases search intensity. Sectoral transitions increase with education, and decrease when more aggregated sector and occupation definitions are chosen. Furthermore, the authors acknowledge that overall, the evidence in the literature on the relationship between density and turnover is inconclusive.

Wheaton and Lewis (2002) test for the role of labor markets in generating increasing returns in agglomerations. Using two different measures of localization, the specialization and concentration of employment in terms of occupation and industry, they test for the idea that the externalities resulting from human capital, like faster human capital formation and improved matching, should be linked to "own" industry and occupational employment. Occupational specialization for a sample of manufacturing employment in US SMAs is found to yield a wage premium of 23 percent. This finding supports both a portfolio and a thick market interpretation of labor market externalities. However, Wheaton and Lewis (2002) find additional evidence for industry localization economies: industry specialization is more pronounced than occupational specialization, and yields a 30 percent

increase in wages. This is, unfortunately, in contrast with the role we attach to industry diversification. However, a different measure of diversity which does not assume diversity to be the opposite to specialization may yield other results.

5.2 Empirical Implications

Our model is consistent with the "stylized facts" observed for agglomerations which include higher wages, higher profits, and a special role for human capital as well as for diversity, and it is further suggestive of faster urban growth. However, this evidence is also consistent with other theories of human capital externalities in cities: assortative matching also raises the incentive to invest in human capital since individuals expect to be better matched in the thick labor markets of cities. And so does the idea of reduced coordination costs in larger markets (Becker and Murphy (1992). In our context, it is particularly difficult to disentangle the effects of pooling from those of knowledge spillovers since the pooling of highly skilled workers facilitates spillovers and spillovers are more likely with highly skilled.

Given our requirement that firms produce for different output markets and are able to share the labor market, a simple specialization index, as it is often used when testing for the existence of urbanization versus localization economies, will not suffice for the measurement of industrial diversity. Any test of human capital externalities should consider interlinkages between industries both in terms of correlation of demand, and in terms of labor input, possibly measured by similarities in occupational and qualification structure. On the output side, it means that firms' demand need to be less than perfectly correlated. A separation by industries might be blurred if, for example, two firms belong to different industries but will be hit by the same demand shocks if they are vertically integrated. Focusing our attention on the pooling effect, it is further difficult to distinguish wage movement in response to changes in skill space (S) from those in response to variations in the shock distribution.

As regards the labor market outcomes in agglomerations, we would expect a higher level of human capital, and a positive correlation between general and specific skills. The incidence of pooling would be confirmed by high rates of labor mobility/job turnover within the region. Referring to our extended case with the possibility of unemployment when negatively symmetric shocks occur, employment should be more stable over time, and unemployment should generally

be lower. Unemployment would also be the result if the change from one firm to the other was not instantaneous, so unemployment would be of the frictional type.

To test the predictions concerning human capital, a thorough distinction between specific and general human capital would be helpful. Here, we would expect a strong positive correlation between the two. Furthermore, there should be a positive relationship between labor mobility, wages, productivity and human capital on the one hand, and diversity, and skills on the other hand. Labor mobility should take place within skill groups irrespective of sectors.

The piece of evidence that is perhaps most in line with our model is the finding by Duranton and Puga (2005) of a shift in the pattern of urban specialization from sectoral towards functional specialization. The authors relate this development to the pervasive role of IT services and intra-firm reorganization strategies which have facilitated such reorganization of specialization patterns. They also present a model of input sharing which causes those functions of a firms that share the same inputs, to concentrate in a particular location/agglomeration. The pooling of specialized labor might then be considered as a variant of input sharing. Headquarter functions, for example, require a small range of similar specialized types of labor, and so the headquarters of firms belonging to a variety of industries will co-locate. An agglomeration with functional specialization resembles the industry and labor market structure that yields the highest pooling advantage in our model.

