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Abstract

In an overlapping-generations model with endogenous birth rates, I design a reform of the pay-as-you-go pension system, which internalises positive externalities of children – their pension contributions. Individuals may differ in their preferences for children and their ability to have children at all. They can choose between the status-quo flat-rate benefits and a new system, in which they get just the benefits that are (on average) financed by their own children, reduced by an amount which is used to subsidise the flat-rate system. Whereas people with low child preferences keep the status quo, people with high child preferences choose the individualised system, having the optimal incentives to raise children and a higher utility.

Keywords: Overlapping-generations models; Pension reform; Pay-as-you-go system; Endogenous population growth

JEL classification: H55; J11; J13; J26

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1 Introduction

In most industrialised countries, the prevailing pension system is the pay-as-you-go (PAYGO) system, in which contributions are immediately paid out to pensioners. These pension benefits are mostly dependent on salaries or wages. But they do not, or only to a low degree, depend on the individual number of children of the respective pensioners. This is the case because, originally, the pension systems were designed as an insurance against infertility and the resulting old-age poverty (Sinn, 2003). But, like any other insurance, the PAYGO pension system implies an incentive problem. It distorts the decision of potential parents about the number of their children downwards. There are empirical studies by Cigno and Rosati (1996), Ehrlich and Zhong (1998) and Cigno et al. (2003), which suggest that fertility is really negatively affected by the level of pension benefits in countries with such PAYGO systems. Low birth rates, however, are just the reason for the current pension crisis in many countries, because the rate of return in a PAYGO system depends heavily on population growth: the more working people there are per pensioner, the higher the pension benefits are for a given contribution rate (cf. e.g. Gaggermeier and Lucke, 2002). Germany, where a woman has only 1.4 children on average (Statistisches Bundesamt, 2000, 2003, Sommer, 2004), is particularly concerned with this problem.

A transition to fully or partially funded systems, where contributions are invested in the capital market and paid out to the respective contributors, has been a hotly disputed issue in the last few years.[1] But, it has been argued that such a transition can never imply a Pareto improvement (unless the transition removes some distortions that are not caused by the nature of the PAYGO system), cf. Fenge (1995), Sinn (2000), Gaggermeier and Lucke (2002). This is because, when the size of the PAYGO system is reduced e.g. in favour of a funded system, one generation has to bear a double burden in satisfying the pension claims of the contemporaneous pensioners and in saving for their own funded pensions. Financing the pensions of the

[1] Cf. e.g. Börsch-Supan (1998), Kotlikoff (1996a, 1996b), Feldstein and Samwick (1998) and Sinn (2000, 2001).

"transition period" by government debt would only redistribute this disadvantage to one or more following generations, cf. Sinn (2000).

Therefore, Sinn (2001, 2003) and von Auer and Büttner (2004) propose to link the individual pension benefits to the individual number of children in order to re-establish (at least partly) the right incentives to rear children. The hypothesis that birth rates are influenced by economic incentives is supported for instance by a birth decline in the Saarland after its re-integration into Germany 1957 (where there was less support for parents with children than before in France) and by a birth increase in the German Democratic Republic after the enacting of family policies in 1976, cf. Sinn (2001).

Von Auer and Büttner (2004) have even shown that, if individuals are homogenous, introducing a PAYGO system with a fixed proportion between individual pension benefits and children can imply a Pareto improvement[2]. I am going to ex-

[2] Sometimes, it is argued that the Pareto criterion cannot be applied if population growth is endogenous, cf. e.g. Golosov et al. (2004). It is true that the *additional* children induced by a reform are not asked if they want to be born and to finance part of pension system; their utility cannot be compared with the situation without the reform (unless they have a well-defined utility even in case they are not born). But, I only compare the well-being of the *existing* individuals of any generation in two scenarios: with and without a pension reform. I understand the Pareto criterion as being met if somebody has a higher utility and no individual of any certain characteristics is worse off with the reform than any individual of the same characteristics without the reform. As this is true for my as well as for von Auer and Büttner's reform proposal, I join von Auer and Büttner in using the term "Pareto improvement". Readers who do not agree may call the outcome of the reform a "quasi Pareto improvement" or a "sequence of Pareto improvements, given the population of the respective generation of potential parents". I am not aware of any term being introduced in the literature for this concept so far. It is similar to Golosov et al.'s concept of "A efficiency" (A like "alive"), where the utility of only those individuals is compared that are born in *all* scenarios. Both concepts avoid that utility for (an arbitrary number of) unborn individuals has to be specified. On the one hand, however, my reform proposal fulfils stricter requirements as I consider *all* individuals, on the other hand, it may not be an A improvement as I do not know *which* individuals were not born without the reform; it might just be those whose utility is increased by the reform. - A Pareto improvement is given anyway under the assumption that being born always yields higher utility than being not born.

pose this approach in the framework of my model. In reality, of course, individuals are different, particularly with respect to their preferences for having children and consuming goods. Some of them are not even able to have children. Those ones would not receive any pension in von Auer and Büttner's system. Such a system would certainly politically and legally not be feasible.

This paper refines von Auer and Büttner's work. In contrast to von Auer and Büttner (2004) and other contributions that deal with reproduction incentives in PAYGO systems,[3] I allow that individuals of any number differ in their preferences for children or are involuntarily childless. They maximise their lifetime utility - dependent on consumption during working age and old age as well as on the number of children - subject to an intertemporal budget constraint, which is affected by the pension system.

When determining the number of their children, individuals equilibrate marginal costs and marginal utility. On the one hand, children require consumption expenditure and time. On the other hand, they cause felicity to their parents and finance their pensions. The first benefit of children is considered by potential parents in a PAYGO system with lump-sum benefits, the second one, however, is an externality, which is to be internalised. I show that this can be - partly - done without disadvantaging anybody by giving to each pensioner the right to choose between the status-quo flat-rate benefits and a new, individualised system. In the new system, pensioners get the pension benefits that are (on average) financed by their own children, but reduced by a certain amount which is independent of the individual children number and which is used to subsidise the flat-rate system. People who have sufficiently high child preferences choose the new system, getting the socially optimal incentive to rear children. I show that this set of individuals is non-empty. They have a higher utility than in the flat-rate system, even if they view a child as an "inferior good" and have fewer children after the reform.