Our model may be best applied to a context of pronounced demand uncertainty where firms rely on highly skilled and specialized labor. An example would be modern high-tech industries, where demand can quickly rush in and out, and where product life cycles, as one determinant of demand, have become ever shorter.

5.3 Policy Implications

According to our model, the optimal structure for an agglomeration is one where firms produce for different output markets but share the labor market, i.e. employ similar skills. While in Simon (1988) labor input is regarded as homogeneous, we capture the heterogeneity of skills and therefore the idea that there maybe costs of mismatch. More heterogeneous labor weakens wage competition between firms,

but will impose excessive adjustment costs on workers and weaken the advantage from the portfolio effect.

Our model imposes a certain structure on industries and skills for the pooling benefit to become effective. In contrast to a number of earlier studies looking at the relationship between diversity and regional stability[15], we do not recommend that policy makers should aim at attracting certain industries and actively create a diverse structure. We rather emphasize market mechanisms as well as the development and strengthening of agglomeration-specific skills. It is a priori rather difficult to determine those industries with the lowest correlation in demand. This is partly a problem of the correct measurement, and partly the problem of whether observed patterns of industry employment growth really do reflect variations in demand or its constraints. Secondly, the performance of certain industries with regions will depend on other factors too, so that the pooling advantage considers only one particular item on the list of possible costs and benefits of a certain industry in a certain region.

We have shown that pooling provides a tool that protects regions against asymmetric shocks to labor demand. It thus improves the efficiency of the allocation of regional resources, raising average productivity, profits and wages, and providing an impetus to long-run growth. As the paper by Magnani (2001) reveals, workers are able to anticipate the risk of demand shocks to their own industry and to respond by moving, job-to-job, to another industry. Such inter-sectoral mobility is then shown to be rising in education. Equally, the paper by Haskel, Kersley, and Martin (1997) shows that firms, if given the ability to deploy their workforce as they wish, will choose to respond to changes in demand by adjusting employment, rather than hours or prices or labor hoarding.

We therefore suggest policy measures which rely upon and strengthen such individual responses by removing obstacles to inter-sectoral labor mobility, disseminating the relevant information, and improving education.

Finally, pooling might be seen as a means to permanently increase a region's growth rate. The work of Lucas (1988) has demonstrated the role of human capital as one of the engines of growth. Pooling increases the return to human capital, and subsequently there will be higher investments. Depending on the mechanism behind (endogenous) growth, growth (rate) effects will follow.

[15]for a survey see Dissart (2003)

Growth may be the result of increasing specialization, as is assumed in Kim and Mohtadi (1992). With constant population growth, there will be more workers and more firms in the local market. Hence, under the assumption of equal spacing, the distance between firms in terms of skill requirements decreases, and average match quality increases. Subsequently, workers invest more into "intensive" human capital (which corresponds to specific human capital in our set-up).

Alternatively, growth could be achieved by (permanent) movements in the other pooling parameters, variance and correlation (σ^2 as summary measure for extent and frequency of shocks). With an increase in σ^2, the incidence or the extent of shocks rises, so the efficiency gain from the reallocation of workers is rising too, and so is the incentive to acquire human capital.

Furthermore, the reallocation of labor between firms in response to shocks may serve as a channel for knowledge spillovers. Such spillovers can be modeled as a function of the human capital level of the previous generation or period, and of the intensity of pooling. The current period match productivity would then be raised by the size and the amount of spillovers.

6 Conclusion

Agglomerations with a pooled labor market may enjoy two advantages: protection against asymmetric shocks and lower mismatch. These advantages raise productivity, and are biggest if there is a range of diverse industries, producing under uncorrelated demand, and if the same firms can share a specific set of skills.

Under a more efficient allocation of labor (and reduced mismatch) firms´ profits will rise. With higher wages workers will find it worthwhile to accumulate more human capital. Our model predicts that specific (directly productive) human capital and adaptive skills are complements. In the long-run, therefore, the workforce in a pooled labor market will not only be more productive but also more flexible in adapting to fluctuations and technological change. Moreover, human capital will augment the pooling advantage: The more skilled a worker, the further the distance in skill space across which she is able to adjust.