[3]E.g. van Groezen et al. (2003), Abío et al. (2004), Cremer et al. (2004a), Cremer et al. (2004b) or Fenge and Meier (2005); Cremer et al. (2004b), for instance, distinguish individuals by their costs for raising children, which seems much less plausible to me than different preferences

In order to estimate roughly the effects of such a reform on population growth, welfare and other economic variables, I have calibrated the model with German data and simulated it with a standard constant-relative-risk-aversion utility function. It has turned out that, as the relative marginal costs of children and consumption change, individuals renounce on consumption in favour of children, in fact to such a degree that the birth rate increases from about 1.4 children per woman to about 1.6.

The rest of the paper is organised as follows: Section 2 presents the basic model. Section 3 explains the reform proposal, which is illustrated by a simulation in section 4. Section 5 concludes.

2 The model

In this overlapping-generations model[4], we have individuals that live three periods, "childhood", "youth" (working age) and "old age". While children do not make any economic decisions, young people decide on their consumption level and how many children to have, and old people (pensioners) just live off their savings and the pension they get. The individuals maximise their lifetime utility subject to an intertemporal budget constraint under perfect foresight. They may differ in their preferences for children and consumption as well as in their ability of having children.

In order to analyse the effects of social security on population growth and welfare, we must look at the individuals' optimisation problem. The preferences of individual i are described by a quasi-concave lifetime-utility function of consumption and children,

$$u_{i\tau} = u_{i\tau}\left(c_{i\tau}^1, c_{i\tau}^2, m_{i\tau}\right). \tag{1}$$

$c_{i\tau}^1$and $c_{i\tau}^2$ is consumption of an agent born in period $\tau - 1$, i. e. whose autonomous economic life begins in period τ when he or she reaches working age; superscript 1 denotes youth (i. e. in period τ) and superscript 2 denotes old age (in period $\tau + 1$). $m_{i\tau}$ is the - by assumption indefinitely divisible - number of children of

[4]Which is developed from a model with exogenous population growth, cf. Bräuninger (1997).

individual i of generation τ. The functional form of $u_{i\tau}$ may be different for different individuals, but generally, it must allow an individual children number of zero, i.e. $u_{i\tau}\left(c^1_{i\tau}, c^2_{i\tau}, 0\right) > -\infty$ if $c^1_{i\tau}, c^2_{i\tau} > 0$, whereas $u_{i\tau}\left(0, c^2_{i\tau}, m_{i\tau}\right) = u_{i\tau}\left(c^1_{i\tau}, 0, m_{i\tau}\right) = -\infty$.

The young generation's budget constraint equals

$$c^1_{i\tau} + c^0_\tau m_{i\tau} + t_\tau + s^1_{i\tau} = w_{i\tau}, \tag{2}$$

where c^0_τ are costs of a child, t_τ are contributions to a pay-as-you-go pension system, $s^1_{i\tau}$ is individual i's private saving, and $w_{i\tau}$ is his or her labour income.[5] Pension contributions are exogenously fixed by the government. For simplicity, they are assumed to be lump-sum contributions. Children' s consumption c^0_τ is also exogenous and independent of the parents' consumption.[6]

When people are old, they consume their savings (including interest) and their pension benefits $g_{i,\tau+1}$:

$$c^2_{i\tau} = \left(1 + r_{\tau+1}\right) s^1_{i\tau} + g_{i,\tau+1}. \tag{3}$$

As pension contributions are exogenous, the benefits are endogenously determined (and potentially individually different). They are computed as follows.

According to the budget constraint of the government, the sum of pension benefits must equal total contributions in each period:

$$\sum_{i=1}^{N_\tau} g_{i,\tau+1} = t_{\tau+1} N_{\tau+1}, \tag{4}$$

where N_τ is the number of working-age people in period τ (that is the number of pensioners in period $\tau + 1$) and $N_{\tau+1} = \sum_{i=1}^{N_\tau} m_{i\tau}$ is the number of working

[5] $w_{i\tau}$ generally depends on $m_{i\tau}$ (opportunity costs of forgone labour income), which is considered in the simulations. For simplicity, in the theoretical part, I assume that $w_{i\tau}$ is independent of $m_{i\tau}$. This is theoretically the same as assuming a linear relationship and summarising both consumption and time costs under c^0_τ, cf. section 4.

[6] c^0_τ is assumed to grow with the exogenous growth rate of labour productivity, as all variables like consumption, social security contributions and pension benefits do in a steady state. In order to keep formulas and explanations as simple and descriptive as possible, I refrain from normalising variables in terms of efficiency units of labour at this point. However, productivity growth must be taken into account in simulations (see below). Net of productivity growth, c^0_τ is constant.

individuals in period $\tau+1$. Dividing both sides by N_τ gives

$$g_{\tau+1} = t_{\tau+1} m_\tau. \tag{5}$$

$g_{\tau+1}$ is the average PAYGO pension per pensioner in period $\tau+1$, and $m_\tau = \frac{N_{\tau+1}}{N_\tau}$ is the average number of children per worker in period τ (such that $m_\tau - 1$ is the population-growth rate from period τ to period $\tau+1$).

Inserting equation (2) for $s^1_{i\tau}$ into equation (3) yields generation τ's lifetime-budget constraint, according to which the present value of total expenditure equals the present value of total revenues:

$$c^0_\tau m_{i\tau} + c^1_{i\tau} + \frac{c^2_{i\tau}}{1+r_{\tau+1}} + t^I_{i\tau} = w_{i\tau} \tag{6}$$

with

$$t^I_{i\tau} = t_\tau - \frac{g_{i,\tau+1}}{1+r_{\tau+1}} \tag{7}$$

$t^I_{i\tau}$ is the present-value surplus of PAYGO-pension contributions over PAYGO-pension benefits, which is an implicit tax caused by the unfunded pension system. This is the channel through which the pension system affects the individuals' choice variables and in particular the number of their children.

The individuals' problem is to maximise the utility function (1) subject to the budget equation (6) and the non-negativity constraints

$$c^1_{i\tau} \geq 0, \tag{8}$$

$$c^2_{i\tau} \geq 0 \tag{9}$$

and

$$m_{i\tau} \geq 0. \tag{10}$$

While the form of the utility function ensures that the constraints (8) and (9) are not binding, $m_{i\tau} = 0$ is allowed, such that we must consider constraint (10) explicitly.

Suppose that each individual gets the same pension $g_{\tau+1}$. Then, inserting (5) in (7), the implicit tax becomes

$$t^I_{i\tau} = t_\tau - \frac{t_{\tau+1} m_\tau}{1+r_{\tau+1}}, \tag{11}$$

where the average m_τ is taken as given.[7]

Maximising (1) subject to (6), (10) and (11) yields the first-order conditions

$$\frac{\partial u_{i\tau}}{\partial c_{i\tau}^1} = (1 + r_{\tau+1}) \frac{\partial u_{i\tau}}{\partial c_{i\tau}^2}, \tag{12}$$

$$\frac{\partial u_{i\tau}}{\partial m_{i\tau}} \leq c_\tau^0 \frac{\partial u_{i\tau}}{\partial c_{i\tau}^1} \tag{13}$$

and

$$m_{i\tau} \left[\frac{\partial u_{i\tau}}{\partial m_{i\tau}} - c_\tau^0 \frac{\partial u_{i\tau}}{\partial c_{i\tau}^1} \right] = 0. \tag{14}$$

The standard Euler equation (12) holds if saving an additional unit of income for the retirement period yields the same utility as consuming it in the working period. Equation (13) states that the marginal utility of a child must be lower than or equal to marginal costs, c_τ^0, evaluated with the marginal utility of income. Because of (14), it can be lower only if $m_{i\tau} = 0$. Equations (12) to (14) together with (6) determine individual i's choice variables $c_{i\tau}^1$, $c_{i\tau}^2$ and $m_{i\tau}$, where $m_{i\tau}$ must fulfil both (13) and (14). Of course, for individuals who are not able to have children, (13) and (14) do not apply; those individuals only choose $c_{i\tau}^1$ and $c_{i\tau}^2$.

3 Reform proposal: individualisation of pension benefits

It can be easily shown that a reduction in the contributions to the pay-as-you-go system decreases the implicit tax in the long run.[8] This means an increase in disposable income, inducing a positive income effect on consumption and birth rates and, thereby, causing higher population growth and higher utility. But, this kind of reform can never bring about a Pareto improvement. When the contributions are decreased, the contemporaneous pensioners are disadvantaged since they get lower

[7] When t grows with the rate of labour productivity, p, such that $t_{\tau+1} = t_\tau (1 + p_{\tau+1})$ (e.g. in a steady state), the implicit tax is positive if (and only if) the growth rate of effective labour, $(1 + p_{\tau+1}) m_\tau - 1$, is lower than the market interest rate. In the long run, this must be the case, see Sinn (2001).

[8] See equation (11) and footnote 7.

pension benefits without having paid less when they were young, see Gaggermeier and Lucke (2002). Financing the pensions of the "transition period" by government debt would only redistribute this disadvantage to one or more following generations, cf. Sinn (2000).

Von Auer and Büttner (2004) show that, if individuals are homogenous, individualising pension benefits achieves a Pareto improvement. Individualisation means that each individual gets exactly the benefits that are financed by their own children:[9]

$$g_{i,\tau+1} = t_{\tau+1} m_{i\tau}. \tag{15}$$

Summing up over N_τ yields equation (4); the budget constraint of the PAYGO pension system is clearly fulfilled. Moreover, the implicit tax becomes

$$t^I_{i\tau} = t_\tau - \frac{t_{\tau+1} m_{i\tau}}{1 + r_{\tau+1}}. \tag{16}$$

It depends on the individual number of children and is not taken as given any more. Now, the first-order conditions (13) and (14) become

$$\frac{\partial u_{i\tau}}{\partial m_{i\tau}} + \frac{t_{\tau+1}}{1 + r_{\tau+1}} \frac{\partial u_{i\tau}}{\partial c^1_{i\tau}} \leq c^0_\tau \frac{\partial u_{i\tau}}{\partial c^1_{i\tau}} \tag{17}$$

and

$$m_{i\tau} \left[\frac{\partial u_{i\tau}}{\partial m_{i\tau}} + \frac{t_{\tau+1}}{1 + r_{\tau+1}} \frac{\partial u_{i\tau}}{\partial c^1_{i\tau}} - c^0_\tau \frac{\partial u_{i\tau}}{\partial c^1_{i\tau}} \right] = 0, \tag{18}$$

respectively. There is an additional term, which represents an additional marginal utility of children: the present value of the pension contribution by an additional child, that is the additional pension benefit for the individual i of generation τ, evalutated with his or her marginal utility of income. This is just the externality of children which is internalised by the reform. Now, potential parents consider all benefits when deciding on the number of their children and, thus, they have the socially optimal incentives in this respect.

If all individuals are identical, such that they all have the average m_τ without the reform, they have the option of achieving the same pension with the same number of children as before, cf. equations (5) and (15). If they do so, nothing changes,

[9] In practice, $t_{\tau+1}$ would be the *average* contribution per child.

including consumption and utility. But, individuals can increase the number of their children and will do so, at least if the original number is positive. This is because for the value of $m_{i\tau}$ that equilibrates equation (13), marginal benefits in terms of utility exceed marginal costs according to equation (17). Increasing $m_{i\tau}$ increases utility. That is, we have a feasible Pareto improvement.

In reality, however, individuals differ in their preferences for having children and consuming goods. Some people are even unable to have children (they could only adopt them). Implementing the above-described reform strictly would disadvantage those individuals whose preference for children is sufficiently low. Infertile individuals or ones whose preference for children is so low that they do not want to have any even with the reform would not get any pension benefits at all. This would politically as well as legally certainly not be feasible. Nor would it be fair, because those people have contributed to the system (even though paying contributions is not enough since children are also required).

Therefore, I propose a modified reform. This reform provides the incentives of individualisation for part of the individuals, but it ensures that nobody is worse off than without the reform.

According to this proposal, individuals get the right to choose between the status-quo flat-rate pension system and a new, "individualised" system. In this new system, pensioners get the benefits that can be financed by their own children, but reduced by a lump-sum amount which is used for "subsidising" the pensioners whose number of children is not sufficient to finance the status-quo pension level for their parents.