We have thus shown the functioning of a mechanism whereby a region (or agglomeration) can adapt to asymmetric shocks in the absence of inter-regional labor mobility if it invests in workers´ human capital. The greater the uncertainty,

and the more important specialized skills, the higher the advantage to a region of creating a polled labor market in the sense of developing region-specific skills - assuming that firms will choose locations as to realize the possible gains from pooling.

Piecewise evidence on firm and worker behavior under uncertainty supports our claim.

A Appendix

A.1 Proof of proposition 1

1. Inserting equilibrium wages and employment from (7) and (8) into (9) and (10) yields:

   - for the expected wage:

$$E[w^T] = E\left[\frac{\hat{s}^B}{S}(w_1^B - \frac{\hat{s}^B}{2x}) + \frac{S-\hat{s}^B}{S}(w_2^B - \frac{S-\hat{s}^B}{2x})\right] \quad (26)$$

$$= E[\tilde{p}_1]y - \frac{5S}{4x} + \frac{xy^2}{18S}\left(E[\tilde{p}_1^2] - E[\tilde{p}_1\tilde{p}_2]\right) \quad (27)$$

$$= py - \frac{5S}{4x} + \frac{xy^2}{18S}\sigma_p^2(1-\rho) \quad (28)$$

     where use is made of $E[\tilde{p}_1] = E[\tilde{p}_2] = p$, $\sigma_p^2 = Var[\tilde{p}_i] = E[\tilde{p}_i^2] - (E[\tilde{p}_i])^2$, $Cov[\tilde{p}_1, \tilde{p}_2] = E[\tilde{p}_1\tilde{p}_2] - (E[\tilde{p}_i])^2$ and $\rho = \frac{Cov[\tilde{p}_1,\tilde{p}_2]}{Var[\tilde{p}_i]}$.

   - similarly, for expected profits:

$$E[\pi_1] = E\left[\frac{\hat{s}^B}{S}(\tilde{p}_1 y - w_1^B)\right] \quad (29)$$

$$= \frac{S}{2x} + \frac{xy^2}{9S}\left(E[\tilde{p}_1^2] - E[\tilde{p}_1\tilde{p}_2]\right) \quad (30)$$

$$= \frac{S}{2x} + \frac{xy^2}{9S}\sigma_p^2(1-\rho) \quad (31)$$

   - for expected total surplus, by adding up:

$$E[TS] = E[w^T] + 2E[\pi_i] \quad (32)$$

$$= py - \frac{S}{4x} + \frac{5xy^2}{18S}\sigma_p^2(1-\rho) \quad (33)$$

2. Inspection of the above results yields $\frac{dE[w^T]}{d\rho} < 0$, $\frac{dE[\pi_i]}{d\rho} < 0$ and $\frac{dE[TS]}{d\rho} < 0$, as well as $\frac{dE[w^T]}{d\sigma_p^2} \geq 0$, $\frac{dE[\pi_i]}{d\sigma_p^2} \geq 0$ and $\frac{dE[TS]}{d\sigma_p^2} \geq 0$.

3. Inspection of the above results yields $\frac{dE[w^T]}{dS} < 0$ and $\frac{dE[TS]}{dS} < 0$.

A.2 Proof of proposition 2

The worker maximizes:

$$\phi(x,y) \equiv E[w^T] - c^h = py - \frac{5S}{4x} + \frac{xy^2}{18S}\sigma_p^2(1-\rho) - \frac{\alpha}{2}(x^2+y^2) \quad (34)$$