Formally, the pension benefits are determined as

$$g_{i,\tau+1} = g_{\tau+1}^{NI} \tag{19}$$

(where $g_{\tau+1}^{NI}$ denotes level in the non-individualised system, i.e. the status-quo level) for all individuals that have chosen the old system and

$$g_{i,\tau+1} = t_{\tau+1} m_{i\tau} - a_{\tau+1} \tag{20}$$

with

$$a_{\tau+1} = \frac{1}{N_\tau^*} \sum_{j=N_\tau^*+1}^{N_\tau} \left(g_{j,\tau+1} - t_{\tau+1} m_{j\tau} \right) \tag{21}$$

for all individuals i that have chosen the new system. Let all individuals be sorted in descending order of children number, such that the individuals $j \in \{1, 2, ..., N_\tau^*\}$ prefer the individualised system and the individuals $j \in \{N_\tau^* + 1, N_\tau^* + 2, ..., N_\tau\}$ choose the flat-rate system. The amount $a_{\tau+1}$ is the sum of the "finance gaps" of all old-system pensioners, divided by the number of all new-system pensioners. For the pension and the children number of the old-system pensioners, the respective status-quo quantities $g_{\tau+1}^{NI}$ and $m_{j\tau}^{NI}$ can be inserted into equation (21). Using equation (5), the following expression for $a_{\tau+1}$ results:

$$a_{\tau+1} = \frac{N_\tau - N_\tau^*}{N_\tau^*} t_{\tau+1} \left(m_\tau^{NI} - \widetilde{m}_\tau^{NI} \right), \tag{22}$$

where $m_\tau^{NI} = \frac{1}{N_\tau} \sum_{j=1}^{N_\tau} m_{j\tau}^{NI}$ is the status-quo average children number and $\widetilde{m}_\tau^{NI} = \frac{1}{N_\tau - N_\tau^*} \sum_{j=N_\tau^*+1}^{N_\tau} m_{j\tau}^{NI}$ is the average children number of old-system pensioners after the reform. $m_\tau^{NI} - \widetilde{m}_\tau^{NI}$ is the lack of children per pensioner in the flat-rate system. This is multiplied with the pension contribution per child and the number of the concerning pensioners, giving the totally required subsidy that must be paid by the N_τ^* pensioners in the individualised system. When calculating the benefits for a pensioner in the new system, $a_{\tau+1}$ must be subtracted from the amount contributed by the respective pensioner's children.

Inserting equation (20) for $g_{i,\tau+1}$ in (7) gives the expression

$$t_{i\tau}^I = t_\tau - \frac{t_{\tau+1} m_{i\tau} - a_{\tau+1}}{1 + r_{\tau+1}} \tag{23}$$

for the implicit tax. Utility maximisation yields exactly the optimality conditions (17) and (18) then. This is because $a_{\tau+1}$ is independent of the children number $m_{i\tau}$ of individual i. Thus, the individuals that have chosen the individualised system have the socially optimal incentives to rear children. They have a higher utility than in the status quo with flat-rate pension benefits, which they could have also got.

a_τ is constructed in such a way that the budget constraint (4) of the social security is fulfilled. This can be easily shown by inserting equation (21) into (20), multiplying both equation sides with N_τ^* and summing up over all new-system pensioners, see appendix 1.

But, who chooses the individualised system? Individuals prefer this system if and only if it provides them with at least the same pension level as the flat-rate system, i.e. if

$$t_{\tau+1}m_{i\tau} - a_{\tau+1} \geq g_{\tau+1}^{NI} = t_{\tau+1}m_{\tau}^{NI}. \tag{24}$$

Let m_{τ}^{*} be the indifference number of children, for which $t_{\tau+1}m_{\tau}^{*} - a_{\tau+1} = g_{\tau+1}^{NI}$ holds. Then, people with $m_{i\tau} \geq m_{\tau}^{*}$ after the reform choose the new system.

This is illustrated in figure 1, where line AA is the status-quo pension-benefit function of children, line BB is the benefit function with full individualisation (running through the "status-quo point" S, which denotes the status-quo average children number and status-quo pension level) and line CC is the benefit function in the individualised part of the partially individualised system.[10] As with partial individualisation people can choose the part which provides them with the higher pension, the bold line ATC is the relevant benefit function .

Figure 2 clarifies the consequences of the different pension systems for the birth rates. First, assume that the utility function (1) can be equivalently (with respect to the outcome of its maximisation) written as a function of only two variables, $c_{i\tau}$ and $m_{i\tau}$, where $c_{i\tau} \equiv c_{i\tau}^{1} + \frac{c_{i\tau}^{2}}{1+r_{\tau+1}}$ is the present value of total consumption.[11] Now, the individuals' decision problem can be illustrated in a two-goods (children and total consumption) diagram.

Present-value net income (labour income minus pension contributions plus discounted pension income, see c axis) can be used for children or consumption. Line AA is the status-quo budget constraint for every individual, cf. equations (6) and (11).[12] Its slope is $-\frac{1}{c_{\tau}^{0}}$, the negative ratio of (marginal) costs of consumption and children. Line BB, in contrast, is the budget constraint for every individual in the

[10] Note that m_i on the x-axis of figure 1 denotes the children number after the reform, not the status-quo number. Through the reform, an individual with an optimal status-quo m below m^* may have got an optimal m *above* m^* because of the higher incentives.

[11] This can be easily shown to be the case for a standard CRRA utility function, cf. appendix 3.

[12] All letters in figure 2 correspond to the same letters in figure 1.

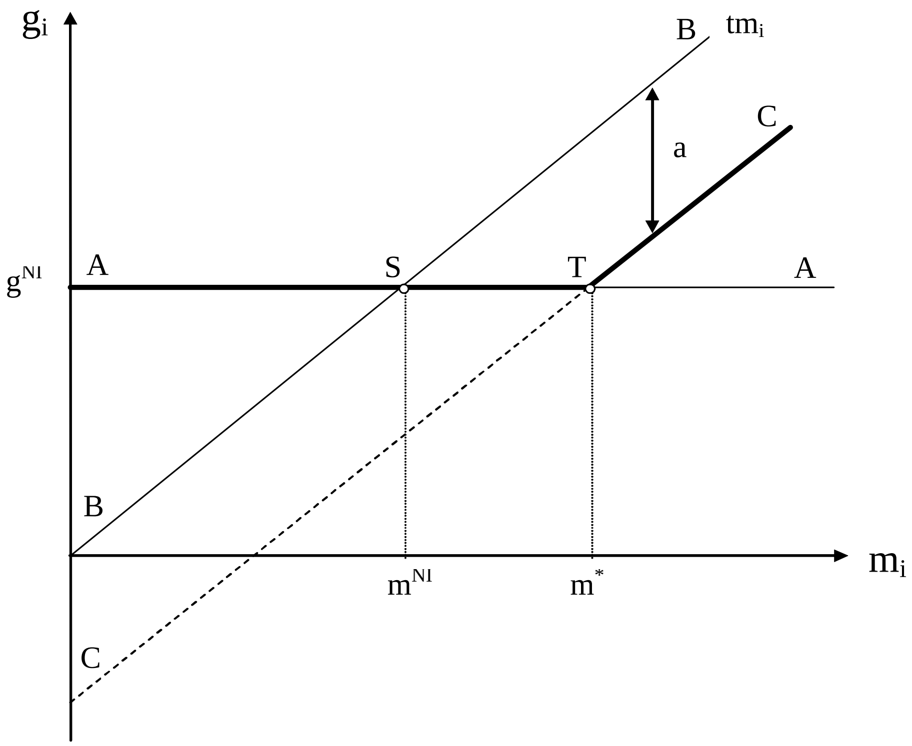

Figure 1: Child-related pension benefits

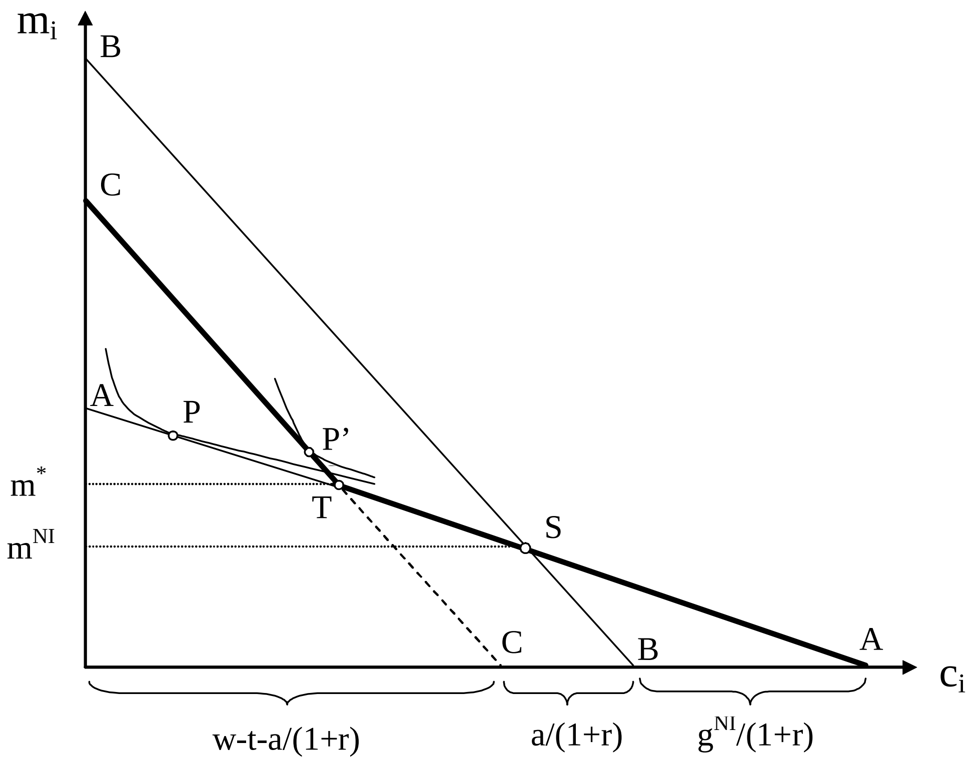

Figure 2: Budget constraints

fully individualised system, cf. equations (6) and (16). It is steeper ($-\frac{1}{c_\tau^0 - \frac{t_{\tau+1}}{1+r_{\tau+1}}}$) because the marginal costs of children are lower (where the additional pension financed by an additional child is interpreted as negative costs). Individualising the pension system, however, does not only imply an upwards turn of the budget line, but also a shift to the left. This is because the flat-rate pension $g_{\tau+1}^{NI}$ is not paid out any more. Line BB must run through the status-quo point S, which combines status-quo average children number and consumption and which can always be achieved in the fully individualised system.

In the individualised part of the *partially* individualised system, marginal costs and thus the slope of the budget line are the same as in the fully individualised system, but the shift goes even further, since the amount $a_{\tau+1}$ must be subtracted additionally. The corresponding budget line CC must cross line AA at the point T or the indifference children number m_τ^*, respectively. But, since people can choose between the individualised and the flat-rate pension, the envelope of the two feasible sets is the relevant constraint. Of course, $a_{\tau+1}$ and thus the position of the CC line itself depend on the individuals' decision regarding the individualised or the flat-rate part of the partially individualised system. View the bold line CTA as the relevant budget constraint for every individual, given the optimal behaviour of all other individuals.

Now, what does this mean for the individuals' decisions? Obviously, for any form of a quasi-concave utility function and convex indifference curves, individuals whose status-quo children number is above m_τ^* - e.g. at point P - always choose the individualised pension. On the other hand, there may be individuals whose *status-quo* tangent solution (or even border solution with $m_{i\tau} = 0$) is right of T and who are better off with the individualised pension nevertheless.

Figure 2 also shows that some individuals in the individualised part may have fewer children than without the reform. Suppose that the indifference curves drawn in figure 2 describe the preferences of an individual i, who takes the budget line CTA as given. For i, children are inferior to such a degree that, as a consequence of the reform, he or she chooses P' instead of P, which implies a lower children number

$m_{i\tau}$; a negative income effect dominates a positive substitution effect on $m_{i\tau}$.

The possibility that the reform provides some individuals with higher pension benefits for the *same* children number, implying a higher consumption level, can be demonstrated as follows. First, assume that only one individual i with the original optimum P chooses the individualised pension. In this case, the line CC runs through point P, since with the original children number, i would get the original pension and can afford the original consumption level; i reaches a higher indifference curve with a *higher* number of children, because i's new budget line is tangent to a higher indifference curve above P, even if i views children as an inferior good. Now, however, assume that another individual j, whose original optimum is right of P, also chooses the individualised pension because his or her originally achieved indifference curve intersects with the CC line through P. This additional individual in the individualised part of the pension system reduces the ratio $\frac{N_\tau - N_\tau^*}{N_\tau^*}$, so that $a_{\tau+1}$ becomes smaller (unless the decrease in $\frac{N_\tau - N_\tau^*}{N_\tau^*}$ is overcompensated by a decrease in $\widetilde{m}_\tau^{NI}$), cf. equation (22), and the CC line shifts to the right, let's say to the points P' and T. Now, i gets higher pension benefits without changing his or her children number because his or her support of other families is lower than originally. - Of course, i's situation cannot apply for *all* individuals in the individualised part of the system. In total, the birth rate must be higher than in the status quo; otherwise, there would be no resources for the larger feasible set of c-m combinations.