Let ϕ_i denote the partial derivative of ϕ with respect to i, and ϕ_{ij} its cross-partial derivative with respect to i and j. Then, the first order conditions for the optimum human capital investments are

$$\phi_y \equiv p + \frac{xy}{9S}\sigma_p^2(1-\rho) - \alpha y = 0 \tag{35}$$

and

$$\phi_x \equiv \frac{5S}{4x^2} + \frac{y^2}{18S}\sigma_p^2(1-\rho) - \alpha x = 0 \tag{36}$$

Additionally, the following second order conditions have to be satisfied:

$$\phi_{yy} \equiv \frac{x}{9S}\sigma_p^2(1-\rho) - \alpha < 0\ , \tag{37}$$

$$\phi_{xx} \equiv -\frac{5S}{2x^3} - \alpha < 0 \tag{38}$$

and

$$\phi_{yy}\phi_{xx} - (\phi_{yx})^2 > 0 \tag{39}$$

where $\phi_{yx} \equiv \frac{y}{9S}\sigma_p^2(1-\rho) \geq 0$.

1. Complementarity of x and y follows from $\phi_{yx} > 0$ for $\rho < 1$.

2. Total differentiation of (35) and (36) with respect to x, y and p yields

$$\frac{dy}{dp} = \frac{-\phi_{yp}\phi_{xx} + \phi_{xp}\phi_{yx}}{\phi_{yy}\phi_{xx} - (\phi_{yx})^2} > 0 \tag{40}$$

and

$$\frac{dx}{dp} = \frac{-\phi_{xp}\phi_{yy} + \phi_{yp}\phi_{yx}}{\phi_{yy}\phi_{xx} - (\phi_{yx})^2} \geq 0 \tag{41}$$

where $\phi_{yp} = 1$ and $\phi_{xp} = 0$. The signs follow immediately.

3. Total differentiation of (35) and (36) with respect to x, y and ρ yields

$$\frac{dy}{d\rho} = \frac{-\phi_{y\rho}\phi_{xx} + \phi_{x\rho}\phi_{yx}}{\phi_{yy}\phi_{xx} - (\phi_{yx})^2} < 0 \tag{42}$$

and

$$\frac{dx}{d\rho} = \frac{-\phi_{x\rho}\phi_{yy} + \phi_{y\rho}\phi_{yx}}{\phi_{yy}\phi_{xx} - (\phi_{yx})^2} < 0 \tag{43}$$

where $\phi_{y\rho} = -\frac{xy}{9S}\sigma_p^2 < 0$ and $\phi_{x\rho} = -\frac{y^2}{18S}\sigma_p^2 < 0$. The signs follow immediately.

4. Total differentiation of (35) and (36) with respect to x, y and σ_p yields

$$\frac{dy}{d\sigma_p} = \frac{-\phi_{y\sigma_p}\phi_{xx} + \phi_{x\sigma_p}\phi_{yx}}{\phi_{yy}\phi_{xx} - (\phi_{yx})^2} \geq 0 \tag{44}$$

and

$$\frac{dx}{d\sigma_p} = \frac{-\phi_{x\sigma_p}\phi_{yy} + \phi_{y\sigma_p}\phi_{yx}}{\phi_{yy}\phi_{xx} - (\phi_{yx})^2} \geq 0 \tag{45}$$

where $\phi_{y\sigma_p} = \frac{2xy}{9S}\sigma_p(1-\rho) \geq 0$ and $\phi_{x\sigma_p} = \frac{y}{9S}\sigma_p(1-\rho) \geq 0$. The signs follow immediately and inequalities are strict for $\rho < 1$.

5. Total differentiation of (35) and (36) with respect to x, y and S yields

$$\frac{dy}{dS} = \frac{-\phi_{yS}\phi_{xx} + \phi_{xS}\phi_{yx}}{\phi_{yy}\phi_{xx} - (\phi_{yx})^2} \leq 0 \tag{46}$$

where $\phi_{yS} = -\frac{xy}{9S^2}\sigma_p^2(1-\rho) \leq 0$ and $\phi_{xS} = \frac{5}{4x^2} - \frac{y^2}{18S^2}\sigma_p^2(1-\rho)$. Now the sign of $\frac{dy}{dS}$ is not obvious. However, rearranging the numerator yields

$$\begin{aligned} -\phi_{yS}\phi_{xx} + \phi_{xS}\phi_{yx} = -\sigma_p^2(1-\rho)\Bigg(\alpha\frac{xy}{9S^2} + \frac{5y}{36Sx^2} \\ + \frac{y^3}{162S^3}\sigma_p^2(1-\rho)\Bigg) \quad \leq 0 \end{aligned} \tag{47}$$

such that $\frac{dy}{dS} \leq 0$ is confirmed.

A.3 Proof of proposition 3

The proof is structurally similar to the proof of proposition 2. Now, the worker maximizes:

$$\begin{aligned} \psi(x,y) \equiv E[w^T] - c^h &= \frac{1+\rho}{4}\left((p+\varepsilon)y - \frac{5S}{4x} + \frac{1}{4S}(p-\varepsilon)^2y^2x\right) \\ &+ \frac{1-\rho}{2}\left(py - \frac{5S}{4x} + \frac{1}{9S}\varepsilon^2y^2x\right) - \frac{\alpha}{2}(x^2+y^2) \end{aligned} \tag{48}$$

Let ψ_i denote the partial derivative of ψ with respect to i, and ψ_{ij} its cross-partial derivative with respect to i and j. Then, the first order conditions for the optimum human capital investments are

$$\psi_y \equiv \frac{1+\rho}{4}\left(p+\varepsilon + \frac{1}{2S}(p-\varepsilon)^2yx\right) + \frac{1-\rho}{2}\left(p + \frac{2}{9S}\varepsilon^2yx\right) - \alpha y = 0 \tag{49}$$

and

$$\psi_x \equiv \frac{1+\rho}{4}\left(\frac{5S}{4x^2}+\frac{1}{4S}(p-\varepsilon)^2y^2\right)+\frac{1-\rho}{2}\left(\frac{5S}{4x^2}+\frac{1}{9S}\varepsilon^2y^2\right)-\alpha x=0 \quad (50)$$

Additionally, the following second order conditions have to be satisfied:

$$\psi_{yy} \equiv \frac{1+\rho}{8S}(p-\varepsilon)^2x+\frac{1-\rho}{9S}\varepsilon^2x-\alpha<0 \quad (51)$$

$$\psi_{xx} \equiv -\frac{3-\rho}{4}\frac{5S}{2x^3}-\alpha<0 \quad (52)$$

$$\psi_{yy}\psi_{xx}-(\psi_{yx})^2>0 \quad (53)$$

where $\psi_{yx} \equiv \frac{1+\rho}{8S}(p-\varepsilon)^2y+\frac{1-\rho}{9S}\varepsilon^2y>0$.

Total differentiation then yields the following system of equations:

$$\begin{pmatrix} \psi_{yy} & \psi_{yx} \\ \psi_{yx} & \psi_{xx} \end{pmatrix}\begin{pmatrix} dy \\ dx \end{pmatrix}+\begin{pmatrix} \psi_{yp} & \psi_{y\rho} & \psi_{yS} \\ \psi_{xp} & \psi_{x\rho} & \psi_{xS} \end{pmatrix}\begin{pmatrix} dp \\ d\rho \\ dS \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (54)$$

where

$$\psi_{yp} \equiv \frac{1+\rho}{4}\left(1+\frac{1}{S}(p-\varepsilon)yx\right)+\frac{1-\rho}{2}>0 \quad (55)$$

$$\psi_{xp} \equiv \frac{1+\rho}{8S}(p-\varepsilon)y^2>0 \quad (56)$$

$$\psi_{y\rho} \equiv \frac{1}{4}\left(p+\varepsilon+\frac{1}{2S}(p-\varepsilon)^2yx\right)-\frac{1}{2}\left(p+\frac{2}{9S}\varepsilon^2yx\right)<0 \quad (57)$$