The set of individuals who prefer the individualised system is non-empty. Assume that at the most one person, like i in the example above, chooses the new system. Then, doing so is indeed worth for that person, which is formally shown in appendix 2. The intuitive explanation is the following. For all other people, everything remains the same. They have the status-quo number of children and get the status-quo pension benefits. That is, with the reform, i's family must subsidise the other pensioners (or are subsidised by them) with the same amount as without the reform. Consequently, the gain of pension contributions through i's additional children can be used exclusively for i's benefits, such that i is better off in the new system than in the old one. In other words, i can always achieve the status-quo combination of children and consumption, but relative costs have changed in favour of children. As explained above, his or her new budget line CC in figure 2 runs through his or her original (interior) consumption/children point, so that he or she has more children for sure.

Note that there is no transition dynamics with a double burden. In the period in which the new system is introduced, both pension contributions and benefits (for the contemporanous pensioners) are still status-quo; one generation later, individualised-system pensioners receive a pension according to the number of their children; higher benefits can be financed by a higher number of contributors.

4 Implemention in a total model and simulation

The above-described pension reform has been implemented in a basic total model[13] of a small open economy with perfect capital mobility and an exogenous interest rate. There is a representative firm, which maximises profits under perfect competition in good and factors markets, producing a homogenous good with a Cobb-Douglas technology and employing labour and capital. Households maximise their lifetime utility as described in section 3, but in addition to consumption costs for children, they face opportunity costs of foregone labour income:

$$w_{i\tau} = \pi_{i\tau} w_\tau, \tag{25}$$

where the individual labour-force-participation rate $\pi_{i\tau}$ is a function of the individual number of children:

$$\pi_{i\tau} = 1 - \psi m_{i\tau}. \tag{26}$$

The general wage w_τ depends only on the exogenous interest rate and on the parameters of the production function[14] (such that marginal costs of children are $\psi w_\tau + c^0_\tau$ instead of c^0_τ only).

[13] Based on Gaggermeier and Lucke (2002).

[14] I assume that the general wage rate w is independent of π and m. It can be shown that this is true with a Cobb-Douglas technology, because in this case, the positive effects of higher labour input on the optimal capital stock and on output for a given capital stock just balance out with

For the utility function, the following CRRA form is assumed, where the weight of children, ε_i, differs between individuals:

$$u_{i\tau} = \log c_{i\tau}^1 + \delta \log c_{i\tau}^2 + \varepsilon_i \log(1 + m_{i\tau}) \tag{27}$$

with δ and ε_i positive. It allows $m_{i\tau} = 0$; in this case, the last term vanishes. With this functional form, both consumption and children are normal goods, as is shown in appendix 3.

For simplicity, the population is grouped into three types of identical individuals with shares $\lambda_j = \frac{N_{j\tau}}{N_\tau}$, $j = 1, 2, 3$. Let $\varepsilon^{(j)}$ and $m_\tau^{(j)}$ denote the child-preference parameter and children number of the type-j individuals, respectively, and assume that $\varepsilon^{(1)} > \varepsilon^{(2)}$ applies and that type 3 is infertile such that $m_\tau^{(1)} > m_\tau^{(2)} \geqslant m_\tau^{(3)} = 0$.

When calculating the individualised pension, two cases must be distinguished:[15]

a) $m_\tau^{(2)} > m_\tau^*$. Then

$$g_{i,\tau+1} = t_{\tau+1} m_{i\tau} - \frac{\lambda_3}{\lambda_1 + \lambda_2} g_{\tau+1}^{NI} = t_{\tau+1} m_{i\tau} - a_{\tau+1} \tag{28}$$

$$for\ all\ i \in T_1 \cup T_2.$$

b) $m_\tau^{(2)} < m_\tau^*$. Then

$$g_{i,\tau+1} = t_{\tau+1} m_{i\tau} - \frac{\lambda_2}{\lambda_1}\left(g_{\tau+1}^{NI} - t_{\tau+1} m_\tau^{(2)}\right) - \frac{\lambda_3}{\lambda_1} g_{\tau+1}^{NI} = t_{\tau+1} m_{i\tau} - a_{\tau+1} \tag{29}$$

$$for\ all\ i \in T_1.$$

T_j is the set of type-j individuals.[16]

The parameters and exogenous variables are calibrated with German data as follows:

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the negative effect on marginal labour productivity, cf. Gaggermeier and Lucke (2002). And (the log of) a Cobb-Douglas function can be viewed as a (linear) approximation of any functional form, such that the effect of varying labour supply on w is supposed to be negligible.

[15] Note that $m_{1\tau} > m_\tau^*$ must necessarily apply, cf. section 3.

[16] Note that $\left(g_{\tau+1}^{NI} - t_{\tau+1} m_{2\tau}\right)$ can be either positive or negative, depending on whether $m_{2\tau}$ is larger or smaller than the average status-quo number of children.

λ_1	λ_2	λ_3	δ	$\varepsilon^{(1)}$	$\varepsilon^{(2)}$
0.67	0.27	0.06	0.31	1.5	0.5

c_τ^0	t_τ	$r_{\tau+1}$	α	β	ζ	ψ	p
125,000	225,000	2.34	0.27	0.73	0.46	0.23	0.64

The shares of the three types have been calibrated as $\lambda_1 = 0.67$, $\lambda_2 = 0.27$ and $\lambda_3 = 0.06$. This reflects the observation (or estimation, respectively) that in Germany about one third of women do not have any children and that about six percent are supposed to be involuntarily childless.

The discount factor δ is chosen in such a way that a steady-state consumption ratio c_τ^2/c_τ^1 of slightly above 1 results; $\delta = 0.31$ corresponds to a yearly rate of time preference of 4%. Given all other parameters, the child-preference parameters $\varepsilon^{(1)}$ and $\varepsilon^{(2)}$ are set such that the observed status-quo birth rate is reproduced. They are also consistent with the fact that women who have at least one child, mostly have two children or more (and about two on average). $\varepsilon^{(3)}$ is irrelevant since type 3 is assumed to be infertile.

$c_\tau^0 = 125,000$ EUR in prices of 1995 is a rough estimate gained from the comparison of consumption data of differently sized households.[17] The pension contribution $t_\tau = 225,000$ EUR correspond to a contribution rate of 19.8%.

$r_{\tau+1}$ is the 1992 to 1999 average real rate of return (4.1% per year) of long-term government bonds, both related to a 30-year interval.

The production elasticities of capital, α, and labour, β, (with $\alpha + \beta = 1$) and the depreciation rate ζ together with the interest rate determine the overall wage rate w_τ. $\beta = 0.73$ is calibrated with the average labour-income share (Lohnquote) of 1991 to 2000. $\zeta = 0.46$ resulted as residual of the capital-demand equation.

ψ is estimated with OLS using German time-series data from 1964 to 1998, where the estimation equation is (standard errors in brackets):

$$\pi_\tau = 1 - (\underset{(0.005)}{0.256} - \underset{(0.005)}{0.024}D90 - \underset{(0.006)}{0.045}D9198)\ m_\tau + \eta_\tau \tag{30}$$

[17] See Statistisches Bundesamt (German Federal Statistical Office, 2000): *Statistical Yearbook for the Federal Republic of Germany*, 547.

with

$$\eta_\tau = \underset{(0.083)}{0.791}\eta_{\tau-1} + \xi_\tau \tag{31}$$

m_τ is calculated as the ratio of people below 30 and between 30 and 60 (yearly average). π_τ is the ratio of labour-force participants between 30 and 60 and total population between 30 and 60, respectively. $D90$ (equals 1 in 1990) and $D9198$ (equals 1 from 1991 on) are dummy variables, which are justified by the German reunion at the end of 1989 and the switch from Western German to German data after 1990. For the error term η_τ, it was allowed for first-order autocorrelation. ξ_τ is white noise. Consequently, for ψ $0.256 - 0.045 \approx 0.21$ results.

p, finally, is growth rate of labour productivity (GDP per worker in prices of 1995) from 1992 to 1999.[18] It is needed to simulate a growing economy. In a steady state, all macroeconomic variables grow with the rate of p, cf. Gaggermeier and Lucke (2002).

The following table contains some selected simulation results. The status quo (”no individualisation”, NI) is compared with the proposed reform of a ”partial individualisation” (PI) and a full individualisation (FI) without Pareto improvement according to von Auer and Büttner (cf. section 3). Clearly, $m_\tau^{(2)} < m_\tau^*$ applies, so that the share of people in the new system is $\lambda_1 = 0.67$. (Subscripts always denote the types.)

[18] $p = 0.64$ corresponds to a yearly rate of 1.7%; Statistisches Bundesamt (2000).

	NI	PI	FI
$m^{(1)}$	1.05	1.21	1.43
$m^{(2)}$	0	0	0.15
$m^{(3)}$	0	0	0
m	0.70	0.81	1.00
Pension rates of return	0.5	1.1	2.9
(yearly, in percent)	0.5	0.5	-4.6
	0.5	0.5	-100
$c^{1(1)}$, percentage change	-	-7.2	-11.5
$c^{1(2)}$	-	0	-10.3
$c^{1(3)}$	-	0	-6.4
Yearly growth rate of pop.	-1.2	-0.7	0.0
Yearly growth rate of GDP	0.5	1.0	1.7

The left column shows the status quo, in which a woman has about 1.4 children on average, so that m equals 0.7. While type-3 individuals are involuntarily childless, type-2 individuals are assumed to not want to have any children. As many values of ε are compatible with a children number of zero, $\varepsilon^{(2)}$ is chosen such that type-2 individuals "almost" wish to have children (but that $m^{(2)}$ is lower than 0.001).

The right column shows the fully individualised PAYGO system, where all individuals are provided with the optimal incentives. These induce more children (normal goods with the underlying CRRA utility function) for both types 1 and 2, implying that the population nearly remains constant ($m = 1$). But, only type 1 is better off in this system than in the non-individualised one. For type 2, in contrast, the return to the pension system is negative, for type 3, it is even minus 100 percent, since he or she contributes to the system without getting any benefits. Consumption during the working age (and equally in the old age) is lower for all individuals. This is for type 3 because of his or her missing pension benefits and for type 1 because of his or her higher costs for more children, in spite of higher pension benefits. For type 2, the decrease in consumption is lower than for type 1, even though type 2 suffers a loss in pension benefits; but, his or her children costs increase by much less than those of type 1.

In the middle column, the effects of the partial individualisation are described. Type 1 has the optimal incentives to have children just as in the fully individualised system, but there is a negative income effect caused by the amount a, decreasing the pension benefits. Thus, type 1 has fewer children than with full individualisation. His or her consumption level is lower than in the status quo because of the higher children costs (and despite the higher pension), but it is higher than with full individualisation because of the lower children costs (and despite the lower pension). For type-2 and type-3 individuals, in contrast, nothing changes compared with the non-individualised (status-quo) system.

Utility corresponds to the pension rate of return, which represents the outcome of the pension reforms for the individuals. Type 1 prefers partial individualisation to no individualisation and full individualisation to partial individualisation; types 2 and 3 are indifferent between no and partial individualisation, but are worse off with full individualisation. The growth rates of population and GDP (level, not per capita) correspond to the average children number.

5 Concluding remarks

In an overlapping-generations model with endogenous birth rates, I constructed a PAYGO-pension-system reform, which implies a Pareto improvement - provided that the Pareto criterion is accepted with endogenous population size and given that peoples' expectations are correct - even if any number of individuals has different preferences and a child is an inferior good. This improvement is based on removing a distortion in PAYGO systems in which efforts for raising children are not appropriately honoured; the children's pension contributions are an externality in so far as the pension benefits do not depend on the individual number of children.

Since a Pareto improvement requires that nobody loses, people get the option of keeping the status quo. People with sufficiently high preferences for children, however, are better off in an individualised pension system, in which they get exactly the amount of pension benefits - before subtracting a lump-sum amount necessary for subsidising part of the status-quo pensioners - which is (expectedly) financed by their own children.

In reality, of course, this calculation would have to be based on contributions *averaged* over all contributors, because children differ in their luck and their abilities, which their parents are not (fully) responsible for. In principle, the (model-exogenous) pension contributions per contributor can be computed by multiplying a fixed contribution rate with the labour income of all contributors within one generation's time and dividing this contribution sum by the (average) number of contributors. Details would remain to be defined. Especially, this reform would not be impeded by the existence of unemployed or self-employed people, who are not liable for contributions to the public PAYGO pension system. In the case of self-employment, e.g., it could be assumed that out of the group of self-employed parents, about the same number of children becomes contributors as the other way around. If there are already reproduction incentives in an existing pension system, this system can be theoretically split up in such a way that the reform is applied only to the fraction of contributions which is used to finance flat-rate benefits in the sense of this paper.