$$\psi_{x\rho} \equiv \frac{1}{4}\left(\frac{5S}{4x^2}+\frac{1}{4S}(p-\varepsilon)^2y^2\right)-\frac{1}{2}\left(\frac{5S}{4x^2}+\frac{1}{9S}\varepsilon^2y^2\right)<0 \quad (58)$$

$$\psi_{yS} \equiv -\frac{1+\rho}{8S^2}(p-\varepsilon)^2yx-\frac{1-\rho}{9S^2}\varepsilon^2yx<0 \quad (59)$$

$$\psi_{xS} \equiv \frac{1+\rho}{4}\left(\frac{5}{4x^2}-\frac{1}{4S^2}(p-\varepsilon)^2y^2\right)+\frac{1-\rho}{2}\left(\frac{5}{4x^2}-\frac{1}{9S^2}\varepsilon^2y^2\right) \quad (60)$$

With the exception of expressions (57) and (58), the above signs can be inferred directly. To confirm the other two signs, rearrange (57) to

$$\psi_{y\rho} \equiv \frac{p-\varepsilon}{8S}\left[(p-\varepsilon)yx-S\right]-\frac{1}{8}(p-\varepsilon)-\frac{1}{9S}\varepsilon^2yx<0 \quad (61)$$

By restriction (16), the term in brackets is negative, confirming the sign. Similarly, rearranging (58) yields

$$\psi_{x\rho} \equiv \frac{(p-\varepsilon)yx+S}{16Sx^2}\left[(p-\varepsilon)yx-S\right]-\frac{S}{4x^2}-\frac{1}{18S}\varepsilon^2y^2<0 \quad (62)$$

With these results, we can proof:

1. Complementarity of x and y follows from $\psi_{yx} > 0$.

2.
$$\frac{dy}{dp} = \frac{-\psi_{yp}\psi_{xx} + \psi_{xp}\psi_{yx}}{\psi_{yy}\psi_{xx} - (\psi_{yx})^2} > 0 \tag{63}$$

and

$$\frac{dx}{dp} = \frac{-\psi_{xp}\psi_{yy} + \psi_{yp}\psi_{yx}}{\psi_{yy}\psi_{xx} - (\psi_{yx})^2} > 0 \tag{64}$$

3.
$$\frac{dy}{d\rho} = \frac{-\psi_{y\rho}\psi_{xx} + \psi_{x\rho}\psi_{yx}}{\psi_{yy}\psi_{xx} - (\psi_{yx})^2} < 0 \tag{65}$$

and

$$\frac{dx}{d\rho} = \frac{-\psi_{x\rho}\psi_{yy} + \psi_{y\rho}\psi_{yx}}{\psi_{yy}\psi_{xx} - (\psi_{yx})^2} < 0 \tag{66}$$

4.
$$\frac{dy}{dS} = \frac{-\psi_{yS}\psi_{xx} + \psi_{xS}\psi_{yx}}{\psi_{yy}\psi_{xx} - (\psi_{yx})^2} < 0 \tag{67}$$

Here, the sign of $\frac{dy}{dS}$ is not obvious. However, rearranging the numerator yields

$$\begin{aligned} -\psi_{yS}\psi_{xx} + \psi_{xS}\psi_{yx} &= \psi_{yS}\alpha \\ &\quad -\psi_{yx}\left(\frac{1+\rho}{16S^2}(p-\varepsilon)^2y^2 + \frac{1-\rho}{18S^2}\varepsilon^2y^2\right) \\ &\quad -\frac{3-\rho}{4}\frac{5}{4x^2}\left(\frac{1+\rho}{8S}(p-\varepsilon)^2y + \frac{1-\rho}{9S}\varepsilon^2y\right) \end{aligned} \tag{68}$$

such that $\frac{dy}{dS} < 0$ is confirmed.

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