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Appendix 1: Social-security budget Proof that the social-security budget is fulfilled after the pension reform of "partial individualisation":

Inserting equation (21) into (20) yields:

$$g_{i,\tau+1} = t_{\tau+1} m_{i\tau} - \frac{1}{N_\tau^*} \sum_{j=N_\tau^*+1}^{N_\tau} \left(g_{j,\tau+1} - t_{\tau+1} m_{j\tau}\right)$$

for all individuals $i \in \{1, 2, ..., N_\tau^*\}$. Re-arranging terms, multiplying both sides with N_τ^*,

$$\left(g_{i,\tau+1} - t_{\tau+1} m_{i\tau}\right) N_\tau^* = - \sum_{j=N_\tau^*+1}^{N_\tau} \left(g_{j,\tau+1} - t_{\tau+1} m_{j\tau}\right),$$

and summing up over all new-system pensioners,

$$\sum_{i=1}^{N_\tau^*} \left(g_{i,\tau+1} - t_{\tau+1} m_{i\tau}\right) N_\tau^* = - \sum_{i=1}^{N_\tau^*} \sum_{j=N_\tau^*+1}^{N_\tau} \left(g_{j,\tau+1} - t_{\tau+1} m_{j\tau}\right),$$

gives

$$N_\tau^* \sum_{i=1}^{N_\tau^*} \left(g_{i,\tau+1} - t_{\tau+1} m_{i\tau}\right) = - \sum_{j=N_\tau^*+1}^{N_\tau} \left(g_{j,\tau+1} - t_{\tau+1} m_{j\tau}\right) \sum_{i=1}^{N_\tau^*} 1,$$

$$\begin{aligned} \sum_{i=1}^{N_\tau^*} \left(g_{i,\tau+1} - t_{\tau+1} m_{i\tau}\right) + \sum_{j=N_\tau^*+1}^{N_\tau} \left(g_{j,\tau+1} - t_{\tau+1} m_{j\tau}\right) &= \sum_{j=1}^{N_\tau} \left(g_{j,\tau+1} - t_{\tau+1} m_{j\tau}\right) \\ &= 0 \end{aligned}$$

or

$$\sum_{j=1}^{N_\tau} g_{j,\tau+1} = t_{\tau+1} N_{\tau+1}$$

The sum of pension benefits equals total contributions in each period, which was to be shown.

Appendix 2: Does anybody choose the new system? Suppose that at most one individual i chooses the individualised system. Then, according to equation (24) and using equation (21) with $N_\tau^* = 1$ (but without ordering the individuals), he or she does so if

$$g_{i,\tau+1} = t_{\tau+1} m_{i\tau} - a_{\tau+1} = t_{\tau+1} m_{i\tau} - \sum_{\substack{j=1 \\ j \neq i}}^{N_\tau} \left(g_{\tau+1}^{NI} - t_{\tau+1} m_{j\tau}^{NI}\right) \geq g_{\tau+1}^{NI}$$

or

$$t_{\tau+1} m_{i\tau} + t_{\tau+1} \sum_{\substack{j=1 \\ j\neq i}}^{N_\tau} m_{j\tau}^{NI} \geq \sum_{j=1}^{N_\tau} g_{\tau+1}^{NI} = N_\tau g_{\tau+1}^{NI} = N_\tau t_{\tau+1} m_\tau^{NI}$$

or

$$m_{i\tau} + \sum_{\substack{j=1 \\ j\neq i}}^{N_\tau} m_{j\tau}^{NI} \geq N_\tau m_\tau^{NI} = \sum_{j=1}^{N_\tau} m_{j\tau}^{NI} = m_{i\tau}^{NI} + \sum_{\substack{j=1 \\ j\neq i}}^{N_\tau} m_{j\tau}^{NI}$$

or

$$m_{i\tau} \geq m_{i\tau}^{NI}.$$

To get at least the same pension in the individualised system as in the status quo, i must have at least as many children. This is the case, since in the individualised system, the individual marginal utility of children is higher than in the status-quo system, cf. equations (13) and (17).

Appendix 3: Utility as a function of total consumption and children For illustration in a two-goods diagram, the CRRA utility function

$$u_{i\tau} = \log c_{i\tau}^1 + \delta \log c_{i\tau}^2 + \varepsilon_i \log (1 + m_{i\tau})$$

can be transformed into a function of the two arguments $c_{i\tau} \equiv c_{i\tau}^1 + \frac{c_{i\tau}^2}{1+r_{\tau+1}}$ and $\widetilde{m}_{it} \equiv 1 + m_{i\tau}$ if the optimal consumption ratio $\frac{c_{i\tau}^2}{c_{i\tau}^1} = \delta (1 + r_{\tau+1})$ is presumed:

$$c_{i\tau}^2 = \delta (1 + r_{\tau+1}) c_{i\tau}^1 \implies c_{i\tau} = (1+\delta) c_{i\tau}^1,$$

such that

$$u_{i\tau} = \log \frac{1}{1+\delta} c_{i\tau} + \delta \log \frac{\delta (1 + r_{\tau+1})}{1+\delta} c_{i\tau} + \varepsilon_i \log \widetilde{m}_{it} = konst + (1+\delta) \log c_{i\tau} + \varepsilon_i \log \widetilde{m}_{it}$$

or

$$\widetilde{u}_{i\tau} = \log c_{i\tau} + \widetilde{\varepsilon}_i \log \widetilde{m}_{it} \tag{32}$$

with $\widetilde{\varepsilon}_i \equiv \frac{\varepsilon_i}{1+\delta}$.

In the following, I show that this utility function implies that both consumption and children are normal goods. Implicitly differentiating (32) gives

$$\frac{d\widetilde{m}_{it}}{dc_{i\tau}} = -\frac{\widetilde{m}_{it}}{\widetilde{\varepsilon}_i c_{i\tau}}$$

for the slope of the indifference curves. This remains constant if children and consumption are increased (or decreased) proportionally, that is on any ray through the origin of a $c_{i\tau}$-$\widetilde{m}_{it}$ diagram. Thus, after an income increase, or a shift of the budget constraint to the right, the ratio of $\widetilde{m}_{it}$ and $c_{i\tau}$ must not change; if $c_{i\tau}$ is increased, $\widetilde{m}_{it}$ and consequently $m_{i\tau} = \widetilde{m}_{it} - 1$ must be also increased;[19] decreasing both quantities, in contrast, would contradict the standard assumption of non-satisfaction.

[19]It can be easily shown that the increase in m_{it} must be even higher than the increase in $c_{i\tau}$.

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